ARCHIWUM ENERGETYKI tom **XLI**(2011), nr 3-4, 3–14

Zygfryd Domachowski*
Marek Dzida
M. Hossein Ghaemi

Gdansk University of Technology
Faculty of Ocean Engineering and Ship Technology
Gdańsk

Influence of electric power generation structure on frequency control — mathematical modelling

A 20% decrease in CO_2 emission is the principal goal formulated by European Union in the "3x20" package. As a result, development of windfarms, nuclear power plants and even small heat and power generating plants are expected in Poland. Electric power generation in windfarms and in nuclear power plants involves specific problems in the electric grid frequency control. Windfarm power generation requires a support from conventional power plants, when a nuclear power plant requires adequate structure for the control and adjustment purposes. A concept of mathematical model of so-called multi-machine power system, based on representative power plant mathematical models, has been presented in this paper. Such a model of electric grid is suggested for primary and secondary frequency control simulations, as well. The purpose is to investigate the influence of power generation structure on power system frequency control.

1 Introduction

Nowadays and in the future, for Poland as well for any other country, the power industry goal is to ensure reliable power supply and to satisfy requirements regarding carbon dioxide emission. The task of decarbonising in electrical energy production and consumption has become a global priority. The world's power producers look to make significant cuts in power plants carbon dioxide emission to avoid global climate change. Power grids will require a far higher proportion of renewable energy capacity than they currently have.

In the European Union the carbon dioxide diminution is addressed in the so-called "3x20" package. It would be attained as a result of 20% increase in the renewable energy application and the same in the efficiency of energy consummation.

---

[*] E-mail: domachow@pg.gda.pl

In Poland, near 95% of electric power is still produced from coal. Thus beyond a renewable energy large application a development of nuclear power plants is also expected. Electric power generation based on renewable energy as well as on nuclear energy involves specific problems in the electric grid frequency control. Hydro, solar, tidal and wind power have varying levels of intermittency. However, among them wind power has entirely different nature than the other ones as the patterns and strength of wind force are extremely unpredictable. In some weeks and months there are extended periods of high output and then extended periods of low input. Moreover, the periods of low input often coincide with periods of high energy demands. Among renewable energy resources wind power is expected to take the most important role in Poland. The sporadic nature of wind power does not mean it cannot be utilized but it means that electric girds must manage this irregular supply to a regular one. One solution is supporting windfarm power generation by the conventional power plants.

Nuclear power plants meet another specific problem in power system control. Increasing of their participation in electric power generation results in the deterioration of power system frequency control, particularly in transient conditions. An adequate structure and adjustment of nuclear power plant controllers enables to mitigate their influence on power system frequency control.

Today, to find the solutions many of the mentioned problems can be investigated using mathematical modelling and simulation tools. Depending on the investigation purpose two types of mathematical model of power generation are used. They are the single machine model and the multi-machine one. Both of them have specific advantages and disadvantages. They differ in the complexity and the domain of applicability. Therefore, their application depends on the nature of the problem to be solved. The single machine model is based on more simplifications than the multi-machine one. Nevertheless it is useful for

- synthesis (optimisation) of generating unit controller's structure and parameters,
- analysis of generating unit control system transient responses,
- preliminary verification of control law and related algorithm.

To investigate the dynamics of the whole power system it is necessary to use the multi-machine mathematical model. Such a model enables

- analysis of power system frequency and power control,
- analysis of cooperation and interactions between generating units of different types in power system control,

- synthesis and optimisation of power system control algorithms.

Both single machine and multi-machine mathematical modelling of power generation control are described below. However, the multi-machine modelling of power system control is the main topic of the paper.

2 Single machine mathematical modelling of power generation control

Analysis of the role of single machine mathematical modelling has been focused on the participation of nuclear and wind power plants in power generation chain. In the seventieth of the last century France decided to develop nuclear power generation. Following that decision, Electricite de France started investigations on the influence of nuclear power plant on the power system frequency control [15]. Basing on a step response of the nuclear power plant control system to the frequency set point the structure and parameters of power plant controller had been optimised. On such a basis the behaviour of a nuclear power plant control system was compared to a conventional one. As a result some disadvantages of nuclear power plant control system transients were observed.

In the next step the above mentioned nuclear power plant control system's mathematical model has been applied for investigations of nuclear generating units influence on power system frequency control. It has been a part of multi-machine model. In addition, two other parts of power system are considered, i.e. thermal (steam) and water conventional power plants.

