

spring-frictional complete set, friction pair,
reactions of constraint, tilt angles of contacting surfaces

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INVESTIGATION OF THE INFLUENCE OF FRICTION PAIRS GEOMETRICAL PARAMETERS OF WAGON TRUCK SPRING - FRICTIONAL SET ON THE EFFECTIVENESS OF THEIR OPERATION

Summary. The article gives an account of the results of the analytical and numerical modeling of force action of friction pairs “truck bolster – frictional wedge” and “frictional wedge – frictional plank”. There have been obtained formulas for determining constraint reactions according to friction pairs geometrical parameters which makes it possible to find their rational values.

ИССЛЕДОВАНИЕ ВЛИЯНИЯ ГЕОМЕТРИЧЕСКИХ ПАРАМЕТРОВ ПАР ТРЕНИЯ ПРУЖИННО-ФРИКЦИОННОГО КОМПЛЕКТА ТЕЛЕЖКИ ГРУЗОВОГО ВАГОНА НА ЭФФЕКТИВНОСТЬ ИХ РАБОТЫ

Аннотация. В статье изложены результаты аналитического и численного моделирования силового воздействия пар трения «надрессорная балка – фрикционный клин» и «фрикционный клин – фрикционная планка». Получены формулы для определения реакции связей в зависимости от геометрических параметров пар трения, которые позволяют найти их рациональные значения.

1. FORMULATION OF A PROBLEM

A spring-frictional set of model 18-100 wagon truck has been structured in such a way that its frictional wedges 2 and 3 contact only with three hardbody elements – truck bolster 1, frictional plank 4 (or 5) and double springs 7 (or 8) (Fig. 1).

In Fig. 1 1 is the tail unit of the truck bolster; 2 and 3 are frictional wedges; 4 and 5 are frictional plank; 6 is spring sets under truck bolster; 7 and 8 are sets of wedged springs; 9 is a solebar.

In [1] the spring-frictional truck set of wagon model 18-100 has been analytically modeled in a simplified manner. At the same time there has been obtained a final analytical formula for finding constraint reactions of frictional wedges on truck bolster without allowing for the constructive features of frictional wedges and inaccuracies in manufacturing their inclined planes.

In [2] there has been considered the model and developed the method of computation of constraint reactions of spring-frictional truck set of wagon model 18-100 allowing for possible edge contact of units rubbing each other from different sides. However, while creating a computational model of frictional wedge 2 (or 3) as a research object it would be advisable according to the principle of freeing

from the constraints [3] to replace the surfaces of its contact with the truck bolster *1* and frictional plank *4* (or *5*), as external constraints only by two constraints reactions in the form of \bar{R}_1 , replacing the truck bolster and \bar{R}_2 , replacing the frictional plank.

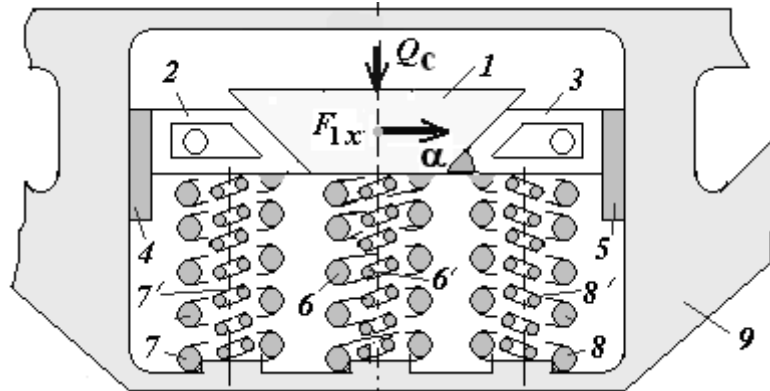


Fig. 1. Spring-frictional truck set loaded with the force from the pressure from the body of the wagon with cargo
Рис. 1. Пружинно-фрикционный комплект тележки, нагруженной силой давления кузова вагона с грузом

In [4] there has been derived the formula of normal reaction on the equivalent inclined surface during the motion of the frictional wedge downwards based on the assumption that the frictional wedge is additionally loaded with the lengthwise force applied to the truck bolster allowing only for wedged springs reaction. The obtained formula does not contain the weight of the wagon with cargo and the reaction of cushioned springs. Moreover, in [4] it is not specified which of frictional wedges are overloaded and which are unloaded.

