Modeling disturbances influencing an Earth-orbiting satellite

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Abstract: Space is one of the most interesting environments where man made objects can function. Satellites, probes, and manned spacecrafts have been exploring it for over six decades now. Particularly interesting characteristics of interplanetary environment from the stand point of satellite stabilization are origins and nature of existing disturbance forces and torques. Some of those effects are even used to complement or replace chemical propulsion to control and stabilize spacecrafts attitude. This article contains a general description, categorization and basic models of forces that can affect control of spacecrafts in Earth orbit.

Keywords: attitude control, orbit control, disturbance torques

One could expect that with lack of air and therefore no external friction there are no disturbances acting upon a spacecraft in the Earth orbit. Even on close examination there are no sources that can be intuitively classified as “large”. However a typical satellite is designed to point its instruments in a very precise manner for relatively long periods of time. For example a famous Hubble Space Telescope was initially designed to maintain pointing accuracy of at least $7/1000^{\text{th}}$ of an arcsecond [4]. This kind of requirement makes it necessary to analyze even smallest disturbance sources, to make sure that control authority is maintained even in worst-case scenario. With telescope’s demanded mission time of 20 years it is also essential to have a precise predictions of attitude control systems fuel consumption. This of course is heavily dependent on disturbance effects accumulating over time.

Disturbances acting upon a satellite can be divided into external and internal. External effects are those characterizing the Space environment. They would act event if a spacecraft itself was a rigid body. Internal disturbances are closely tied witch spacecraft structure, in particular: internal moving parts and mass or radiation being emitted.

1. External disturbance sources

1.1. Gravitational torque

Gravitational force between the Earth and a satellite in an orbit is obviously the dominant interaction. It causes spacecraft to obey Keppler’s laws of planetary motion. Irregularities of mass distribution in Earth’s crust, impact of gravitational attraction by the Sun and the Moon and tidal movements of oceans that cause deviations from ideal elliptical orbit. Those irregularities usually do not need to be corrected by spacecraft’s propulsion but rather their effects are taken into account while interpreting scientific measurements or performing other satellite’s tasks

There is, however, another meaningful effect caused by non-uniformity of gravitational field around the Earth’s center of mass. In uniform field center of mass of a spacecraft would become a center of gravity as well. In Earth orbit, when a spacecraft’s mass distribution is not spherically symmetrical this results in a non-zero torque about the center of mass. Because of this torque some (up to 24 according to [1]) stable equilibria may emerge. This phenomenon was used by Lagrange in 1780 to explain why the Moon always faces the Earth with the same hemisphere (fig. 1).

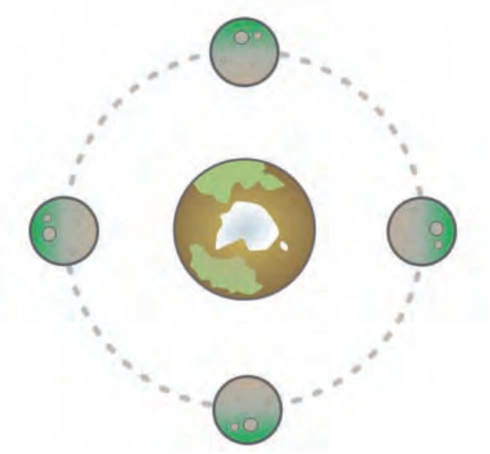

Fig. 1. Gravity torque stabilization of Moon attitude in the Earth-Moon system
Rys. 1. Grawitacyjna stabilizacja orientacji Księżyca względem Ziemi

Some satellites use this principle for passive attitude stabilization. By deploying gravity gradient booms and somehow dumping initial post-launch angular momentum those spacecrafts can maintain Earth-oriented position through their orbit [1]. Usually additional spin stabilization is employed by forcing space vehicle to slowly spin around the axis passing through both it’s center of mass and center of gravity. This approach has a limited precision, but does not require any fuel to maintain stable attitude. This configuration was for instance used by series of Transit satellites that where part of positioning system that preceded GPS.

