WYBRANE PROBLEMY INŻYNIERSKIE
NUMER 2
INSTYTUT AUTOMATYZACJI PROCESÓW TECHNOLOGICZNYCH
I ZINTEGROWANYCH SYSTEMÓW WYTWARZANIA

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A PROBLEM OF PREDICTIVE SCHEDULING OF JOBS IN A PRODUCTION SYSTEM

Abstract: In the paper a classical model of failures is considered in that successive failure-free times are supposed to have Weibull distributions and are followed by exponentially distributed times of repairs. It is assumed that parameters of these distributions, in general, change with time. Basing on information about the number of failures, failure-free times, repair times, in a number of periods of the same duration in the past parameters of the model are estimated. Next, predictions of the most important reliability characteristics are found using classical regression technique.

1. Introduction

Reliability parameters in a production process depend on occurrence of disturbances that cause changes in a basic schedule. The basic schedule becomes unrealizable after the disturbances have appeared. In the basic schedule, any event which is possible to forecast should be planed. The more changes in the basic schedule are the lower robustness of the schedule is. Cost of reorganization of the production schedule increases and time is wasted.

A method for elaborating the robust basic schedule is searched. Analysis of historical data of a machine failures frequency, a number of the machine failures and data acquisition for forecasting a future time of the machine failure are essential. In the literature the machine failure and repair are described by: mean time between the machine failures and mean time of repair [1].

The goal of the paper is to answer for the question what distribution describes: a failure time of a machine, a repairing time of the machine, basing on information about failure free times and repairing times. For elaborating the robust schedule, the predicted failure time of the machine is needed.

2. A production scheduling model of failures

For historical data of machine M_i failure frequency and machine M_i repairing time histograms are built. Observing the histograms successive failure-free times are supposed to have Weibull distributions and are followed by exponentially distributed times of repairs. It is assumed that parameters of these distributions, in general, change with time. Basing on information about the number of failures and failure-free times in a number of periods of the same duration in the past predictions of the reliability characteristics are searched.

Let us consider a classical model of failures in that successive periods of reliable work of a production system are followed by times of repair. Such the system, firstly, is observed on m successive time periods

$$[0,T),[T,2T),...,[(m-1)T,mT) \tag{1}$$

of the same durations, for which the information about numbers of detected failures or failure-free times is known. The prediction of system behavior is being built for the next period $[mT,(m+1)T)$. We assume that failure-free times $X_{i,1},...,X_{i,N_i}$ in the ith period $[(i-1)T,iT)$, $i=1,...,m+1$ have Weibull distribution. Here N_i denotes a random number of failures detected in $[(i-1)T,iT)$. At the end of reliable work period $X_{i,k}$, as the failure occurs, a repair time $Y_{i,k}$ begins immediately and so on. Repair times $Y_{i,1},...,Y_{i,N_i}$ for $i=1,...,m+1$ are supposed to be exponentially distributed.

The evolution of the system can be observed on successive cycles $Z_{i,k}=X_{i,k}+Y_{i,k}$, $i=1,...,m+1$, $k=1,...,N_i$ which are independent random variables with DFs (=distribution functions) of the form

$$H_i(t)=\int_0^t h_i(y)dy=\alpha_i\lambda_i p_i\int_0^t e^{-\alpha_i y}\int_0^y x^{p_i-1}e^{\alpha_i x-\lambda_i x^{p_i}}dxdy,\ \ t>0\,. \tag{2}$$

3. Estimation of unknown parameters

In [2,4] the application of Maximum Likelihood Principle and Empirical Moments Approach to estimate unknown parameters λ_1, p_1 of Weibull distribution is presented. In the paper the parameter α_i for Exponential distribution is estimated.