In [5, 6] there have been presented the results of the analytical and numerical modeling of force action of friction pairs “truck bolster – frictional wedge” and “frictional wedge – frictional plank” of spring-frictional truck set of 18-578 wagon model. The equations of truck bolster equilibrium as a physical object have been solved analytically. There have been obtained formulas for determining constraint reaction according to geometrical parameters (tilt angle of contacting surfaces) of friction pairs. However, in [5, 6] the investigation has been performed without taking into consideration the lengthwise force acting from the adjacent leading wagon and applied to the truck bolster.

1.1. Man-made assumption

To be able to consider asymmetrical allocation of hardbody cargo with respect to symmetry axis as in [5, 6] it is necessary to obtain analytical formulas for determining constraint reaction in friction pairs “truck bolster – frictional wedge” and “frictional wedge – frictional plank” according to the lengthwise force acting from the adjacent leading wagon towards the truck bolster and their geometrical parameters.

Let us assume that through the wagon frame vertical towards truck bolster center bowl there are being transferred pressure forces from the frame of the wagon with cargo [5, 6]. The pressure force of the truck bolster with cargo in the form of Q_c (or Q_D) will act upon on the truck spring sets of the wagon. Elasticity forces of spring sets exert pressure on the truck bolster and frictional wedges and through those also on the truck solebars. Furthermore, we will take into consideration only one segment F_{1x} of lengthwise force F_x applied to the truck bolster from the automatic coupling devices of the leading wagon because the other segment F_{2x} will be applied to the truck bolster of the rear truck [7] and the third one – to the automatic coupling devices of the driven wagons.

Let us consider a computing model of the mechanical system “truck bolster – frictional wedges – frictional planks” in the way it is shown in Fig. 2. Here F_6 is reactions (elasticity force) of wedged spring sets exerting resistance to the downward transition of the truck bolster *1* (5 units) (Fig. 3a) and

F_7 and F_8 reactions (elasticity force) of wedged spring sets exerting resistance to downward transition of frictional wedges 2 and 3 (Fig. 2b, d).

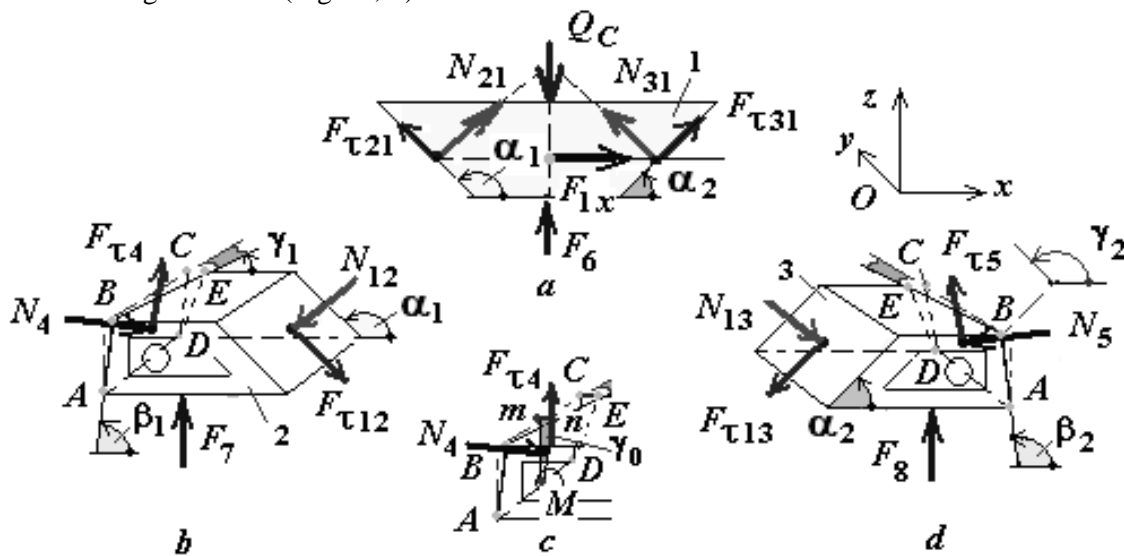


Fig. 2. Computing models of truck bolster 1 and frictional wedges 2 and 3
Рис. 2. Расчётные модели надрессорной балки 1, и фрикционных клиньев 2 и 3

