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Laboratory investigations of dynamic properties of accelerometers with fractional orders for application in telematic equipment

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ABSTRACT

The paper presents laboratory studies on measuring accelerometers, which were modelled in the classical differential equations, as well as the fractional calculus. Measurement errors were examined and the classical and fractional models in terms of dynamic properties were compared. The advantages of fractional calculus in modelling dynamic elements were also indicated.

KEYWORDS: fractional calculus, measuring transducer, measurement errors

1. Introduction

The recent dynamic development of the research into the use of fractional calculus for the analysis of dynamic systems encouraged the author to attempt its use for the analysis and modelling of transducers and measurement systems. The differential equation describing an absolute movement of the transducer's seismic mass [4], [6], [7], [8] takes the form:

$$\frac{d^2}{dt^2}y(t) + 2\zeta\omega_0\frac{d}{dt}y(t) + \omega_0^2 y(t) = 2\zeta\omega_0\frac{d}{dt}x(t) + \omega_0^2 x(t) \qquad (1)$$

A relative shift of the seismic mass is introduced in equation (1):

$$w(t)=y(t) - x(t) \qquad (2)$$

changes it into:

$$\frac{d^2}{dt^2}w(t) + 2\zeta\omega_0\frac{d}{dt}w(t) + \omega_0^2 w(t) = -\frac{d^2}{dt^2}x(t) \qquad (3)$$

Taking into consideration the assumption that the dynamic behavior of the element responsible for damping is better described by the fractional derivative, equation (3) is written down as:

$$\frac{d^2}{dt^2}w(t) + 2\zeta\omega_0\frac{d^{(\nu_1)}}{dt^{(\nu_1)}}w(t) + \omega_0^2 w(t) = -\frac{d^2}{dt^2}x(t) \qquad (4)$$

Generalizing equation (4) in view of the fact that integer order derivatives in the integral equation derivative are a special case of non-integer derivatives, we can write down:

$$A_2\frac{d^{(\nu_2)}}{dt^{(\nu_2)}}y(t) + A_1\frac{d^{(\nu_1)}}{dt^{(\nu_1)}}y(t) + A_0\frac{d^{(\nu_0)}}{dt^{(\nu_0)}}y(t) = B_2\frac{d^{(\mu_2)}}{dt^{(\mu_2)}}x(t) + B_1\frac{d^{(\mu_1)}}{dt^{(\mu_1)}}x(t) + B_0\frac{d^{(\mu_0)}}{dt^{(\mu_0)}}x(t) \qquad (5)$$

2. Identification of transducer dynamics

In order to identify sensor dynamics, a measurement system was constructed (Figure 1).

The DelataTron accelerometer, Type4507, manufactured by the Bruel&Kjaer company, characterized by sensitivity of 10.18 mV/ms^{-2} was examined. The sensor was placed on the electrodynamic inductor.

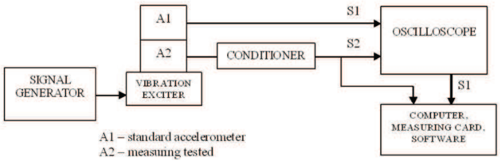

Fig.1. Block diagram of the laboratory measurement system for examining accelerometers

A model accelerometer produced by VEB Metra, type KB 12, sensitivity of 317 mV/ms^{-2} was aligned in one axis with the examined sensor. The input signal was the vibrations of the inductor plate actuated by a sinusoidal signal from the generator. The model signal was the one from the KB 12 sensor, whereas the signal examined was the signal from the 4507 sensor.

The main objective of the study was identification of the mathematical model of the 4507 sensor on the basis of signals received from the sensors. The identification method applied here was ARX [1], [2], [9] – the examined signal was compared with the model signal and on the basis of the comparison discreet transmittance of the examined sensor was determined.

Signals were collected at a sampling frequency of 10 000 Hz each with the use of the measurement card. The sampling time used in the ARX method was 0.0001 s. The voltage-source signals were examined. Then they were converted/translated into acceleration. Identification was accomplished with the use of the MATLAB&Simulink package [10] (Fig. 2.).