This approach is based on the assumption that numbers, durations of repairing periods have been measured and are known. Suppose that in each of intervals $[0,T),[T,2T),...,[(m-1)T,mT)$ we have given sample values of random variables $Y_{i,k}$, $k=1,...,N_i$ for any $i=1,...,m$, where n_i is the observed value of N_i, so we have the following observations:

$$(y_{1,1},\ y_{1,2},\ ...,y_{1,n_1}),...,(y_{m,1},\ y_{m,2},\ ...,y_{m,n_m}). \tag{3}$$

Consider firstly the interval $[0,T)$. Since $y_{1,1}$, $y_{1,2}$, ..., y_{1,n_1} are i.i.d. (independent and identically distributed) random variables one can apply the Maximum Likelihood Principle to estimate unknown parameter α_i for Exponential distribution. Define the likelihood functions as follows:

$$L\left(\alpha_1 \middle| y_{1,1},\ y_{1,2},\ ...,y_{1,n_1}\right)=\prod_{k=1}^{n_1}\alpha_1 e^{-\alpha_1 y_{1,k}}=\alpha_1^{n_1} e^{-\alpha_1\sum_{k=1}^{n_1} y_{1,k}} . \tag{4}$$

Logarithming and differentiating on variable α_1 for (4) one obtain the equation:

$$\frac{\partial \ln L}{\partial \alpha_1}=\frac{n_1}{\alpha_1}-\sum_{k=1}^{n_1} y_{1,k}=0 \tag{5}$$

$$nL=n_1\ln\alpha_1-\alpha_1\sum_{k=1}^{n_1} y_{1,k} \tag{6}$$

and differentiating both sides of (6) by $\sum_{k=1}^{n_1} y_{1,k}$ and multiplying by $\left(\frac{1}{n_1}\right)$ one can find values of $\tilde{\alpha}_1$

$$\tilde{\alpha}_1=\frac{n_1}{\sum_{k=1}^{n_1} x_{1,k}} . \tag{7}$$

The same equation (7) one can reach using Empirical moments approach. After finding estimators $\hat{\alpha}_1,...,\hat{\alpha}_m$ one can extrapolate values $\hat{\alpha}_{m+1}$ for the period $[mT,(m+1)T)$ for which one have no observation, using the regression method.

4. Numerical example

Let us consider the production shop of $W=4$ machines and $V=3$ products. The production routes are defined in MP (8), the operation times are defined in MOT (9). Butch sizes of the products are unlimited. It is assumed that there are $m=5$ successive time periods of the same durations, for which the information about numbers of detected failures, failure-free times $X_{i,3,1},...,X_{i,3,N_{i,3}}$ and repair times $y_{i,3,1},...,y_{i,3,N_{i,3}}$ of machine $w=3$ in the ith period $[(i-1)T,iT)$, $i=1,...,5$ are given. Basing on information about numbers of detected failures and failure-free times $X_{i,3,1},...,X_{i,3,N_{i,3}}$ of machine $w=3$ in the ith period $[(i-1)T,iT)$, $i=1,...,5$ presented in [2,4], parameters for Weibull distribution $\hat{p}_6$ and $\hat{\lambda}_6$ for 6th period have been estimated. $\hat{p}_{6,3}=2{,}919$ and $\hat{\lambda}_{6,3}=0{,}002$ using the empirical moments approach.

$$MP=\begin{bmatrix}3,2,1,0\\2,0,0,1\\0,1,0,2\end{bmatrix},\ MOT=\begin{bmatrix}3,5,10,0\\3,0,0,1\\0,5,0,1\end{bmatrix}. \tag{8,9}$$

The histogram for repair times $y_{i,w,1},\dots,y_{i,w,N_{i,w}}$ for $i=1,\dots,m+1$ for wth machine is built (Fig. 1.). Basing on the histogram one can make the hypothesis that H$_i$:{the cumulative distribution function of the repair time Y$_i$ is $g_i(t)$}, where $g_i(t)$ is an exponential distribution function.

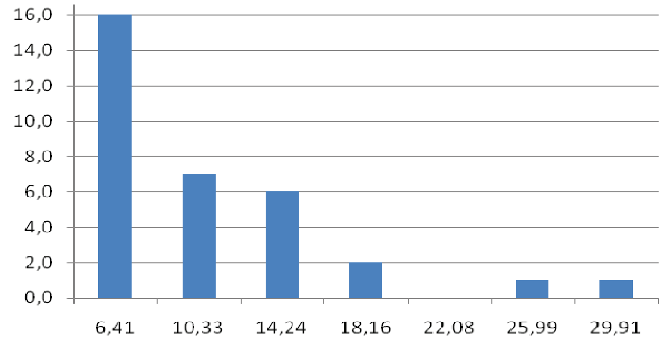

Fig. 1. The histogram of repair times $y_{i,3,1},\dots,y_{i,3,N_{i,3}}$ *of 3rd machine*

In order to verify the hypothesis, a Kołogomorow's test of goodness of fit between the valuated distribution of the sample and the theoretical distribution (1) is realised [5].