In Fig. 2a the following designations are accepted: N_{21} , N_{31} and $F_{\tau 21}$, $F_{\tau 31}$ are normal and tangent components of constraint reactions (frictional wedges) 2 and 3; α_1 and α_2 - are the tilt angles of the truck bolster surfaces to the horizon, rad. ($\alpha = 45^\circ + 1^\circ$); in Fig. 2 b, d: N_{12} , N_{13} and $F_{\tau 12}$, $F_{\tau 13}$ are normal and tangent components of constraint reaction on wedges 2 and 3, N_4 , N_5 and $F_{\tau 4}$, $F_{\tau 5}$ - are normal and tangent components of constraint reaction (frictional planks), $\gamma_1 \leq \frac{\pi}{2}$ and $\gamma_2 \geq \frac{\pi}{2}$ - tilt angle of rear surface $ABED$ of frictional wedges 2 and 3, contacting with frictional planks 4 and 5, rad. ($\gamma_1 \approx 88^\circ - 1^\circ$, $\gamma_2 \approx 92^\circ + 1^\circ$).

Let us study truck bolster equilibrium 1 (Fig. 2a). The truck bolster 1 experiences: reaction R_{21} and R_{31} of frictional wedged 2 and 3 which are disintegrated into normal and tangent components - N_{21} , N_{31} and $F_{\tau 21}$, $F_{\tau 31}$; active force Q_c , equal to half of the reaction falling on truck bolster 1, and reactive force in the form of the resultant of the reaction of spring sets $6 F_6$. We assume that the inclined surfaces of the truck bolster are manufactured with inaccuracies, i.e. $\alpha_1 \neq \alpha_2$, where α_1 and α_2 are tilt angles of the surfaces of truck bolster 1 to the horizon, rad. ($\alpha_1 \approx 134^\circ 30' + 1^\circ$, $\alpha_2 \approx 45^\circ 30' + 1^\circ$).

During analytical investigation let us assume that just as in case of [5, 6] tilt angles of surfaces (α_1 and α_2) of truck bolster 1, frictional wedges 2, 3 and frictional planks (β_1 , β_2 and γ_1 , γ_2) have different values ($\alpha_1 \neq \alpha_2$, $\beta_1 \neq \beta_2$, and $\gamma_1 \neq \gamma_2$) which corresponds either to the fact of their being manufactured with inaccuracies or allows for nonuniform wear of their surfaces. We also assume that gliding friction coefficients f between contacting surfaces of the truck bolster (f_1 and f_2), frictional wedges and planks (f_3 and f_4) have different values.

For solving the set applied problem we will use a kinetostatic method from the course of theoretical mechanics [3].

2. SOLUTION RESULTS

Let us set up two equilibrium equations of planar force system equating to zero the sum of projects of all forces on axes x and z .

$$N_{21} \cos(\alpha_1 - \frac{\pi}{2}) + F_{\tau 21} \cos(\alpha_1) + N_{31} \cos(\alpha_2 + \frac{\pi}{2}) + F_{\tau 31} \cos(\alpha_2) + F_{1x} = 0; \quad (1)$$

$$N_{21} \sin(\alpha_1 - \frac{\pi}{2}) + F_{\tau 21} \sin(\alpha_1) + N_{31} \sin(\alpha_2 + \frac{\pi}{2}) + F_{\tau 31} \sin(\alpha_2) - Q_C + F_6 = 0 \quad (2)$$

Here we have two independent equilibrium equations and four unknown ones: N_{21} , N_{31} and $F_{\tau 21}$, $F_{\tau 31}$. To solve the problem it is enough to add to equations (1) and (2) the equation resulting from the Coulomb's law.

$$F_{\tau} \leq fN, \quad (3)$$

where: f – is a coefficient of gliding friction between contacting surfaces of truck bolster 1 and frictional wedges $2, 3$ and also between frictional wedges $2, 3$ and frictional planks.

Substituting equations (3) for (1) and (2) after equation rearranging we have:

$$\begin{aligned} aN_{21} + bN_{31} &= -F_{1x}; \\ cN_{21} + dN_{31} &= Q_C - F_6, \end{aligned} \quad (4)$$

where: a, b, c and d are constant coefficients:

$$\begin{aligned} a &= \cos(\alpha_1 - \frac{\pi}{2}) + f_1 \cos(\alpha_1); & b &= \cos(\alpha_2 + \frac{\pi}{2}) + f_2 \cos(\alpha_2); \\ c &= \sin(\alpha_1 - \frac{\pi}{2}) + f_1 \sin(\alpha_1); & d &= \sin(\alpha_2 + \frac{\pi}{2}) + f_2 \sin(\alpha_2); \end{aligned} \quad (5)$$