Using a single machine mathematical model the following aspects of nuclear power plants control system are analysed:

- influence of controller's structure and parameters on nuclear power plant dynamics [5],
- nuclear power plant controller's structure and parameters adjustment [5],
- comparison of nuclear power plant control system with conventional one [8], and
- specific regulation problems in nuclear power plant [3].

It is assumed that the wind power plant produces as much power as possible. Thus a single machine mathematical modelling can be applied only to optimise the wing angle setting to achieve the maximum possible efficiency of wind power transformation [12].

3 Multi-machine mathematical modelling of power system control

Starting with a single machine mathematical model the next step will be a double machine one. Now, in the preliminary investigations of the wind power generation its influence on the electric grid frequency control should be considered. In this regard, support of wind turbine by gas turbine within a so-called separate power system has been assumed. Such a double machine power system has been analysed for both simple [9] and combined [10,11] gas turbine cycles. It turned out that for economy reasons two or more power plants should be applied to support a windfarm [10,11]. As a general concept of supporting a windfarm, when starting to analyse the problem, a set of conventional power plants has been suggested [7]. Then a multi-machine model, which is an adequate method of mathematical modelling of power system, is applied.

As it has been mentioned in introduction, a triple-machine mathematical model has been used by Electricite de France to analyse the influence of nuclear power generation on the power system frequency control [15]. Similarly, when decision for construction of the first Polish nuclear power generation made, such a model was assumed. During preliminary study, several variants for different parts of nuclear power plants in power system have been analysed [6].

Today, a hybrid power generation in the Polish power system should be considered. Coal and lignite still are the principal resources of electric power. More than 15% of electric power is produced in heat and power cogeneration systems. Approximately, 5% of electric power comes from water energy. Wind power generation is evolving step by step. Installation of combined cycle power plants (about 5000 MW of total power) is also announced [14]. In the near future, development of nuclear power generation is expected. Moreover, a distributed power generation is also under developing. Therefore, a triple–machine model is not enough for modelling the Polish power system.

4 Mathematical model of assumed multi-machine Polish power system

In a general view let us assume the Polish power sector composed of:

- coal fuelled,
- combined cycle,
- nuclear,

- water, and
- wind

power generation plants. In such a way five types of power generating units have been assumed. A simplified model of such a power system structure is represented in Fig. 1.

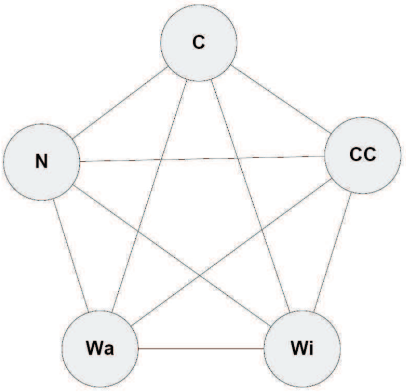

Figure 1. Simplified model of Polish power system structure: C – coal, CC – combined cycle, N – nuclear, Wa – water, Wi – wind power generating unit.

In a preliminary and simplified study it can be assumed that the part of water power generation will not be changed. The part of wind, combined cycle and nuclear power generation will increase within the time leading to decreasing of participation of the coal power generation in Polish power system. Furthermore, assume that each type of power generation illustrated in Fig. 1 is described by so-called representative power plant. In this way, Fig. 1 represents a set of five types of power generating units cooperating and interacting in power system frequency control. To be able to formulate a proper mathematical model of such a set of generating units by focusing on the frequency control, let us start with writing the power balance equation for one of them.

4.1 Mathematical modelling of generating unit power balance

For the i-th generating unit (i ={C, CC, N, Wa, Wi} — see Fig.1), a power balance change, i.e. ΔP_{ig} – ΔP_{ic}, where index g denotes generated power, and index c indicates power consumed by the receivers, involves the frequency change. As a result the following variations occur:

- change of kinetic energy of generating unit's mass,
- change of power consumed by the receivers,
- change of power transmitted to other generating units.

Change of kinetic energy of generating unit's mass Denoting with index 'o'a steady state of generating unit, the change of its kinetic energy E_i, resulting from power balance, takes the following form:

$$\begin{aligned}\frac{dE_i}{dt} &= \frac{d}{dt}\left(E_{io}\frac{E_i}{E_{io}}\right) = \frac{d}{dt}\left[E_{io}\left(\frac{f_i}{f_{io}}\right)^2\right] = \frac{d}{dt}\left[E_{io}\left(\frac{f_{io}+\Delta f_i}{f_{io}}\right)^2\right] \\ &\approx \frac{d}{dt}\left[E_{io}\left(1+2\frac{\Delta f_i}{f_{io}}\right)\right] = 2\frac{E_{io}}{f_{io}}\frac{d}{dt}\left(\Delta f_i\right) \ , \qquad (1)\end{aligned}$$

where f_i denotes i-th generating unit frequency.