In simplistic model shown in fig. 2 we can see a satellite immersed with the Earth’s inverse square gravitational field. Satellite consists of two identical balls of fixed mass $\boldsymbol{m}$. They are connected with rod of negligible mass and with length equal $\mathbf{l}$. Satellite is oriented in such a way that second ball lies in a distance $\mathbf{R_2}$ from the Earth's mass center which is greater than the corresponding distance $\mathbf{R_1}$ of a first ball. As size of the balls is very small in comparison to those distances they can be treated as point masses. Therefore, one can describe forces acting on those two parts as:

$$F_1 = -G\frac{mM}{R_1^2}, \quad (1)$$

$$F_2 = -G\frac{mM}{R_2^2}, \quad (2)$$

where G is gravitational constant and $\mathbf{M}$ is the mass of the Earth. Assuming that angle is very small due to $\mathbf{R} >> \mathbf{l}$ resulting torque $\mathbf{T_G}$ along satellite's center of mass will be described by following relationship:

$$T_G = (F_2 - F_1)\frac{l}{2}\sin\alpha. \quad (3)$$

From (1) and (2) and the fact that $\mathbf{R_1} > \mathbf{R_2}$ it becomes clear that $\mathbf{F_1} < \mathbf{F_2}$. Therefore as long as $\sin(\alpha)$ does not equal zero (which happens in equilibria) there will be a non zero torque along the center of the rod.

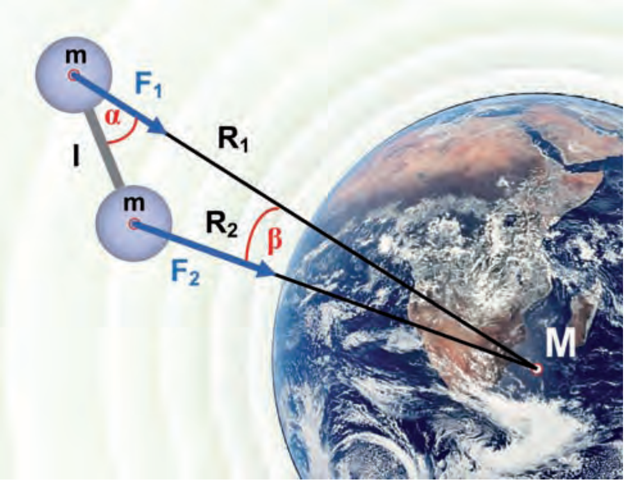

Fig. 2. Gravity gradient torque example

Rys. 2. Moment siły wynikający z gradientu pola grawitacyjnego

1.2. Aerodynamic torque

Intuitive assumption of ideal vacuum being one of the characteristics of space environment is not precise. The boundary of space is defined in several ways. Kármán line 100 km above sea level is often used because roughly at this altitude a vehicle would have to travel faster than orbital velocity in order to derive sufficient aerodynamic lift from the atmosphere to support itself [5]. In USA every person who flew higher than 80 km is considered an astronaut. At the same time altitude of 122 km was recognized as the space shuttle reentry boundary because at this point atmospheric drag becomes a dominant force. Picture in fig. 3, taken by astronauts from the International Space Station, shows that there is no easily distinguishable boundary between outer space and the Earth's atmosphere but rather a smooth transition.

Fig. 3. Picture of upper atmosphere taken by International Space Station crew over the South China Sea (source: NASA)

Rys. 3. Zdjęcie górnych warstw atmosfery wykonane przez załogę Międzynarodowej Stacji Kosmicznej nad Morzem Południowochińskim (źródło: NASA)

At the altitude of 700 km the average air density is on the order of 10^{-16} g/cm^3. There are two main reasons why such a small amount of air can have impact on satellite attitude and trajectory. First of all air drag effects accumulate for the whole lifetime of a spacecraft. Secondly, it is crucial to understand that objects orbiting the Earth need to be traveling with very large velocities. High relative speed causes significant momentum transfer between spacecraft and colliding air particles.

Value of orbital velocity of a satellite in circular orbit is represented by following relationship:

$$V_o \approx \sqrt{\frac{M^2G}{(m+M)R}}, \quad (4)$$

where $\mathbf{M}$ is a mass of the Earth, $\mathbf{m}$ is a mass of satellite, $\mathbf{G}$ is a gravitational constant and $\mathbf{R}$ is a distance between centers of mass. For a spacecraft of negligible mass orbiting the Earth at altitude of 700 km (plus the Earth's radius) this velocity will be approximately 7,5 km/s.