According to the Kramer rule [8] from the system (4) we find normal components of constraint reaction (frictional wedges 2 and 3) when inclined surfaces of truck bolster 1 are manufactured with inaccuracies

$$N_{21} = -\frac{1}{ad - bc} \left[\begin{aligned} &F_{1x} \left(\sin(\alpha_2 + \frac{\pi}{2}) + f_2 \sin(\alpha_2) \right) + \\ &+ (Q_C - F_6) \left(\cos(\alpha_2 + \frac{\pi}{2}) + f_2 \cos(\alpha_2) \right) \end{aligned} \right]; \quad (6)$$

$$N_{31} = \frac{1}{ad - bc} \left[\begin{aligned} &(Q_C - F_6) \left(\cos(\alpha_1 - \frac{\pi}{2}) + f_1 \cos(\alpha_1) \right) + \\ &+ F_{1x} \left(\sin(\alpha_1 - \frac{\pi}{2}) + f_1 \sin(\alpha_1) \right) \end{aligned} \right]. \quad (7)$$

Matrix which is set up from coefficients under the unknown of system (4) and calculated in a symbolic way (9) is equal to:

$$\begin{aligned} ad - bc &= \sin(\alpha_1) \cos(\alpha_2) + \sin(\alpha_2) f_2 \sin(\alpha_1) + \\ &+ f_1 \cos(\alpha_1) \cos(\alpha_2) + f_1 \cos(\alpha_1) f_2 \sin(\alpha_2) + \\ &+ \sin(\alpha_2) \cos(\alpha_1) (-1) + \sin(\alpha_2) f_1 \sin(\alpha_1) + \\ &+ f_2 \cos(\alpha_1) \cos(\alpha_2) - f_2 \cos(\alpha_2) f_1 \sin(\alpha_1). \end{aligned} \quad (8)$$

Taking into consideration the fact that if the angle is negative then the function is reduced to the function of the positive angle according to formulas $\sin(\alpha - \frac{\pi}{2}) = -\sin(\frac{\pi}{2} - \alpha) = -\cos(\alpha)$ and $\cos(\alpha - \frac{\pi}{2}) = \cos(-(\frac{\pi}{2} - \alpha)) = \sin(\alpha)$ [8], correlations (6) with a glance of expression (8) will be presented in the form:

$$N_{21} = -\frac{1}{\sin(\alpha_1)\cos(\alpha_2) + \sin(\alpha_2)f_2\sin(\alpha_1) + f_1\cos(\alpha_1)\cos(\alpha_2) + f_1\cos(\alpha_1)f_2\sin(\alpha_2) + \sin(\alpha_2)\cos(\alpha_1)(-1) + \sin(\alpha_2)f_1\sin(\alpha_1) + f_2\cos(\alpha_1)\cos(\alpha_2) - f_2\cos(\alpha_2)f_1\sin(\alpha_1)} \times$$

$$\times \left[\begin{array}{l} F_{1x}(\cos(\alpha_2) + f_2\sin(\alpha_2)) - \\ -(Q_C - F_6)(\sin(\alpha_2) - f_2\cos(\alpha_2)) \end{array} \right]; \quad (9)$$

$$N_{31} = \frac{1}{\sin(\alpha_1)\cos(\alpha_2) + \sin(\alpha_2)f_2\sin(\alpha_1) + f_1\cos(\alpha_1)\cos(\alpha_2) + f_1\cos(\alpha_1)f_2\sin(\alpha_2) + \sin(\alpha_2)\cos(\alpha_1)(-1) + \sin(\alpha_2)f_1\sin(\alpha_1) + f_2\cos(\alpha_1)\cos(\alpha_2) - f_2\cos(\alpha_2)f_1\sin(\alpha_1)} \times$$

$$\times \left[\begin{array}{l} (Q_C - F_6)(\sin(\alpha_1) + f_1\cos(\alpha_1)) - \\ -F_{1x}(\cos(\alpha_1) - f_1\sin(\alpha_1)) \end{array} \right]. \quad (10)$$

In a particular case when $f_1 = f_2$ and $\alpha_1 = \alpha_2 + \frac{\pi}{2}$ which correspond to the condition of inclined surfaces of truck bolster 1 without inaccuracies. Expressions (9) and (10) will be presented in the following way:

$$N_{21} = -\frac{1}{1 - f_1^2} \left[\begin{array}{l} F_{1x}(\cos(\alpha_2) + f_1\sin(\alpha_2)) - \\ -(Q_C - F_6)(\sin(\alpha_2) - f_1\cos(\alpha_2)) \end{array} \right]; \quad (11)$$

$$N_{31} = \frac{1}{1 - f_1^2} \left[\begin{array}{l} (Q_C - F_6)(\cos(\alpha_2) - f_1\sin(\alpha_2)) + \\ + F_{1x}(\sin(\alpha_2) + f_1\cos(\alpha_2)) \end{array} \right]. \quad (12)$$

i.e. $N_{31} = N_{21}$ under $F_{1x} = 0$ which agrees with the results [1] and provides evidence of the correctness of obtained analytical expressions.