Change of power consumed by the receivers Let us denote:

$$D_{io} = \left.\frac{\partial P_{ic}}{\partial f_i}\right|_{f_i=f_{io}} \ , \qquad (2)$$

then the approximate change of power consumed by receivers is given with the following expression:

$$\Delta P_{ic} \approx D_{io}\Delta f_i \ . \qquad (3)$$

Change of power transmitted to other generating units Denote ΔP_{tij} as the change of power transmitted from i-th to j-th generating unit. Then the change of total power transmitted from i-th generating unit to all other cooperating and interacting ones is given with the following sum:

$$\Delta P_{ti} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \Delta P_{tij} \ , \qquad (4)$$

where n denotes the number of cooperating generating units.

The power transmitted from i-th voltage node up to j-th voltage node, with the transmission line of reactance X_{ij}, is described as follows:

$$P_{tij} + \mathbf{j}Q_{tij} = U_i\overline{\frac{U_i - U_j}{\mathbf{j}X_{ij}}} \ , \qquad (5)$$

where:

$$\begin{aligned} U_i &= |U_i|\, e^{\mathbf{j}\delta_i}\ , \\ U_j &= |U_j|\, e^{\mathbf{j}\delta_j} \end{aligned} \tag{6}$$

denote voltage in the node i-th and j-th respectively, and δ_i, δ_j denote the angle between voltage vector and reference vector. Therefore, the relationship (5) can be represented in the following form:

$$P_{tij} + \mathbf{j}Q_{tij} = \mathbf{j}\frac{|U_i|^2}{X_{ij}} - \mathbf{j}\ \frac{|U_i|\,|U_j|}{X_{ij}} e^{\mathbf{j}(\delta_i - \delta_j)} \tag{7}$$

and consecutively

$$P_{tij} = \frac{|U_i|\,|U_j|}{X_{ij}}\ \sin\left(\delta_i - \delta_j\right)\ . \tag{8}$$

Furthermore denote $\delta_i = \delta_{io} + \Delta\delta_i$, and $\delta_j = \delta_{jo} + \Delta\delta j$; then approximately

$$\begin{aligned} \Delta P_{tij} &\approx \left.\frac{\partial P_{tij}}{\partial\left(\delta_i - \delta_j\right)}\right|_{\substack{\delta_i = \delta_{io} \\ \delta_j = \delta_{jo}}} \Delta\left(\delta_i - \delta_j\right) = \\ &= \frac{|U_{io}|\,|U_{jo}|}{X_{ij}} \cos\left(\delta_{io} - \delta_{jo}\right) \Delta\left(\delta_i - \delta_j\right)\ . \end{aligned} \tag{9}$$

Finally, taking into account that $\delta = \int \omega dt = 2\pi \int f dt$, the relationships (9) and (4) take the following forms:

$$\Delta P_{tij} = \frac{1}{T'_{ij}} \left(\int \Delta f_i dt - \int \Delta f_j dt \right) \tag{10}$$

and

$$\Delta P_{ti} = \sum_{\substack{j=1 \\ i \neq j}}^{n} \frac{1}{T'_{ij}} \left(\int \Delta f_i dt - \int \Delta f_j dt \right)\ , \tag{11}$$

where $T'_{ij}$is denoted as follows:

$$\frac{1}{T'_{ij}} = 2\pi \frac{|U_{io}|\,|U_{jo}|}{X_{ij}} \cos\left(\delta_{io} - \delta_{jo}\right)\ . \tag{12}$$

4.2 Differential equation of generating unit power balance

According to the relationships (1), (3) and (11) the differential equation of i-th generating unit power balance can be written as follows:

$$2\frac{E_{io}}{f_{io}} \frac{d}{dt} \left(\Delta f_i\right) + D_{io}\Delta f_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{T'_{ij}} \left(\int \Delta f_i dt - \int \Delta f_j\, dt \right) = \Delta P_{ig} - \Delta P_{ic}\ . \tag{13}$$