Air drag causes low altitude satellites orbits to decay. Notice that from equation (4) square of orbital velocity is reverse proportional to orbit's altitude. This means that on the contrary to common misconception satellites are not slowed down by atmospheric drag but rather they gain velocity with decreasing orbit altitude. This is true of course only until atmospheric density becomes high enough. At certain altitude orbital mechanics equations cease to be a good estimation and satellite breaks up in atmosphere or deorbits.

Classical fluid dynamics drag equation gives an estimation of an atmospheric drag force acting upon a satellite. This force will act in the opposite direction to a speed vector and of a value approximated by following relationship:

$$F_{AD} = \frac{\rho C_D A V_o^2}{2}, \quad (5)$$

where $\boldsymbol{\rho}$ is an atmospheric density, $\mathbf{C_D}$ is a drag coefficient of the satellite, $\mathbf{A}$ is the reference area and $\mathbf{V_o}$ is the velocity of a satellite.

Apart from decaying orbit of the spacecraft it is also observed that atmospheric drag can introduce torques impacting spacecraft's attitude. Of course shape of the spacecraft is the deciding factor. If a spacecrafts center of pressure lies far from center of mass drag force will introduce torque accordingly to following equation:

$$\vec{T}_A = \vec{l}_{CM,CP} \quad \vec{F}_{AD}, \tag{6}$$

where $\mathbf{I}_{\mathbf{CM,CP}}$ is a distance vector between center of mass and drag force application point. Diagram in fig. 4 describes this situation.

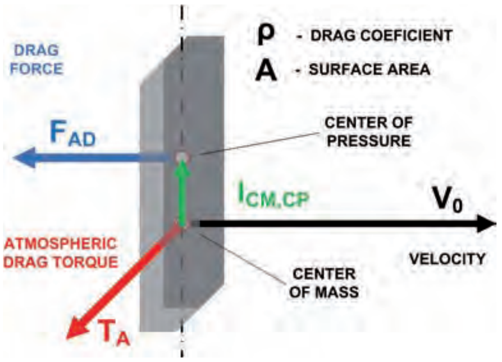

Fig. 4. Simplistic example of atmospheric drag torque

Rys. 4. Prosty przykład momentu siły wynikającego z tarcia o atmosferę

Unfortunately this model is in many cases too simple to provide precise enough estimation. For greater precision at orbital altitudes atmosphere should be modeled as individual particles colliding witch spacecraft because mean free path (average distance traveled by gas particle before hitting other particle) is on the order of kilometers. This approach is known as *free-molecular flow model* and in this case is more precise than conventional *continuum flow* model.

It was found that probability of collision with atmospheric particle is larger for the parts of the satellite that are closer to the Earth, as shown for Hubble Space Telescope in fig. 5. This creates *atmospheric drag torque*. When satellite orbits the Earth this torque will constantly act along the same axis, perpendicular to orbital plane. If not compensated for this effect may cause spacecraft to spin along this axis with increasing angular velocity.

This is one of the reasons why low orbit spacecrafts are very often symmetrical. Designers seek to avoid creating aerodynamically stable orientations and thus minimize the atmospheric drag torques.

At typical *Low Earth Orbit* attitude of 700 km the difference in atmospheric density between solar minimum and solar maximum can be as large as two orders of magnitude [2] ranging from 10^{-17} g/cm^3 to 10^{-15} g/cm^3. Day-Night cycle and local weather conditions also have a major impact on atmosphere. Many atmosphere density profile models have been constructed [3]. MSISE-90 is the one recommended by *ECSS* (European Cooperation for Space Standardization) [10]. Unfortunately, the biggest difficulty lies in predicting solar activity. This uncertainty makes it very hard to precisely predict impact of the atmospheric drag on a satellite.