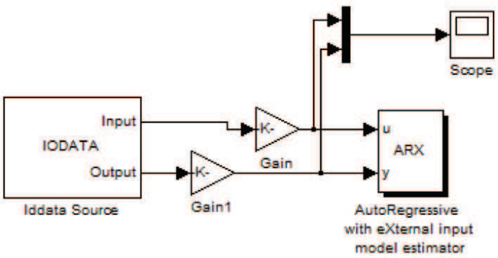

Fig.2. Measurement system for the examined sensor identification

As a result of the ARX identification method, the examined sensor transmittance looks as follows:

$$G(z) = \frac{0{,}79196z^2 + 0{,}51435z}{z^2 - 0{,}6439z + 0{,}048034} \qquad (6)$$

Figure 3 depicts frequency characteristics of the sensor's amplitude and phase

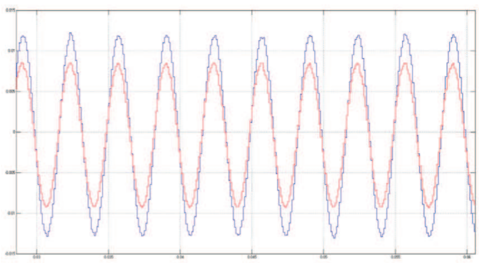

Fig.3. Amplitude and phase characteristics of the sensor model

Figure 4 shows the signals entering the system identification block. The model signal amplitude differs from the amplitude of the identified signal.

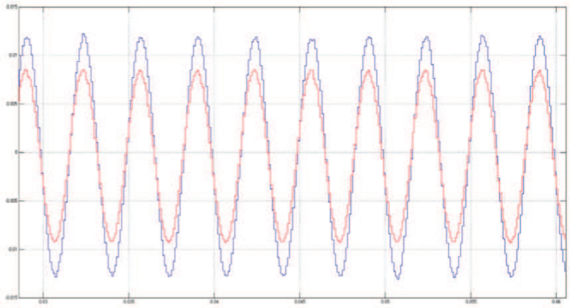

Fig.4. Signals entering the identification block: blue – model signal, red – identified signal

As a result of the ARX identification method, the identified signal and the signal characteristics in the model have the same amplitude and there is no phase shift between these signals (Figure 5.)

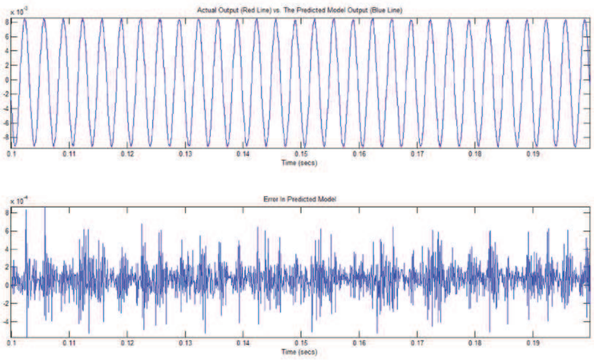

Fig.5. ARX block functioning: top: the characteristics being identified (red) and the characteristics from the model (blue); bottom: error characteristics during identification

In order to compare characteristics from the model sensor, examined sensor and the examined sensor model in the MATLAB&Simulink environment [10], the system presented in figure 6 was built.

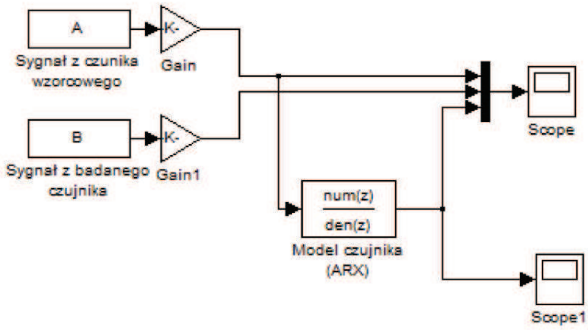

Fig.6. The system comparing characteristics from the model sensor, examined sensor and the obtained model of the examined sensor

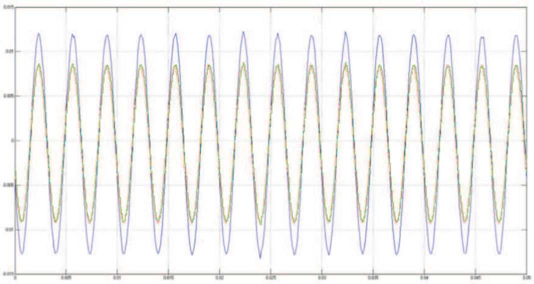

Fig.7. Comparison of signals: blue – from the model sensor; red – from the examined sensor; green - from the examined sensor model. Axes: X – time, Y – acceleration

The comparison of the characteristics indicates that signals from the examined sensor and the examined sensor model have identical amplitudes.