Now by denoting with index n a nominal state of generating unit the relationship (13) can be rewritten in the following form:

$$2\frac{E_{io}}{P_{in}}\frac{f_{in}}{f_{io}}\frac{d}{dt}\left(\frac{\Delta f_i}{f_{in}}\right)+\frac{D_{io}f_{in}}{P_{in}}\frac{\Delta f_i}{f_{in}}+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{f_{in}}{T'_{ij}P_{in}}\left(\int\frac{\Delta f_i}{f_{in}}dt-\int\frac{\Delta f_j}{f_{jn}}dt\right)= \tag{14}$$

$$=\frac{\Delta P_{ig}}{P_{in}}-\frac{\Delta P_{ic}}{P_{in}}\ ,$$

where P_{in} denotes rated power of i-th generating unit, and $f_{in} = f_{jn}$ denotes nominal steady state frequency. To simplify the graphical notation of relationship (14) let us denote:

$$\begin{aligned} 2\frac{E_{io}}{P_{in}}\frac{f_{in}}{f_{io}} &= T_{im}\ ,\\ \frac{D_{io}f_{in}}{P_{in}} &= z_i\ ,\\ \frac{f_{in}}{T'_{ij}P_{in}} &= \frac{1}{T_{ij}}\ . \end{aligned} \tag{15}$$

Moreover, for the same purpose let us replace the notations of frequency and power related changes with frequency and power symbols only. Finally, the mathematical model of i-th generating unit power balance takes the following form:

$$T_{im}\frac{d}{dt}f_i+z_if_i+\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{1}{T_{ij}}\left(\int f_idt-\int f_jdt\right)=P_{ig}-P_{ic}\ , \tag{16}$$

and its block diagram is represented in Fig. 2.

5 Representative generating units

The block diagram of both conventional and nuclear generating unit control system is represented in Fig. 3. Mathematical model parameters of both conventional and nuclear power plant have been assumed on the basis of previous simulation investigations [8]. Parameters of steam pressure controller have been optimised based on steam pressure response to a step change of energy input signal, F_B — see Fig. 3. Similarly, parameters of steam turbine controller have been optimised regarding to the turboset power output response to power set point step signal, P_{REF} — see Fig. 3. In Tab. 1 all parameters of power plant control system (see Fig. 3) of both conventional and nuclear representative generating units are given.

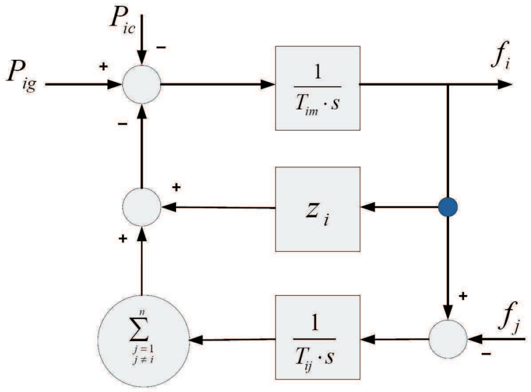

Figure 2. Block diagram of i-th generating unit power balance.

Table 1. Values of parameters of conventional and nuclear power plants.

Power plant parameters	T_0 [s]	T_1 [s]	T_2 [s]	T_4 [s]	w	T_6 [s]	k_{wp}	T_8 [s]	T_9 [s]	T_{11} [s]	z
Nuclear	3	30	30	3	6	0.3	0.5	6	0.2	18	25
Conventional	20	100	200	50	6	0.2	0.3	15	0.2	8	25

Power plant parameters	T_{13} [s]	T_{14} [s]	k_{14} [s]	T_{15} [s]	k_F	k_5	T_5 [s]	K_{16}	T_{16} [s]	T_{17}^2 s^2
Nuclear	0.002	0.2	1.2	0.3	25	5.4	170	-0.1	1.6	5
Conventional	0.002	0.2	2.5	0.3	25	6.0	500	-0.1	1.6	5

In Fig. 4 the block diagram of hydraulic generating unit control system is represented. It is made based on the recommendations of IEEE [16]. Its parameters have been assumed as it is shown in Tab. 2. The applied mathematical models of combined cycle and wind power plant control systems are similar to those presented in [9].