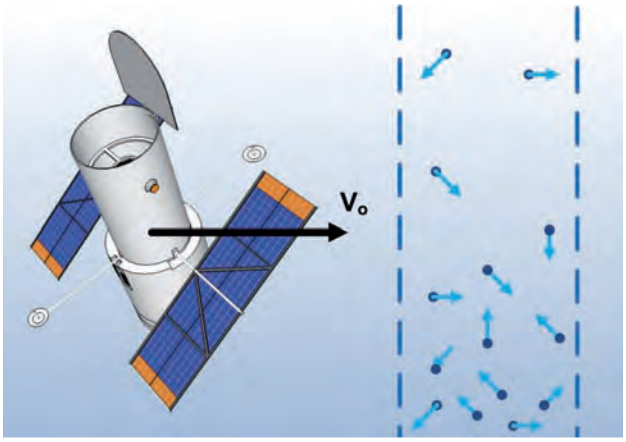

Fig. 5. Gradient in atmospheric particles distribution results in unbalanced drag on quickly moving satellites

Rys. 5. Gradient w dystrybucji cząstek atmosfery powoduje niezrównoważone tarcie o powierzchnie szybko poruszających się satelitów

1.3. Environmental radiation torques

Environmental radiation in near Earth space comes from three main sources. First of all it is direct radiation from the Sun of average 1371 W/m^2. This amount varies seasonally by about ±3 % due to Earth's orbit eccentricity. Secondly, about 30 % of sunlight gets reflected from the Earth and creates albedo. Rest of the solar radiation hitting Earth is absorbed and emitted back with some delay in form of infrared radiation. This process is schematically shown on diagram in fig. 6.

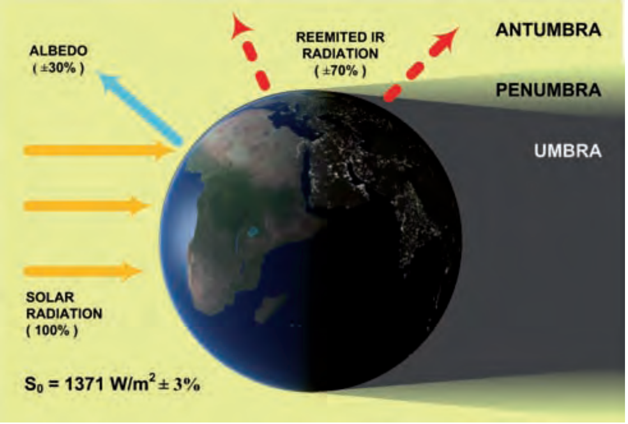

Fig. 6. Earth's energy balance and shadowing

Rys. 6. Bilans energetyczny Ziemi i zacienianie

It is important to note that satellites are not always subjected to direct sunlight due to shadow casted by the planet. Region where they are fully exposed to solar radiation is called *antumbra* and they are in complete shadow while in *umbra*. In between lies *penumbra* where only part of the Sun is visible from behind the Earth.

Even since Nichols invented his radiometer in 1901 it is known that radiation causes a pressure on surfaces. This pressure can be easily explained in terms of corpuscular nature of radiation. Each photon carries some momentum that is partially transferred on impact. Efficiency of this transfer depends on surface's absorption, reflection, and transmission characteristics. Depending on those values proper statistical analysis of absorbed, diffusely reflected and secularly reflected photons should be performed. If high precision is not needed rough approximation of radiation torque can be calculated in a similar fashion as in case of atmospheric drag. Assuming non-transparent surface with area A, unit normal vector n, reflectance **r**, normal radiation energy flux onto unit of area per unit of time $\mathbf{S_0}$, speed of light **c**, normalized solar radiation incidence vector **S**, and distance between centers of mass and radiation pressure $\mathbf{l_{CM,CR}}$ approximation of torque can be given by relationship:

$$\vec{T}_R = \vec{l}_{CM,CR} \frac{AS_0}{c}(1+r)(\vec{n}^T \circ \vec{S})\vec{S}. \tag{7}$$

This method was used for estimating worst case scenario solar pressure affecting Compass 1 satellite [9].

One of the first observed examples of solar pressure affecting spacecraft attitude was Alouette 1 satellite. This spin-stabilized spacecraft was equipped with exceptionally long (45,7 m) dipole antennas. It turned out that antennas where flexible enough to be slightly bent away from the Sun by solar pressure (fig. 7). As satellite was spinning around it's center of mass deforming antennas stayed a little longer under solar pressure's influence while swinging toward the sun and shorter while moving away. Over the course of two years spin ratio of Alouette decreased from 1,4 to 0,3 rpm significantly undermining attitude stabilization efficiency [7].