3. Measurement error analysis

As a result of conducted measurements (section 1) high values of error peaks were observed. They are a consequence of the determination of error for the characteristics of variables over time, which change their values from positive to negative. Sensitivity of the examined sensor is much lower that that of the model sensor – thus we deal with the cases when the model sensor displays the acceleration value close to zero, whereas the examined sensor, due to its low sensitivity, indicates zero. Hence peaks in the characteristics of errors. The lowest error value is reached at values close to the amplitude, the highest – at those close to 0.

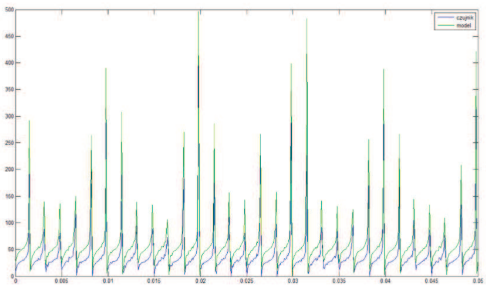

Fig.8. Error characteristics: X – time, Y – relative error (%)

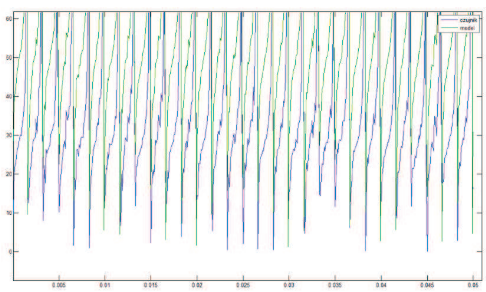

Fig.9. Error characteristics in the changed Y scale

It was assumed that the measure of accuracy of the dynamic characteristics reproduction by the examined sensor and the model of this sensor shall be the median relative error. In the examined case (the sinusoidal characteristics of 300 Hz in frequency), the median for the error characteristics is:

- for the sensor's relative error: 29.1945%
- for the model's relative error: 29.5564%

It can be concluded that the sensor's model reproduces the model signal with the relative error larger by 0.3619% than the sensor's error. This value occurs at examining the characteristics of the same frequency as in the case of the examined sensor identification. When the frequency of the examined characteristics is different from that at identification, then the error values will be higher. The relative error values for the sensor and its model for different frequencies are shown in Table 1.

Table 1. Relative error values for the sensor and its model

Frequency [Hz]	Sensor's relative error [%]	Model's relative error [%]
100	45.2213	30.8089
200	22.9227	30.2997
300	29.1945	29.5564
400	70.60.78	28.3097
500	90.5626	26.0184

The bigger the difference between frequencies of the examined characteristics and the characteristics at which identification was accomplished, then the bigger the difference between the median relative error of the sensor and of the model is.

4. Comparison of the integer and fractional order models of the accelerator

In order to check whether the model based on the fractional order equation describing the dynamic behavior of the object reproduces the model signal better than the "classical" model, on the basis of the sensor transmittance model (6) determined by the ARX method, a group of models was determined by means of fractional order equations. Our investigations started from one ν_2 fractional order responsible for damping. The order of the ν_2 derivative changes the range of values from 0.94 to 2.08 by a 0.02 step.

Frequency characteristics of the models' amplitude and phase are depicted in figures 10 and 11.

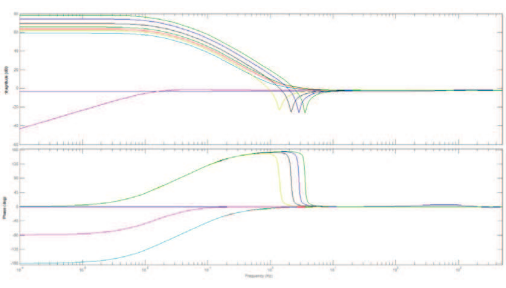

Fig.10. Amplitude and phase characteristics for different ν_2

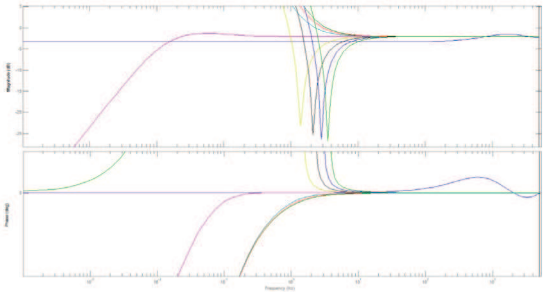

Fig.11. Amplitude and phase characteristics for different ν_2 in a changed Y scale

