discriminant analysis, wheel pair, wheel-axle assembly

Sergey KROTOV*, Viktor KROTOV
Rostov State Transport University
sq. Rostovskogo Strelkovogo Polka Narodnogo Opolcheniya 2, Rostov-on-Don, 344038, Russia
**Corresponding author.* E-mail: svk-19587@yandex.ru

APPLICATION OF THE DISCRIMINANT ANALYSIS AT RESEARCH OF BEARING ABILITY OF THE WHEEL PAIR OF THE CAR

Summary. In this article the opportunity is given to the proof of application the discriminant analysis of bearing ability of the wheel pair of the car. In particular, we offer to classify parameter of bearing ability of the wheel-axle assembly of the car. This parameter is the area of sliding zones of the connection elements, using the calculation results of the deflected mode of the wheel-axle assembly. The method allows to use mathematical models for studying the parameters of the wheel-axle assembly reliability of the wheel with an axis of the wheel pair of the car.

ПРИМЕНЕНИЕ ДИСКРИМИНАНТНОГО АНАЛИЗА ПРИ ИССЛЕДОВАНИИ НЕСУЩЕЙ СПОСОБНОСТИ КОЛЕСНОЙ ПАРЫ ВАГОНА

Аннотация. В статье обосновывается возможность применения дискриминантного анализа к исследованию несущей способности колесной пары грузового вагона. В частности, предлагается классифицировать параметр несущей способности прессового соединения колесной пары – площадь зон скольжения элементов соединения, используя результаты расчета напряженно-деформированного состояния прессового соединения. Методика позволяет использовать математические модели для исследования показателей надежности прессового соединения колеса с осью колесной пары вагона.

The modern development of transport and the effective assurance of transportations by increasing the speed motion and weight of trains, along with the objective tendency of increasing of normative loading of the exploited cars, with the simultaneous increasing of axial load, is predetermined by increasing the operational loading of the rolling stock and wagon construction. The increase of stress loading causes the necessity in ensuring reliability of the wheel pair, as the major bearing element for the wagon's construction providing safety and stability of motion, and also the safety of transported cargoes.

This requirement concerns also to a bearing ability of a wheel connection with an axis and of its reliability depends the integrity of the wheel pair as the basic element of a wagon's construction.

The scientific and practical ability of developing the calculation mode much more surpasses expensive and long full-scale tests. These models are responsible for possible increasing of the axial loadings on condition of durability of a wheel connection. This allows forecasting the exhaustion of bearing ability of their connection. The traditional theoretical calculation of the wheel pairs' durability includes three basic stages: the definition of the operating forces and the choice of the estimated model, press and deformation calculation and the estimation durability by relevant criterion. Introduction of numerical methods gives such possibility [1, 2].

The important place in studying the stress condition the elements of the wheel pair of the car and a rolling stock of railway transportation takes the contacting conditions interfaced with a tightness of a wheel and axle, defining the durability of connection.

There are some calculation complexes for such problems allowing automatically during the decision to define the seizes of the surfaced contact details, the pressure distribution in the zone contact, in the dense coupling area or in the mutual motion area (slipping) of the contacted points, for example, RSFem [3]. This calculated complex allowed to receive all the listed parameters in contact [4]. On fig. 1 you can see the distribution of the sliding zones in press-fit connection of a wheel and an axis, the modern wheel pair of the Russian railways at a vertical axle loading 250 kN, horizontal 60 kN.