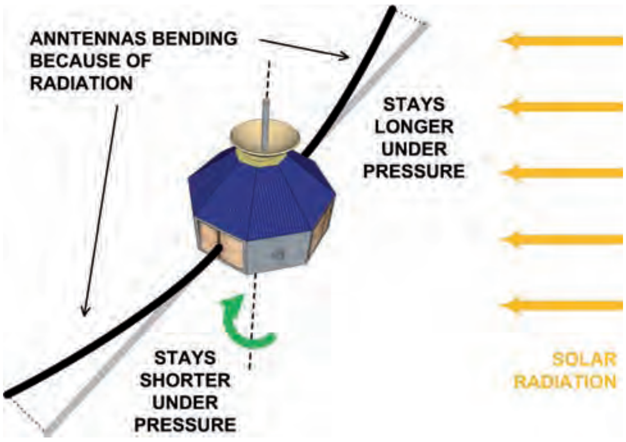

Fig. 7. Alouette 1 satellite with solar pressure affecting flexible antennas

Rys. 7. Ciśnienie słoneczne oddziaływujące na elastyczne anteny satelity Alouette 1

Often engineers can take advantage of this otherwise harmful effect. Solar radiation torque was used to stabilize Mariner 4 probe attitude on its way to Mars. Four adjustable solar vanes where added at the top of spacecrafts solar panels as shown in fig. 8. Those black plates where positioned in such a way that point of application of solar radiation force was moved behind the spacecraft's center of mass. This made Mariner 4 statically stable in terms of attitude [7].

Other interesting phenomenon has been observed to affect trajectories of GPS satellites [6]. Spacecrafts cannot dissipate heat produced by on-board electronics to environment by the means of thermal conduction, and therefore they usually keep thermal balance by employing radiators. Those devices emit heat energy into space in form of infrared radiation. As photons have non-zero momentum this radiation imposes additional force on satellite. This effect is sometimes referred to as *thermal trust.*

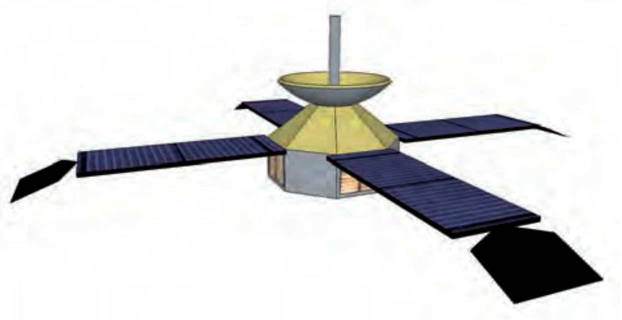

Fig. 8. Mariner 4 probe with visible solar vanes used for attitude stabilization

Rys. 8. Sonda Mariner 4 z widocznymi łopatkami słonecznymi służącymi do stabilizacji orientacji

1.4. Magnetic torques

There are several mechanisms that can affect attitude of a spacecraft immersed in magnetic field. Unarguably most important interaction occurs when spacecraft has non-zero net magnetic moment **m**. Given the external magnetic field has magnetic flux density **B**, the torque affecting satellite can be calculated from simple formula:

$$\vec{T}_m = \vec{m} \quad \vec{B}. \tag{8}$$

The same phenomenon occurs in compass and forces it's needle to always point to the magnetic north.

Earth is surrounded with magnetic field that can be approximated by the field of magnetic dipole. Magnetic potential satisfies Laplace equation and can be expanded into series of spherical harmonics. International Geomagnetic Reference Field model does exactly that and provides periodically updated spherical coefficients that allow to calculate value of magnetic flux density B for a specific point in time and space in the vicinity of the Earth. However, the Earth's magnetic field is very vulnerable to the randomly occurring solar magnetic storms which undermines practical applicability of this and similar models.

Fortunately, in case of magnetic torque engineers have full authority over satellites magnetic moment m. As seen from equation 7 this allows to eliminate the problem by adding permanent magnet to trim the net moment to zero. Some spacecraft are even designed to take advantage of this torque. For example Compass 1 [9] satellite utilized magnetic actuators to alter its net magnetic moment in real-time allowing for three-axis attitude stabilization.