Particularly, the mathematical model of cooperation between different representative power plants depends on time constants T_{ij} — Fig. 2. For simplification, when the same transmission line reactance and mode voltage for all cooperating

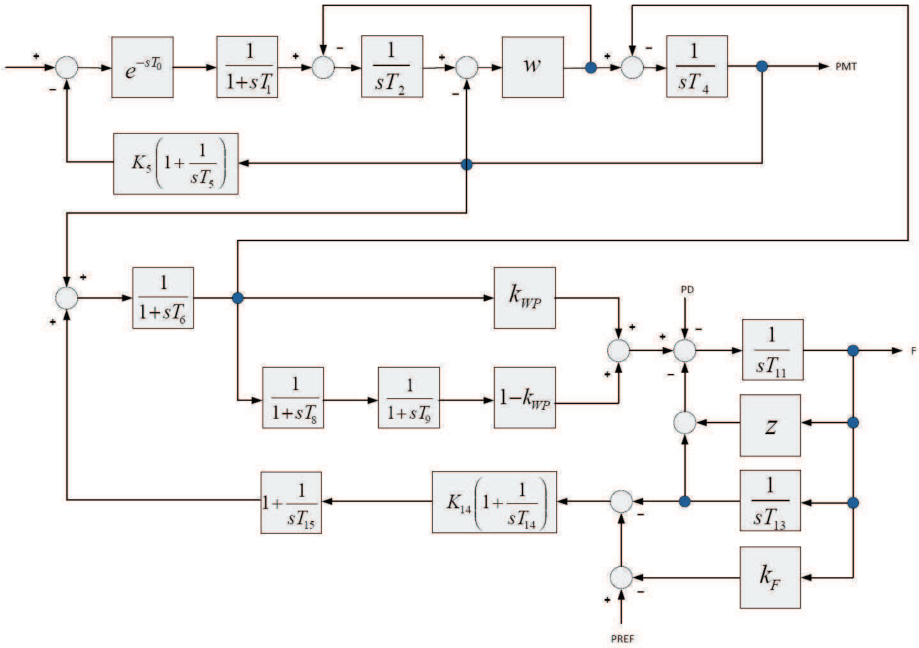

Figure 3. Combined simulation model of control system of conventional and nuclear power plants.

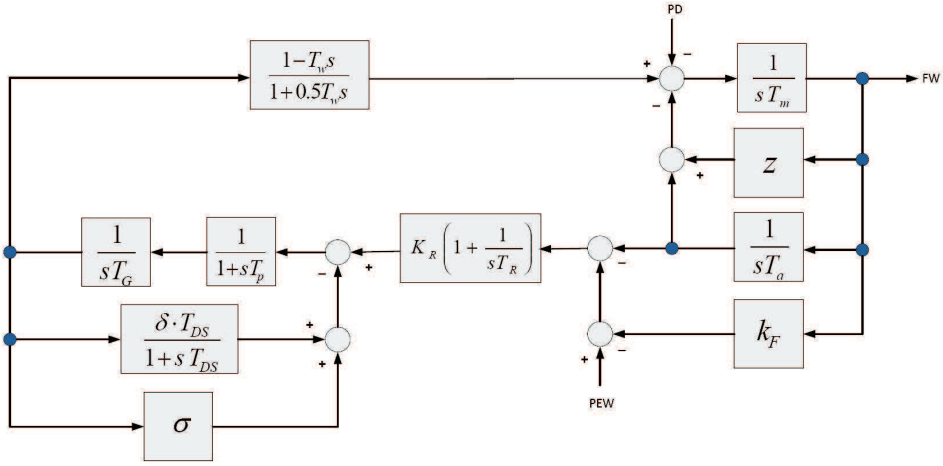

Figure 4. Simulation model of control system of water power plant.

power plants assumed, the following result can be achieved, see (15) and (12):

$$\frac{T_{ij}}{T_{ji}} = \frac{P_{in}}{P_{jn}} \; . \tag{17}$$

If one of these time constants is known or can be approximated, and if the part of different power plants in the total rated power is known then the relationship (17) enables us to determine all other time constants T_{ij}.

Table 2. Values of parameters of water power plant.

Power plant parameters	T_m [s]	T_w [s]	T_p [s]	T_G [s]	δ	σ	T_a [s]	k_F	z	T_D [s]	k_R	T_R [s]
Option 'A'	15	6	0.04	0.2	0.5	0.04	0.02	25	25	100	0.03	1.0
Option 'B'	10	3	0.04	0.2	0.4	0.04	0.02	25	25	50	0.02	1.0

Received in October 2011

References

[1] Akhmadov V., Knudsen H., Nielsen A. M.: *Advanced Simulation of Windmills in the Electric Power Supply.* Electrical Power and Energy System 22, 2000.