Let us consider the first sample $y_{1,3,1},\dots,y_{1,3,33}$ with values of {6, 5.5, 8, 5.6, 6, 4.5, 6.7, 8, 6, 8, 15, 11, 12, 14, 6, 28, 15, 27, 16, 16, 8, 19.5, 10, 11.5, 11, 15, 10.5, 18, 7, 9.6, 4.5, 6.7, 8}. Making the Kołogomorow's test of goodness of fit for the first sample, for the level of significance equaled to $\varepsilon=0.01$, the hypothesis H_1 is not denied. The conclusion is drawn comparing the condition: $d_{1,k}<d_{1,k}(1-\varepsilon)$, where $d_{1,k}=\max(d_{1,k}^{+},d_{1,k}^{-})$, $d_{1,k}^{+}=\max_{1\le k\le n_i}\left|\frac{k}{n_i}-F_o(y_{1,3,k})\right|$ and $d_{1,k}^{-}=\max_{1\le k\le n_i}\left|F_o(y_{1,3,k})-\frac{k-1}{n_i}\right|$, and $F_o(y_{1,3,k})$ is a value of hypothetical distribution for a given ε and $1\le k\le n_i$ [3]. The maximal absolute difference is $d_{1,1}=0.9889$. For $\varepsilon=0.01$ and $k=1$, critical value of Kołogomorow's statistics $d_{1,1}(1-0.01)$ equals to 0.995. The condition $0.9889<0.995$ is fulfilled, so the hypothesis H_1 for the level of significance equaled to $\varepsilon=0.01$ is not denied.

The Maximum Likelihood estimator $\hat{\alpha}_{i,w}$ (7) for the ith period $[(i-1)T,iT)$, $i=1,\dots,m$ and for wth machine is counted. Values $\alpha_{i,3}$ for the ith period $[(i-1)T,iT)$, $i=1,\dots,5$ and for 3rd machine are presented in Tab. 1.

After finding estimators $\hat{\alpha}_1,\dots,\hat{\alpha}_m$ one can extrapolate values $\hat{\alpha}_{m+1}$ for the period $[mT,(m+1)T)$ for which we have no observation, using the regression method. $\hat{\alpha}_{6,3}=0.11$ for the 6th scheduling period for *3rd* machine. The average value of repair time $EY_{6,3} = \frac{1}{\hat{\alpha}_{6,3}}$ for the 6th scheduling period for *3rd* machine equals to 9,09.

Tab.1. Values $\hat{\alpha}_{i,3}$ for ith scheduling horizon and for 3rd machine

i	$N_{i,w}$	$\sum^{n_1} y_{1,k}$	$\hat{\alpha}_{i,3}$
1	33	363,6	0,090759
2	33	380,2	0,086796
3	33	384,6	0,085803
4	31	273,5	0,113346
5	31	298,6	0,103818

Basing on information about numbers of detected failures, failure-free times $X_{i,3,1},\ldots,X_{i,3,N_{i,3}}$ and repair times $y_{i,3,1},\ldots,y_{i,3,N_{i,3}}$ of machine w=3 in the ith period $[(i-1)T, iT)$, $i = 1,\ldots,5$ parameters for Weibull distribution $\hat{p}_{6,3} = 2{,}919$ and $\hat{\lambda}_{6,3} = 0{,}002$ $\hat{p}_6$ [4] and parameter of Exponential distribution $\alpha_{6,3} = 0.11$ for 6th period have been estimated.

Having values of Weibull parameters and Exponential parameter one can compute:

- the probability that the first failure of 3rd machine occurs after 5 unit of time is 0.99, $R_w(5) = P_w\{X_{6,3,1} > 5\} = 0.99$.
- the $MTTFF_3$ for *3*rd machine $MTTFF_3 = 7.54$,
- the $MTBF_3$ for *3*rd machine $MTBF_3 = 0.909$,
- the probability P_w that in the interval $[5,10]$ there occurs at least one failure of *3*rd machine is very small $P_3 = 1.239 \times 10^{-4}$

Predictive scheduling consists in placing time window in the schedule at time $\left[MTBF_3, MTBF_3 + Y_{m+1,3}\right]$ the time period $[mT,(m+1)T)$.