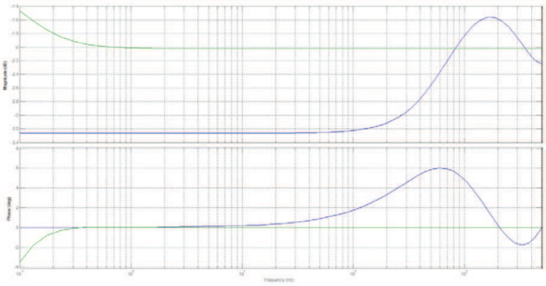

Fig.12. Amplitude and phase characteristics for the fractional notation $\nu_2 = 2$ – green and for the classical notation - blue

Table 2. Median relative error values for the fractional order model

Frequency [Hz]	Median relative error for the fractional order model [%]	The difference by which the median relative error value for the fractional order model decreased in relation to the classical model [%]
100	20.8040	10.0049
200	20.8041	9.4956
300	20.8042	8.7522
400	20.8039	7.5058
500	20.8040	5.2144

Similar investigations were carried out for the ν_1 fractional order.

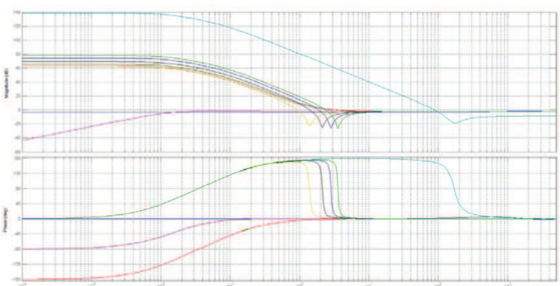

Fig.13. Amplitude and phase characteristics for changed ν_1

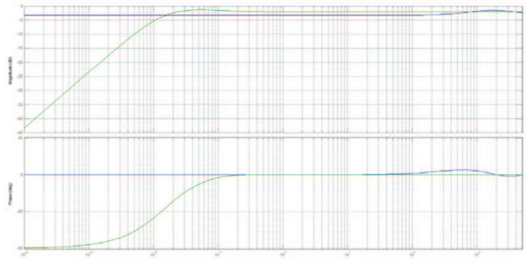

Fig.14. Amplitude and phase characteristics for $\nu_1 = 1$ (green) and the "classical" notation

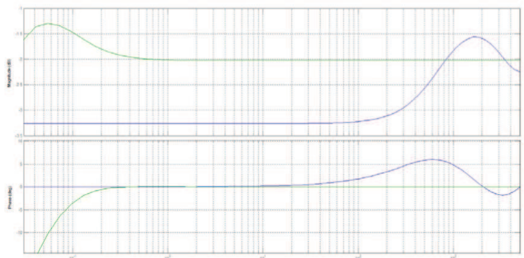

Fig.15. Amplitude and phase characteristics for $\nu_1 = 1$ (green) and "classical" notation – enlarged scale in figure 14

Table 3. Median relative error values for the fractional order model

Frequency [Hz]	Median relative error for the fractional order model [%]	The difference by which the median relative error value decreased with regard to the "classical" model [%]
100	20.8040	24.4173
200	20.8041	2.1186
300	20.8042	8.3903
400	20.8039	49.8039
500	20.8040	69.7586

In summary, we can conclude that out of the group of characteristics of the ν_2 fractional order the closest to the ideal one with reinforcement and phase shift equal 0 is the characteristic for the order equal 1. Due to the way of transmittance determination of fractional coefficients describing the sensor's dynamic behavior, the amplitude and phase characteristics differ from the same characteristics determined for the "classical" notation of dynamic behavior (transmittance is different).