Other worth mentioning influences on a spacecrafts attitude resulting from presence of external magnetic field are torque resulting from Eddy currents and magnetic-hysteresis torque. Both of them can act as momentum damping factors in spin-stabilized satellites. The former is result of electrical currents being induced by the movement of conducting parts of satellite in magnetic field. As those conductors have less than infinite conductivity electrons will meet with resistance that will create equal and opposite reaction force in material itself. The latter influence occurs when high-hysteresis materials inside spacecraft periodically magnetize and demagnetize influenced by spinning magnetic flux vector. This process dissipates kinetic energy of satellite's spin. Hysteretic materials are sometimes intentionally used in form of custom shaped hysteresis rods to provide passive momentum damping [11]. It serves the purpose of detumbling spacecraft at the beginning of mission just after it has been ejected from the rocket's payload compartment.

1.5. Other environmental forces

There are also other environmental phenomena that may affect spacecrafts orbits and attitude such as micrometeoroidal impacts or Lorentz force. As many others also this second effect might someday be used to control spacecrafts trajectories. So called *Lorentz drive* is currently in conceptual phase. It is possible to achieve a meaningful change in spacecrafts course by intentionally charging it before passing through a strong magnetic field and utilizing Lorentz force as a mean of propulsion.

2. Internal disturbance sources

2.1. Internal momentum

According to the principle of conservation of an angular momentum the angular momentum around the center of satellite's mass is constant. This is, of course, true when no propulsion is used and there are no external torques present. However, it is worth noting that scientific instruments and communication antennas that require precise attitude control are usually mounted on the external structure of a satellite. Any non-uniformly moving, especially spinning, parts of satellite can cause their misalignment despite the fact that net momentum of the whole spacecraft does not change.

Because space inside rocket's fairing is limited satellites very often have some deployable elements. Those may include solar panels, communication antennas and gravity gradient booms. Extending those devices causes force opposite to the one used for deployment and applied to the structure of the satellite. Moreover, satellite effectively changes it's shape which affects position of center of mass and changes spacecraft's moment of inertia.

Every spinning element of a spacecraft contributes to the net spin ratio. When spin rate of an element changes the rest of the spacecraft also changes its angular velocity to preserve net momentum. Many spacecrafts use this into their advantage by carrying *reaction wheels*. Those electrical motors with maximized moment of inertia can be precisely controlled to adjust satellites attitude.

Part of spacecraft that obviously needs to move around is fuel. It rapidly travels inside fuel system when propulsion is needed. It also moves inside fuel tank. Because of the state of weightlessness it does not simply settle inside the tank, but rather sticks to it's inner walls and moves in a complicated fashion stimulated by every move of the spacecraft. Navier-Stokes equation are nearly impossible to solve for real life boundary conditions. For this reason fuel sloshing is currently often modeled with finite-element methodology [12].

Last but not least some crewed satellites like International Space Station are subjected to anther disturbance: crew movement. Being unpredictable in nature this disturbance is also limited in value. Weight of a spacecraft is usually at least two orders of magnitude bigger than weight of a crew member. This ratio is even greater for ISS which weights over 400 t. Movements of an astronaut inside a spacecraft are usually very careful which further decreases influence of this problem. However, example of International Space Station shows that it sometimes does need to be taken into account. Space station caries number of experiments demanding very high quality microgravity conditions. Those conditions can easily be spoiled by crew members simply performing their daily activities. Actively stabilized experiment racks (*Active Rack Stabilization System*) had to be employed to minimize influence of those perturbations on experiments

Fig. 9. Vladimir Dezhurov performing test of ARIS ICE-3 experiment stabilization rack by hitting it with a hammer (source: NASA)

Rys. 9. Vladimir Dezhurov przeprowadzający test stabilizowanej szafy instalacyjnej ARIS ICE-3 polegający na uderzaniu w nią młotkiem (źródło: NASA)

2.2. Mass expulsion

Perhaps the most obvious source of trajectory and attitude change is mass expulsion. Cold gas thrusters are commonly used to propel and stabilize spacecrafts. There are however other cases where mass expulsion might occur.

