

# Applying ad hoc algorithms for highway traffic management

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## ABSTRACT

The number of cars participating in highway traffic is still growing as well as the necessity of quick and fluent transport between different locations. Therefore, ensuring the fluency and safety in highway traffic is becoming an essential problem in recent times. If only the driver knew about a danger waiting behind the bend, he would slow down; if someone got the information about a traffic jam 10 km earlier, he would probably take a different road or stop.

This paper aims at providing transport telematics solutions which ensure simple, fast and efficient means for highway traffic management. The ad hoc algorithms presented concern broadcasting within highway traffic and assume the collisions detection and a single hop network. Installing a simple transmitter and receiver on every car is considered. Energy efficient size approximation algorithms and a leader selection procedure are presented and discussed, including simulations.

**KEYWORDS:** transport telematics, leader election, single hop network, size approximation

## 1. Introduction

The ad hoc algorithms presented concern broadcasting in the highway traffic. The highway traffic is still growing. On the very crowded highways, especially in Germany and Italy, but recently also in other countries, tragic accidents take place quite often. Sometimes, even a dozen cars are involved in a crash. Main reasons for those situations include: bad weather conditions, speeding, dangerous driving, other unexpected situations like, for example, a traffic jam. In many cases, those accidents could have been omitted. If only the driver knew about a danger waiting behind the bend, he would slow down. If someone got the information about a traffic jam 10 km earlier, he would probably take a different road or stop for a lunch.

The consideration of those facts reveals that a very urgent solution is needed. One way to solve the problem is

to create an information system that will alert the drivers to the dangers of traffic jam or other road difficulties on time.

Nowadays, transport telematics is a very quickly developing branch [5,6]. It provides many useful algorithms and solutions within Intelligence Transport Systems [1,6]. However, a significant number of them are based on a video analysis. The camera takes a picture of the road and a statistical analysis is performed in order to estimate the number of cars within the photograph. It is a common knowledge that such a procedure may provide us with false estimates in the case of a bad weather or during night. Present solutions very often are based on many different parameters (like for example in [2]), which are difficult to estimate without significant bias. Such complex algorithms are very fragile. Systems implemented within highways are also quite inert – if an accident happens far from the detector gate, it will be discovered far too late to prevent a traffic jam.

## 2. Model

Therefore, a more flexible and quick solution is needed. In order to ensure this, communication between cars must be enabled.

A simple solution is to install on every car a small transmitter and receiver. Of course, these devices cannot be expensive. Therefore, they must be as simple as possible. Let us concern devices that have a small range (few hundred meters). Furthermore, let us assume that three cases may be distinguished during broadcasting: NULL (no broadcast), SINGLE (only one car broadcasts) and COLLISION (more than one car broadcast). It is a quite reasonable assumption, because the transmitter can be located on the front bumper of a car, whereas the receiver – on the rear. This will allow driver to check, whether he succeeded in sending the information or not. For a no collision detection case, see [4]. Furthermore, it can be stated that the highway is divided into small sections, such that different frequencies should be used within neighbouring sections. Therefore, only cars within the same sector can interfere. A reasonable length of such a sector is around 500 meters.

Firstly, the number of cars in a particular section should be estimated. The next step is to choose a leader in each sector that will be responsible for spreading the information to the next sector.

### 2.1. Car's quantity estimation

In this phase, the number of cars within a given section is estimated. Considering the sections are 500 meters long and assuming a traffic jam, the maximum number of cars is equal to 150 (3 lines of cars, one by one, 3 cars every 10 meters). Car's quantity estimation procedure will last 150 time slots.

Let us assume that time is divided into small intervals named time slots. For the purpose of this algorithm we may assume that there are a few dozen of time slots within one second. In each time slot each car sends some information (could be a single bite) with the probability equal to 1/150. Each car listens and counts the number of NULLs during first 150 time slots of the procedure. Based on that number, the car's quantity estimate is derived.