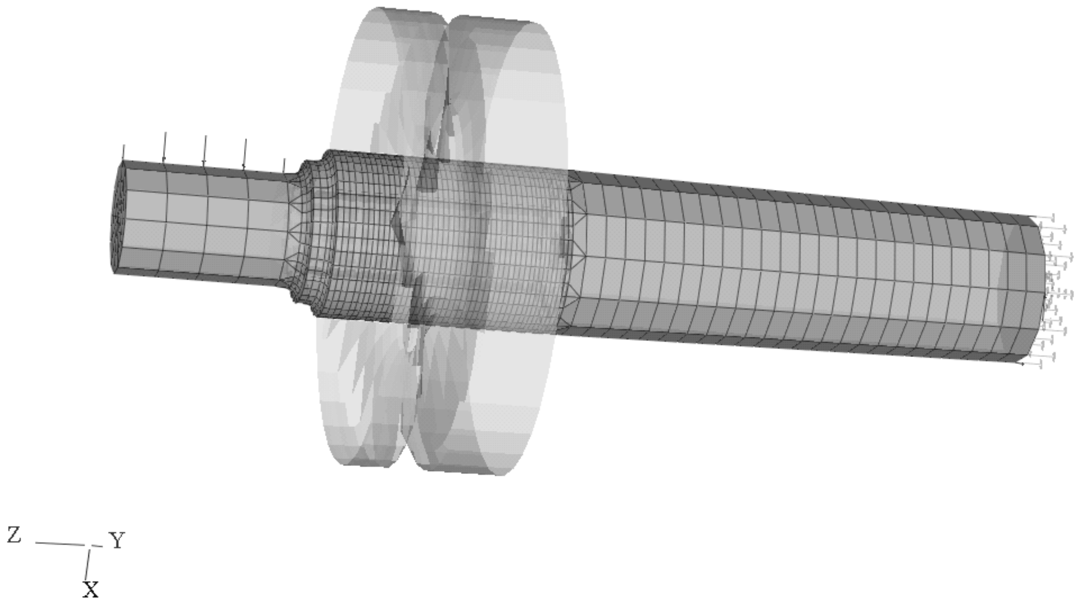

Fig. 1. Diagram of sliding zones in the wheel-axle assembly (shown half of axle)
Рис. 1. Схема зон скольжения в соединении колесо – ось (показана половина оси)

The estimation influence of loading factors, geometrical parameters of a wheel pair, physico - mathematical properties of a wheel and an axis, contact in a the wheel-axle assembly is represented as difficult for the following reasons. The wheel pair in working conditions tests a constant reversed bending, in the diagram of sliding zones in the wheel and axle assembly take place, the zones of mutual sliding and coupling of elements in connection, its size defines bearing ability and finally durability and reliability of the connection. The area size of sliding and coupling is not due to be measured, especially in dynamic loading conditions but can be measured some parameters of the loading of a wheel pair and its the wheel-axle assembly considering the results of calculation of the wheel-axle assembly of the wheel pair (3,4) by means of the discriminant analysis it is possible to classify the degree of the connection's reliability.

The main task of the discriminant analysis is to find principle that allows to attribute certain element to one of k groups $W_1, W_2, \dots, W_k$ on the basis of measurement (calculation) p of parameters $x_1, \dots, x_p$. The procedure of classification for a case p of continuous variables assumes that supervisions belong to one of the populations having multivariate normal distributions. The question is to which of these populations to carry new values of the investigated parameter proceeding from having the information, the question rests against selection of such linear combination of variables that optimum would divide groups and named by discriminant functions

$$z = \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_p x_p ,$$

where $\alpha_1, \ldots, \alpha_p$ - some constants minimizing probability of erroneous classification, it is possible to choose and show that procedure of classification is successful enough [5]. If the result x gets in W_1 the size z has normal distribution with an average $\xi_1 = \sum_{j=1}^{p} \alpha_j \mu_{1j}$ and a dispersion $\sigma_z^2 = \sum_{i=1}^{p} \sum_{j=1}^{p} \alpha_i \sigma_{ij} \alpha_j$. Similarly for *x*, carried to W_2, the size *z* has normal distribution with an average $\xi_2 = \sum_{j=1}^{p} \alpha_j \mu_{2j}$ and with the same dispersion σ_z^2. It makes sense to choose such $\alpha_1, \ldots, \alpha_p$ that ξ_1 and ξ_2 would be as it is possible removed from each other relatively further to σ_z^2. For this purpose takes the Mahalanobis distance $\Delta^2 = \frac{(\xi_1 - \xi_2)^2}{\sigma_z^2}$, the size measuring distances between populations. Thus, $\alpha_1, \ldots, \alpha_p$ should maximize Δ^2. After the reception α_i to each object $x_1, \ldots, x_p$ the value of discriminant function [5] is put in conformity.

At calculation of bearing ability of the wheel pair of the car, its wheel-axle assembly in particular, one of problems of the discriminant analysis can become the following. After carrying out of the full factorial experiment including the influence of six variables, varied at different levels, sizes of zones of sliding and coupling in connection for 96 variants of loading [4] are certain for the first time. The zones size of sliding in the wheel-axle assembly shares for example on three subgroups: less than 20% - a subgroup with a high degree of reliability of connection, from 20% up to 40% - a subgroup with average reliability, over 40% (in results of calculation and more than 60%) - a subgroup with a low degree of reliability. Further methods of the discriminant analysis define such linear combination of variables that leads to this or that group of reliability in our terminology. Actually we get formulas for definition of the zones size of sliding and due to it, it is possible with a certain degree of accuracy as a result for example, measuring actions on the basis of the diagnostic equipment applied in modern conditions of operation - for tensometring, definitions of temperature of a wheel, deterioration of a wheel, etc. not breaking integrity of connection to answer the question concerning the reliability of connection.