Unintentional mass expulsion may happen when spacecraft experiences a leak or venting. Hole in a spacecraft fuel or life support system can cause gas or liquid to rapidly leave the spacecraft providing unwanted trust. This happened during the famous Apollo 13 moon mission, when explosion caused a leak in oxygen tank of Service Module. Entire oxygen stored there vented into outer space over the course of the next 130 minutes. Fortunately, there was enough supplies left in Spacecraft's lunar module to bring the crew safely back to the Earth.

2.3. Radiation thrust

Some satellites carry high power radio transmitters. Energy radiated away by typical communication satellite is on the order of several kilowatts. This radiation creates reaction

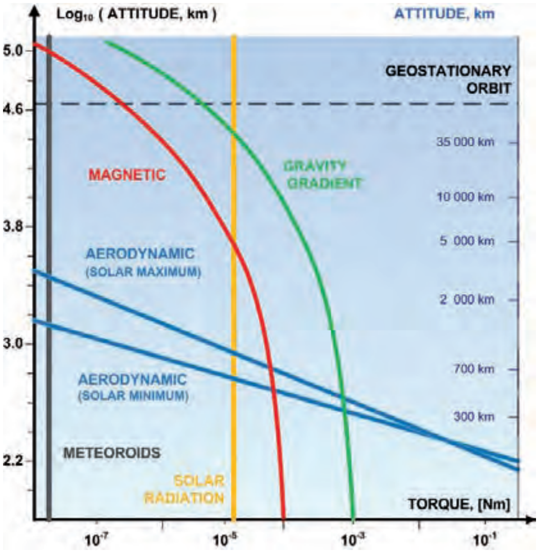

Fig. 10. Diagram presenting influence of several common torques on a typical spacecraft, vs. Altitude

Rys. 10. Diagram przedstawiający wpływ kilku typowych źródeł momentu siły na przykładowy pojazd kosmiczny w zależności od wysokości jego orbity

force of approximately 0,33·10^{-5} N per kilowatt. Such satellites typically have long lifespan between 10 and 20 years, and therefore effects of this *radiation trust* need to be taken into account.

With recent developments in field of high power lasers a Photonic Laser Thruster has been proposed. Basic idea assumes repeatedly bouncing high energy laser beam between two spacecrafts. It is theoretically possible to achieve thrust levels comparable to current chemical propellers, but with significantly higher specific impulse Feasibility of this solution was demonstrated by Dr. Young K. Bae in December 2006.

3. Conclusion

There are many torques affecting satellite in Earth orbit. Their character is also very diverse. As shown by the mentioned examples sometimes those torques may be used to control and stabilize a spacecraft. More often engineers designing satellites treat them as perturbations and by clever design minimize their impact on space vehicle. Fig. 10 shows diagram with comparison of common torques values on a typical spacecraft vs. attitude. Analysis is based on [8] and it should only be treated as an example. Each torque is heavily dependent on a satellite's shape design and designated orbit. However, it is worth noticing that gravity gradient torque varies as $\mathbf{R}^{-3}$, where $\mathbf{R}$ is the distance to the Earth's gravitational center. This is also to some extent true for magnetic torque, although it is additionally heavily dependent on orbit inclination and local anomalies. Aerodynamic torque is dependent on atmosphere density and decreases approximately exponentially with altitude. Because of their nature torques from meteoroidal impacts and solar pressure are almost constant with altitude and do not depend on the distance from Earth.

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Modelowanie zakłóceń ruchu satelity na orbicie okołoziemskiej

Streszczenie: Kosmos jest jednym z najbardziej interesujących środowisk, w których funkcjonować mogą wykonane ręką ludzką urządzenia. Satelity, sondy oraz załogowe statki badają przestrzeń kosmiczną już od sześciu dekad. Jednym z najciekawszych, z punktu widzenia teorii sterowania, zagadnień są źródła i natura występujących tam zakłócających sił i momentów siły. Niektóre z tych efektów są używane, by uzupełnić lub zastąpić napędy chemiczne w kontroli i stabilizacji orientacji pojazdów kosmicznych. Niniejszy artykuł zawiera skrótowy opis i klasyfikację najważniejszych oddziaływań, jakie wpływają na statki kosmiczne na orbicie Ziemi.

Słowa kluczowe: sterowanie orientacją, sterowanie orbitą, momenty zakłócające

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