The character of constraint reaction changing (of frictional wedges 2 and 3) according to tilt angle variation (α_1 and α_2) of friction pair inclined surfaces “truck bolster – frictional planks” with and without allowing for inaccuracies of their manufacturing in accordance with formulas (9) and (10) is presented in Fig. 3a- d.

The analysis of dependences $N_{21}(\alpha_1)$ shows that with increasing angle α_1 which corresponds to decreasing α_2 , constraint reaction increases while with its decreasing the situation is reverse. In case of allowing for inaccuracies of manufacturing of inclined surfaces of friction pairs the value of the constrain reaction is larger than in case when they are not allowed for.

Comparing values $N_{21}(\alpha_1)$ and $N_{31}(\alpha_2)$ it should be noted that $N_{21}(\alpha_1) < N_{31}(\alpha_2)$ due to the action of lengthwise force of the adjacent wagon F_{1x} on the truck bolster. This is most like the reason of more complete contacting of friction pair “frictional wedge 3 – frictional plank 5” than in case of friction pairs “frictional wedge 2 – frictional plank 4”.

Now let us consider equilibrium of frictional wedge 2 (Fig. 2b).

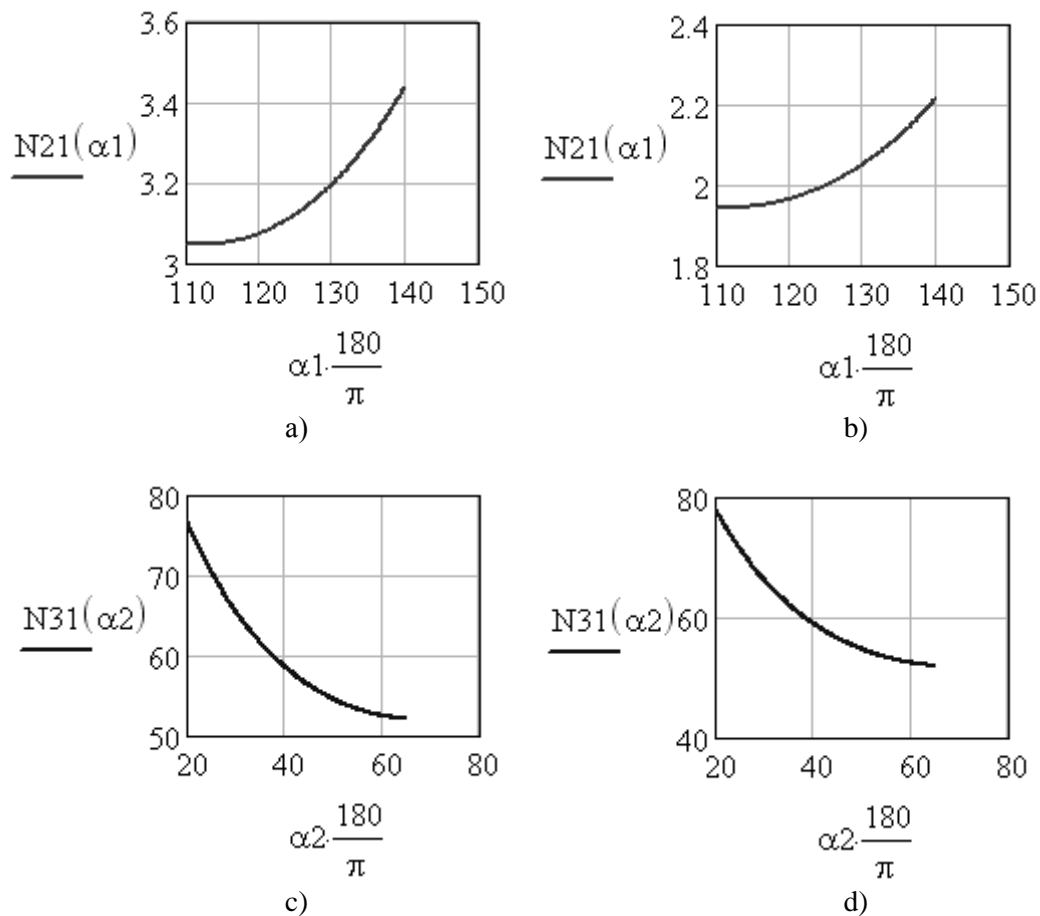


Fig. 3. The character of changing of constraint reactions $N_{21}(\alpha_1)$ and $N_{31}(\alpha_2)$: a), c) and b), d) – without allowing and with allowing for inaccuracies of manufacturing of friction pair inclined surfaces