Performs the following steps: select grouping variable, in this case, the variable «zone of sliding» (designation ZSKOL), and also mutually independent six variables which levels of a variation do not influence against each other: vertical loading on a wheel pair (designation VERT), horizontal reaction of a rail - cross-section loading (POPER), enclosed to a rim of a wheel, temperature influence owing to interaction of a wheel and a braking clamp (TORM), the twisting moment owing to non-uniform pressing braking clamps to wheels of a wheel pair (MOKR), eccentricity appendices of vertical reaction of a rail in relation to the central cross-section section of the wheel-axle assembly (EXCENT), size of diameter of a wheel rolling circle, set by means of parameter – rim thickness of a wheel (TOLOBOD) [4].

The variable « a zone of sliding » is coded. If the value of sliding zones throughout the area of contact in connecting the wheel and axle below 20%, to this group of values are appropriated an index «*b*»; if the size of sliding zones from 20% to 40%, to this group of values are appropriated an index «*c*»; if the size of sliding zones above 40% to this group of values are appropriated an index «*d*». Indexes are coded by a computer like this: «*b*» =100, "*c*" =101, «*d*» =102 [6].

It is possible to apply the step by step discriminant analysis with the purpose to get the classification model, using thus statistics of inclusion or exclusion Fisher's - table 1. So for the set initial conditions such variables as vertical loading, a rim thickness of a wheel or temperature influence can be excluded from model, but from a physical viewpoint it is necessary to leave them. It is known [7], that the appearing of sliders due to braking and wheel gripping can overload a wheel pair on 40-45 kN, but it is known cases of fixation of size of loading 79 kN [8]. The same concerns also to others variables. Wilks' Lamda magnitude lays within the limits of [0,1], and in this case it is possible to tell that capacity of discrimination is equal to 50%. The tolerance resulted in the table represents 1 minus a square of plural correlation of the given variable with other variables in the model. In this case all variables are included in the model as they have high tolerance.

Table 1

Summary: variables in the model

N=96	Discriminant Function Analysis Summary (bcd) No. of vars in model: 6; Grouping: ZSKOL (3 grps) Wilks' Lambda: ,50074 approx. F (12,176)=6,0598 p< 0,0000					
	Wilks' Lambda	Partial Lambda	F-remove (2,88)	p-level	Toler.	1-Toler. (R-Sqr.)
VERT	0,514790	0,972707	1,23461	0,295939	0,988554	0,011446
POPER	0,670756	0,746530	14,93934	0,000003	0,927421	0,072579
MOKR	0,572940	0,873982	6,34428	0,002668	0,984331	0,015669
TORM	0,522981	0,957472	1,95435	0,147757	0,990569	0,009431
EXCENT	0,639658	0,782825	12,20672	0,000021	0,929574	0,070426
TOLOBOD	0,531607	0,941935	2,71235	0,071932	0,976871	0,023129

For checking the hypothesis $H_0 : \Delta^2 = 0$, that means the equality of averages in the groups: H_0 : $\mu_1 = \mu_2 = \mu_3$; value of the F - statistics: $F(12{,}179) = 6{,}058$, that is significant with $p < 0{,}001$.

Table 2 shows the classifying functions for a three groups of areas of sliding zones. The resulted classification functions represent formulas and it becomes possible to calculate classification attributes for new cases.

Table 2

Classification functions for grouping variable ZSKOL

Variable	Classification Functions		
	b p=,30208	c p=,54167	d p=,15625
VERT	1,06931	1,09506	1,11213
POPER	0,10991	0,15195	0,18266
MOKR	0,00143	0,00032	0,00104
TORM	0,01098	0,00933	-0,00023
EXCENT	0,80766	0,25341	0,03102
TOLOBOD	0,62528	0,41948	0,24897
Constant	-132,52542	-136,16165	-142,97967

Thus the discriminant function describes a combination of variables that leads to the occurrence of sliding zones size below 20% of the contact area of a wheel and an axle in the wheel-axle assembly, it looks as follows:

$$b(100) = -132{,}52542 + 1{,}06931 \cdot VERT + 0{,}10991 \cdot POPER + 0{,}00143 \cdot MOKR +$$
$$+ 0{,}01098 \cdot TORM + 0{,}80766 \cdot EXCENT + 0{,}62528 \cdot TOLOBOD$$

When zones of sliding value from 20% up to 40% of the contact area of a wheel and an axle in wheel-axle assembly the discriminant function has the following form:

$$c(101) = -136{,}16165 + 1{,}09506 \cdot VERT + 0{,}15195 \cdot POPER + 0{,}00032 \cdot MOKR +$$
$$+ 0{,}00933 \cdot TORM + 0{,}25341 \cdot EXCENT + 0{,}41948 \cdot TOLOBOD$$