Having the values of $MTBF_3$ parameter for Weibull distribution and the average value of repair time $EY_{6,3}$ the predictive schedule using the ED system is generated. The production model is presented in Fig. 2.

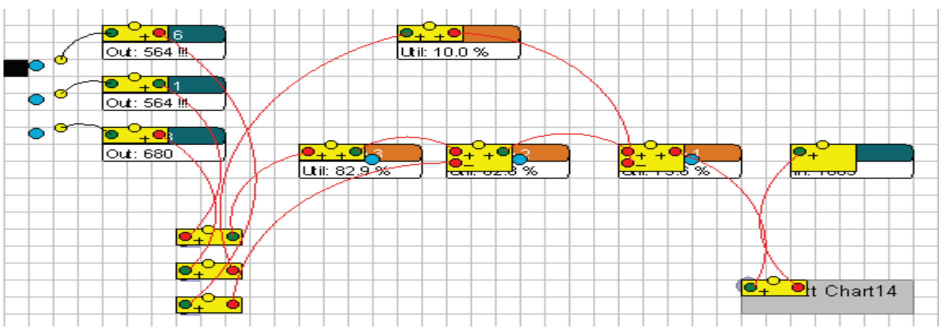

Fig.2. The production system modeled in ED system

Let us assume that the time unit for $MTBF_3$ parameter is 1 hour. The $MTBF_3$ for 3rd machine $MTBF_3 = 7.54h = 452\min$. The time unit of $MTTR_3$ is a minute. The average value of repair time $EY_{6,3}$ for the 6th scheduling period for 3^{rd} machine equals to 9.09 min. For the production process described by MP and MOT (6,7) and $MTBF_3 = 452\min$ and $MTTR_3 = 9.09\min$ the predictive Gantt chart (Fig. 3) is generated. The duration of the fist operation of 13st batch of 1st product is longer because of the machine failure.

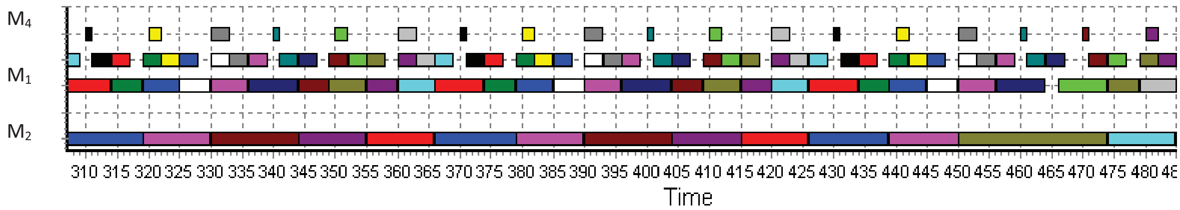

Fig 3. The Gantt chart with failure of 3^{rd} machine

5. Conclusion

In the paper the numerical example for the production model with failures is presented where successive failure-free times are supposed to have Weibull distributions and are followed by exponentially distributed times of repairs. Basing on information about the number of failures, failure-free times and repair times in a number of periods of the same duration in the past unknown parameters are estimated. Having values of parameters: MTTF and MTTR, the predictive schedule is generated using ED system. The paper is the answer for the question about correctness of values of the parameter of exponential distribution appeared in [4].

References

1. Gurel S., Korpeglu E., Akturk M.S.: An anticipative scheduling approach with controllable processing times. "Computers and Operations Resesarch", 2010, Vol. 37, Is. 6, p. 1002-1013.
2. Kempa W. M., Wosik I., Skołud B.: Estimation of reliability characteristics in a production scheduling model with time-changing parameters – first part, theory. Management and control of manufacturing processes, Lublin 2011, p. 7-18.
3. Krysicki W.: Theory of probability and mathematical statistic with examples (in Polish), I and II part, Warsaw: PWN, 2002.
4. Skołud B., Wosik I., Kempa. W. M., Kalinowski K.: Estimation of reliability characteristics in a production scheduling model with time-changing parameters –second part, numerical example. Management and control of manufacturing processes, Lublin 2011, p. 19-30.