[2] Anaya-Lara O., Nicholas J., Cartwright Ph.: *Power Quality Aspects of Windfarms.* International Power Generation, November 2005.

[3] Domachowski Z.: *Specific regulation problems in nuclear power plant.* In: Proc. the VIII Conference "Steam Turbine of Large Output, especially for nuclear power stations", October 30 – November 2, 1984, Karlovy Vary, Czech Republic.

[4] Domachowski Z.: *Power System Frequency Control Characteristics as Function of Nuclear Power Plant Participation.* Mathematical Research, Band 47, Vol. II, Akademie – Verlag, Berlin 1988.

[5] Domachowski Z.: *Control structure influence on nuclear power plant behaviour.* In: Proc. the ISMM International Conference MICRO'90, Montreal, Canada, 22–24 May 1990 (Z. Jacyno, ed.), Acta Press, Anaheim, Calgary, Zurich 1990.

[6] Domachowski Z.: *Influence of turboset automatic control on nuclear power plant safety and electric power grid supply reliability.* Archiwum Energetyki (Archives of Energetics) **XXXVI**(2006) (in Polish).

[7] Domachowski Z.: *Counteracting windpower fluctuations within a separate electric power grid.* In: Proc. the 5th ASME Turbo – Expo 2007. Power For Land, Sea and Air, Montreal, May 14–17, 2007, Paper no. GT 2007-28218.

[8] Domachowski Z., Bugała G.: *Simulation as a tool of comparison between nuclear and fossil fuel generating unit control systems*. In: Proc. the IMACS European Simulating Meeting on Simulation in Research and Development, Eger, Hungary, 27–30 August 1984 (A. Javor, ed.), North-Holland, Amsterdam 1984.

[9] Domachowski Z., Dzida M., M. H. Ghaemi: *Supporting windpower within a separate electric power grid*. In: Proc. the ASME Turbo – Expo 2011, 6–10 June 2011, Vancouver, Canada, Paper no. GT2011-46494.

[10] Domachowski Z., Ghaemi M.H.: *Cooperation of windfarm and conventional power plant in electric power system frequency control*. Systems **13(**2008), Special Issue 1/2 (in Polish).

[11] Domachowski Z., Ghaemi M.H.: *Some problems of heat and power distributed generation control*. Archiwum Energetyki (Archives of Energetics) **XL**(2010), 3.

[12] Lubośny Z.: *Wind Power Plants in Electic Power System*. WNT, Warsaw 2006 (in Polish).

[13] Meier S.: *Grid Interaction of Wind Energy Conversion System*. John Wiley & Sons, Chichester 1998.

[14] Pawlik M.: *EU climate package and fuel structure of domestic power stations*. Archiwum Energetyki (Archives of Energetics) **XL**(2010), 3 (in Polish).

[15] Pioger G., Testud G.: *Etude du comportement dynamique du reseau sur des temps longs. Apliccation au reglage frequence – puissance*. Electricite de France. Bulletin de la Direction des Etudes et Recherches, serie B, 1978, No. 1.

[16] Dynamic Models for Steam and Hydro Turbines in Power System Studies. IEEE Committee Report Paper T73 089-0, 1973.

Wpływ struktury wytwarzania energii elektrycznej na regulację automatyczną częstotliwości — modelowanie matematyczne

Streszczenie

Podstawowym celem pakietu „3x20” Unii Europejskiej jest zmniejszenie o 20% emisji dwutlenku węgla. Z tego powodu oczekuje się w Polsce rozwoju energetyki wiatrowej, podobnie jak energetyki jądrowej, a także mini elektrociepłowni. Elektrownie wiatrowe i jądrowe wnoszą specyficzne problemy w regulacji automatycznej częstotliwości systemu elektroenergetycznego. Farmy wiatrowe wymagają pod tym względem wspomagania przez elektrownie konwencjonalne; bloki jądrowe wymagają odpowiedniego dostosowania struktury i nastawień ich układów regulacji automatycznej. Przedstawiono koncepcję wielomaszynowego modelu matematycznego systemu elektroenergetycznego, opartą na modelach matematycznych tzw. reprezentatywnych turbozespołów i bloków. Taki uproszczony model matematyczny jest proponowany do symulacyjnych badań porównawczych regulacji automatycznej częstotliwości systemu elektroenergetycznego. Ich celem jest analiza wpływu struktury wytwarzania energii elektrycznej na regulację automatyczną częstotliwości systemu elektroenergetycznego.