Рис. 3. Характер изменения реакции связей $N_{21}(\alpha_1)$ и $N_{31}(\alpha_2)$: а), с) и б), д) – без учёта и с учётом погрешностей изготовления наклонных поверхностей пар трения

According to the axiom of action and reaction law frictional wedge 2 experiences the action: normal $N_{12} = -N_{21}$ and tangent $F_{\tau 12} = -F_{\tau 21}$ components of truck bolster reaction $R_{12} = -R_{21}$; normal N_4 and tangent $F_{\tau 4}$ components of frictional plank 4 reaction R_4 as well as resultant of spring set 7 reaction F_7 .

Let us set up force equilibrium equation for frictional wedge 2:

$$N_{12} \cos\left(\alpha_1 + \frac{\pi}{2}\right) + F_{\tau 12} \cos(\alpha_1 + \pi) + N_4 \cos(\gamma_0) \cos\left(\beta_1 + \frac{3}{2}\pi\right) + F_{\tau 4} \cos(\gamma_0) \cos(\beta_1) = 0, \quad (13)$$

where γ_0 – is the angle resulting from computation as in [5, 6].

Taking into consideration correlation (3) we will rewrite the above expressions

$$N_{12} \left(\cos\left(\alpha_1 + \frac{\pi}{2}\right) + f_1 \cos(\alpha_1 + \pi) \right) + N_4 \cos(\gamma_0) \left(\cos\left(\beta_1 + \frac{3}{2}\pi\right) + f_3 \cos(\beta_1) \right) = 0; \quad (14)$$

From correlation (14) taking into consideration (9) after all interim transformations we will find normal component of frictional wedge 4 reaction.

$$N_4 = \frac{1}{\sin(\alpha_1) \cos(\alpha_2) + \sin(\alpha_2) f_2 \sin(\alpha_1) + f_1 \cos(\alpha_1) \cos(\alpha_2) + f_1 \cos(\alpha_1) f_2 \sin(\alpha_2) + \sin(\alpha_2) \cos(\alpha_1) (-1) + \sin(\alpha_2) f_1 \sin(\alpha_1) + f_2 \cos(\alpha_1) \cos(\alpha_2) - f_2 \cos(\alpha_2) f_1 \sin(\alpha_1)} \times$$

$$\times \left[F_{1x} (\cos(\alpha_2) + f_2 \sin(\alpha_2)) - (Q_C - F_6) (\sin(\alpha_2) - f_2 \cos(\alpha_2)) \right] \times \frac{(\sin(\alpha_1) + f_1 \cos(\alpha_1))}{\cos(\gamma_0) (\sin(\beta_1) + f_3 \cos(\beta_1))}. \quad (15)$$

The character of constraint reaction (of frictional plank 4) changing according to tilt angle variation (α_1) of inclined surfaces of friction pair “truck bolster – frictional planks” under $\alpha_2 = \text{const}$ without and with allowing for inaccuracies of their manufacturing in accordance with formula (15) are presented in Fig. 4a, b.