On the basis of amplitude and phase characteristics of the sensor's model obtained by the ARX method and the sensor's model determined by the "fractional order method" it can be concluded that the fractional order model reproduces the sensor's dynamic behavior far more accurately:

- amplitude and phase characteristics are closer to the linear characteristic in a larger scope of signal processing;
- in the case of frequency above 1 Hz, amplitude and phase frequencies are almost linear: magnitude is within the boundaries from -2.02 to -2.03 dB, and the phase shift is within the range from 0.04^0 to 10^{-6}. In the case of the "classical"

model of the transducer model, one cannot think about such great linearity.

Determination of the median relative error for the examined characteristics, let us claim that:

- median relative error in the case the fractional order model's response is examined is constant up to the third decimal place (this is confirmed by linearity of earlier obtained Bode's plot of frequency characteristics);
- in each examined case there is an advantage when the sensor's fractional model is used, the more so, the higher the difference between the frequency at which the "classical" model was determined (300 Hz) and the frequency of the examined characteristics. For the cases examined, the percentage decrease in the median error ranges from 5.2144 to 10.0049 %.

5. Examination of the accelerometer models of ν_1 and ν_2 fractional orders

Bode's plot of frequency characteristics for the ν_1 and ν_2 order combinations was examined (Table 4).

Table 4. Order combinations of orders in equation (5)

ν_1	0.94	0.96	0.98	1	1.02	1.04	1.06	1.08
ν_2	1.94	1.96	1.98	2	2.02	2.04	2.06	2.08

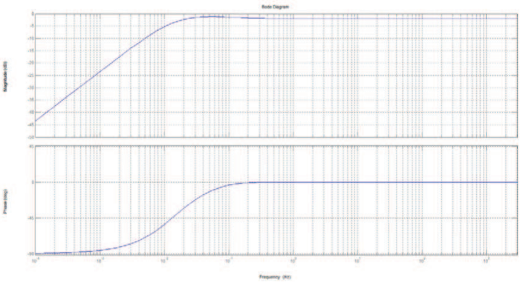

Fig.16. Bode's characteristics for $\nu_1 = 1$ and $\nu_2 = 2$ determined by the method for fractional orders

Linearity starts from ca. 0.5 Hz at magnitude equal -2.02 dB and the maximum phase shift equal 0.05^0 (the peak in the next figure). The phase shift for the frequency of 1 Hz equals 0.005^0.

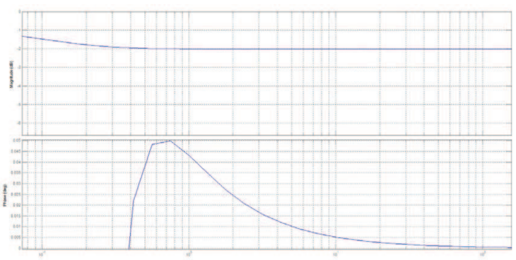

Fig.17. Characteristics from figure 14 in enlarged scale

It is worth noticing that Bode's characteristics in the case of fractional ν_1 only and ν_1 and ν_2 are of a different shape when it comes to low frequencies. Above 1 Hz it is practically of no importance whether it is only ν_1 which is non-linear, or ν_1 and ν_2. Frequency characteristics are practically identical. Thus, the use of non-linear ν_1 only has the same effect as using fractional (non-integer) ν_1 and ν_2. The very method of determining the sensor model's dynamic behavior affects accuracy of such a model processing. In the case of classical and fractional models for identical (integer) orders the observed processing accuracy is higher in the case of the "fractional" type model.

6. Conclusion

The use of the fractional calculus for describing characteristics of dynamic systems seems justified for the following reasons:

- Global research into numerous physical phenomena (description of properties of viscoelastic materials, liquid permeation through porous substances, electric load transfer through an actual insulator, heat transfer through a heat barrier, or descriptions of friction, [3], [11], [12], [14]), showed that fractional calculus describes this type of phenomena more accurately than classical mathematical analysis.
- Continuous physical phenomena of the real world should be described "intuitively" by means of differential equations of orders taken from the set of real numbers and not only, integer numbers, i.e. discrete. Classical integrals and integer order derivatives are only specific cases of the fractional calculus.
- The fact that in previous decades researchers from different areas of science did not use the fractional calculus is accounted for by the author by the lack of IT tools having great computing potential which in our times are widely accessible.

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