If sliding zones size above 40% of the contact area of a wheel and an axle in the wheel-axle assembly the discriminant function looks like:

$$d(102) = -142{,}97967 + 1{,}11213 \cdot VERT + 0{,}18266 \cdot POPER + 0{,}00104 \cdot MOKR +$$
$$- 0{,}00023 \cdot TORM + 0{,}03102 \cdot EXCENT + 0{,}24897 \cdot TOLOBOD$$

To estimate the carrying capacity of press connect the wheel and axle for the new loading conditions, it is necessary to substitute the variables value in formulas and to calculate classification values of the sliding zones in codes and to carry them to matchable group - «*b*», "with", «*d*». The size of the sliding zones will belong to such class to which classification value is maximal.

Let's define probability of an accessory of the counted area of the sliding zone to various groups.

Table 4

Posterior probabilities for each case

Case	Posterior Probabilities (bcd) Incorrect classifications are marked with *			
	Observed Classif.	b p=,30208	c p=,54167	d p=,15625
*	b	0,182444	0,782460	0,035097
*	b	0,070108	0,841369	0,088523
	b	0,756453	0,239766	0,003781
	b	0,520901	0,462007	0,017092
	c	0,228621	0,763327	0,008052
	c	0,094571	0,883566	0,021863
	b	0,801493	0,197773	0,000734

Each considered case concerns to group with maximal posteriori probability. Tables 3 and 4 are resulted fragmentary - full tables contain 96 classifications.

So for possible loading conditions VERT=300 kN/axle, POPER=160 kN, TOLOBOD=2,2 sm, MOKR=0, EXCENT=2,8 sm, TEMP=39,44 kW (a thermal stream at long braking action - [9]) Mahalanobis distance from new supervision up to the center of groups

Supervision	***b***	***c***	***d***
---	**53,67793**	**37,27156**	**31,03219**

Shows the minimal size for group "*d*" Posteriori probabilities have turned out equal

Supervision	***b***	***c***	***d***
---	**0,000020**	**0,132790**	**0,867190**

New supervision with probability 0,867 concerns to the group «*d*». Under these conditions of the wheel-axle assembly of bearing ability will possess low reliability.

With the purpose to increase the discriminant level we can choose any weight variable that allows to set weights for various cases (proportionally to variables value). At definition for example a variable «TORM» as weight, Wilks's Lambda parameter becomes equal 0,37 at significant statistics $F(12,12208) = 649{,}14$ at $p < 0{,}0001$ and it is possible to speak about higher degree of discrimination.

As shown in fig. 2 the calculation results division of the wheel-axle assembly in the form of percentage parities of the sliding zone area is allocated for two groups - with the sliding area up to 20% and more than 40% of the total contact area of a wheel and an axis. It is presented the populations division with a high degree of reliability connection «*b*» and with a low degree the wheel-axle assembly reliability «*d*».

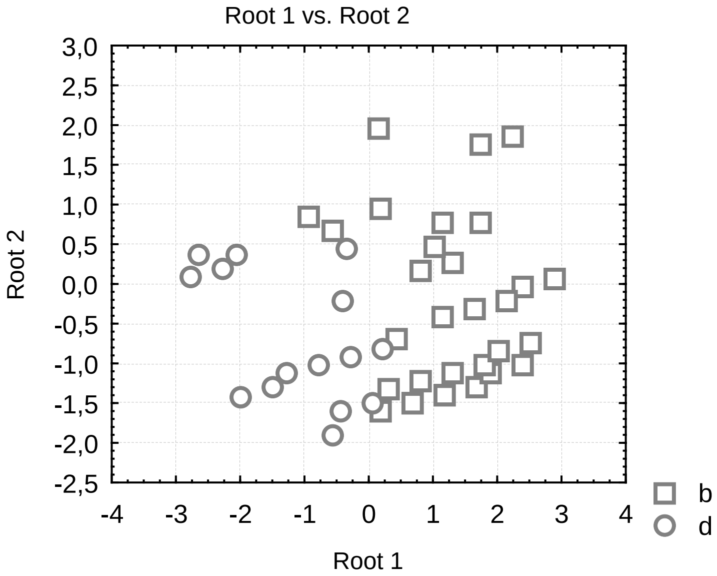

Fig. 2. Scattering diagram of the canonical values (the division of degrees of reliability)
Рис. 2. Диаграмма рассеяния канонических значений (разделение степеней надежности)

Parameters of the original variables can be measured with the motion of the wheel pair of the car. Using a database of calculating extrusion of a wheel and axle connection, as well as intervals of values classified area of the sliding zones we can estimate the reliability of the wheel-axle assembly with its operation.

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Received 6.01.2010; accepted in revised form 21.03.2011