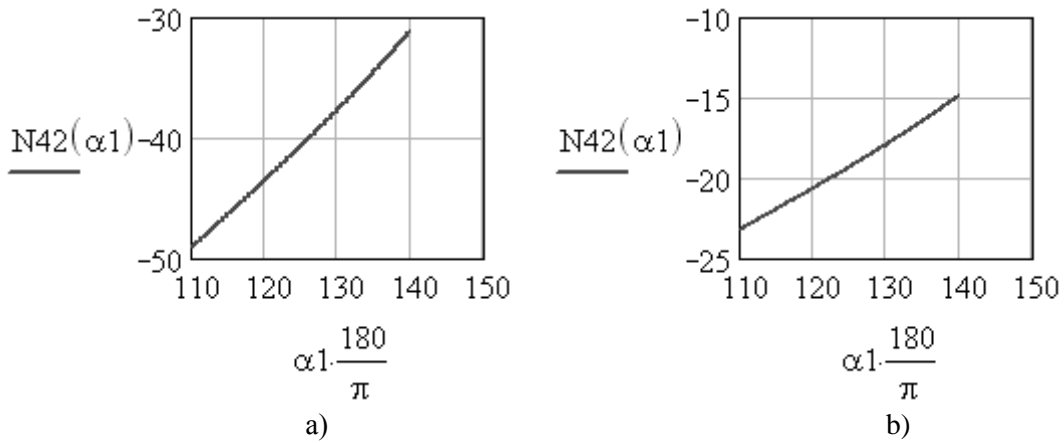


Fig. 4. Character of changing constraint reactions $N_{42}(\alpha_1)$: a) and b) – without and with allowing for inaccuracies of manufacturing friction pair inclined surfaces

Рис. 4. Характер изменения реакции связей $N_{42}(\alpha_1)$: а) и б) – без учёта и с учётом погрешностей изготовления наклонных поверхностей пар трения

The analysis of the dependencies $N_{42}(\alpha_1)$ shows that with increasing angle α_1 constraint reaction according to modulus decreases while with its decreasing the picture is the reverse. When allowed for inaccuracies of manufacturing of friction pair inclined surface the value of the constraint reaction is smaller than when it is not allowed for.

Let us get down to considering the conditions of frictional wedge 3 equilibrium (Fig. 2 d)

According to the axiom of action and reaction law frictional wedge 3 experiences: normal $N_{13} = -N_3$ and tangent $F_{\tau 13} = -F_{\tau 3}$ components of truck bolster reaction $R_{13} = -R_3$; normal N_5 and tangent $F_{\tau 5}$ component of reaction of frictional plank 5 R_5 as well as resultant of spring sets 8 reaction F_8 . We should take into account that in Fig 2 d: $\gamma_2 \geq \frac{\pi}{2}$ is tilt angle of rear plane $ABED$ of frictional wedge 3 contacting with frictional plank 5, $\text{rad}(\gamma_2 = 92^\circ + 1^\circ)$.

Let us set up force equilibrium equation for frictional wedge 3 by analogy to frictional wedge 2:

$$N_{13} \cos(\alpha_2 + \frac{3}{2} \pi) + F_{\tau 13} \cos(\alpha_2 + \pi) +$$

$$+ N_5 \cos(\gamma_0) \cos(\beta_2 + \frac{\pi}{2}) + F_{\tau 5} \cos(\gamma_0) \cos(\beta_2) = 0. \quad (16)$$

Taking into consideration correlations (3) from expression (16) and with a glance to (10) after all transformations we will find normal component of frictional wedge 4 reaction

$$N_5 = \frac{1}{\sin(\alpha_1) \cos(\alpha_2) + \sin(\alpha_2) f_2 \sin(\alpha_1) + f_1 \cos(\alpha_1) \cos(\alpha_2) + f_1 \cos(\alpha_1) f_2 \sin(\alpha_2) + \sin(\alpha_2) \cos(\alpha_1) (-1) + \sin(\alpha_2) f_1 \sin(\alpha_1) + f_2 \cos(\alpha_1) \cos(\alpha_2) - f_2 \cos(\alpha_2) f_1 \sin(\alpha_1)} \times$$

$$\times \left[\frac{(Q_C - F_6)(\sin(\alpha_1) + f_1 \cos(\alpha_1)) - (-F_{1x}(\cos(\alpha_1) - f_1 \sin(\alpha_1)))}{\cos(\gamma_0)(\sin(\beta_2) - f_4 \cos(\beta_2))} \right]. \quad (17)$$

The character of constraint reaction (frictional plank 5) changing according to tilt angle variation (α_2) of inclined planes of friction pair “truck bolster – frictional plank” under $\alpha_1 = \text{const}$ without and with allowing for inaccuracies of their manufacturing in accordance with formula (17) is presented in Fig. 5a, b.

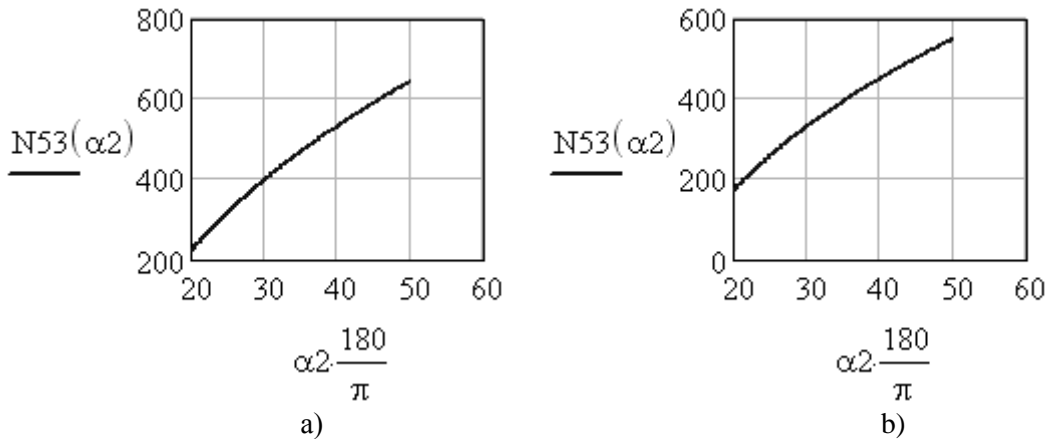


Fig. 5. Character of changing constraint reaction $N_{53}(\alpha_2)$: a) and b) without and with allowing for inaccuracies of manufacturing friction pair inclined surfaces

Рис. 5. Характер изменения реакции связей $N_{53}(\alpha_2)$: а) и б) – без учёта и с учётом погрешностей изготовления наклонных поверхностей пар трения

The analysis of dependences $N_{53}(\alpha_2)$ shows that with increasing angle α_2 constraint reaction according to modulus increases while with its decreasing the picture is the opposite. When allowed for inaccuracies of manufacturing friction pair inclined surfaces the constraint reaction value is smaller than when not allowed for.

When comparing values $N_{42}(\alpha_1)$ and $N_{53}(\alpha_2)$ it is necessary to note that $N_{42}(\alpha_1) \ll N_{53}(\alpha_2)$ due to the action of lengthwise force of the adjacent wagon F_{1x} . Caused by this fact premature wear of friction pairs “frictional wedge 3 – frictional plank 4” as compared to friction pair “frictional wedge 2 – frictional plank” is only natural.

3. SUMMARY

Summing up the results of performed analytical and numerical investigations it is necessary to note that the developed computed and constructed mathematical model of force action of friction pair “truck bolster – frictional wedge” and “frictional wedge – frictional plank” with allowing for the segment of lengthwise force acting upon truck bolster enabled us to obtain analytical formulas for determining constraint reaction according to friction pair geometrical parameters which makes it possible to

find their rational values affecting their operation effectiveness. There has been analytically proved premature wear of friction pairs “frictional wedge 3 – frictional plank 5” as compared to friction pair “frictional wedge 2 – frictional plank 4”.

References

1. Добычин И. А.: *Основы теории механического удара с приложением к задачам железнодорожного транспорта: учеб. пособие*. Изд-во Уральского Университета, Екатеринбург 1994.
2. Глушко М. И., Антропов А. Н.: *Работа пружинно-фрикционного комплекта тележки грузового вагона*. Вестник ВНИИЖТ, № 5, 2004, с. 41 – 44.
3. Туранов Х. Т., Бондаренко А. Н.: *Теоретическая механика в задачах погрузки-выгрузки и перевозки грузов в вагонах*. УрГУПС, Екатеринбург 2006.
4. Орлова А. М.: *Влияние конструктивных схем и параметров ножек грузовых вагонов на их устойчивость, ходовые качества и нагружённость*. Дисс. на соис. уч. степени д-ра техн. наук. ПГУПС, Санкт–Петербург 2007.
5. Туранов Х. Т., Черепов О. В.: *Обобщённая модель пружинно-фрикционного комплекта тележки грузового вагона*. Транспорт: Наука, техника и управление, 2009, с. 32 – 36.
6. Туранов Х. Т., Черепов О. В.: *Численное моделирование пружинно-фрикционного комплекта тележки грузового вагона*. Транспорт: Наука, техника и управление, № 1, 2010, с. 10 – 13.
7. Черепов О. В.: *Методология оценки технического состояния ходовых частей грузового вагона, как устройства преобразования сил*. Транспорт Урала, 2009, № 6, с. 61 – 64.
8. Бронштейн И. Н., Семендяев К. А.: *Справочник по математике для инженеров и учащихся втузов*. Наука, Москва 1964.
9. Кирьянов Д.: *Самоучитель MathCAD 13*. БХБ-Петербург, Санкт–Петербург, 2006.

Received 02.03.2010; accepted in revised form 20.03.2011