It is a common knowledge that the number of NULLs has the Bernoulli distribution:

$$\# NULL \sim B(N, p) \quad (1)$$

where N is the number of time slots and p is equal to 1 - sending probability. Maximum likelihood estimate for parameter p has the following form:

$$\hat{p} = \frac{\# NULL}{N}. \quad (2)$$

In our case,

$$p = \left(1 - \frac{1}{N}\right)^n, \quad (3)$$

where n represents the number of cars.

Therefore, one can estimate n from the following equation:

$$\left(1 - \frac{1}{N}\right)^n = \frac{\# NULL}{N}. \quad (4)$$

Transforming, we obtain:

$$\begin{aligned} \hat{n} &= \frac{1}{\log\left(1 - \frac{1}{N}\right)} \log\left(\frac{\# NULL}{N}\right) = \\ &= \frac{1}{\log\left(1 - \frac{1}{N}\right)} (\log\# NULL - \log N). \end{aligned} \quad (5)$$

This formula works if #NULL>0. If there is no NULL during the first phase, we take 150 as the value of the estimate (maximum number of cars that may appear within a given section). In order to ensure that the estimate value will be an integer we should round the obtained value to the nearest integer.

### 2.2. Estimate's bias

The characteristic function for a Bernoulli variable is given by the following formula:

$$\Psi_X(t) = \sum_{k=0}^{\infty} e^{itk} \binom{n}{k} p^k (1-p)^{n-k}. \quad (6)$$

Therefore, the characteristic function for the variable log#NULL is given by:

$$\Psi_{\log\#NULL}(t) = \sum_{k=0}^n e^{i \log k} \binom{n}{k} p^k (1-p)^{n-k}, \quad (7)$$

Therefore,

$$\Psi'_{\log\#NULL}(t) = \sum_{k=0}^n i \log(k) e^{i \log k} \binom{n}{k} p^k (1-p)^{n-k} \quad (8)$$

Based on this formula we may calculate the first moment for the variable log#NULL:

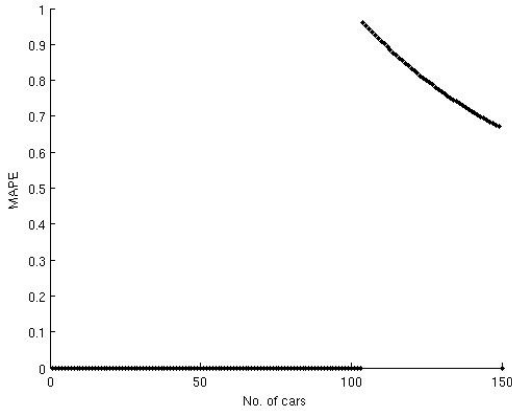


Fig. 1. Mean Absolute Percentage Error in the case of N=150  
Source: [own work]

$$E(\log\#NULL) = \frac{\Psi'_{\log\#NULL}(0)}{i} = \sum_{k=0}^n \log(k) \binom{n}{k} p^k (1-p)^{n-k} \quad (9)$$

Having that, a mean value of the car's quantity estimate may be calculated:

$$E(\hat{n}) = \left\{ \frac{E(\log\#NULL) - \log N}{\log(1 - \frac{1}{N})} \right\} = \left\{ \frac{\sum_{k=0}^n \log(k) \binom{n}{k} p^k (1-p)^{n-k} - \log N}{\log(1 - \frac{1}{N})} \right\} \quad (10)$$

In order to evaluate the proposed estimate its mean percentage error is calculated:

$$MAPE(\hat{n}) = \left| \frac{E(\hat{n}) - n}{n} \right| \cdot 100\% \quad (11)$$

MAPE for the proposed estimate is presented in Fig. 1.

As we can clearly see, the estimate's bias is equal 0 for  $n < 100$  and is less than 1% for  $100 < n < 150$ . As we know a detailed distribution for NULLs, we may construct confidence intervals for the car's quantity estimate:

$$\mathcal{C}_{\xi\%} = [E(\hat{n}) - n_{\xi.5\%}, E(\hat{n}) - n_{2.5\%}] \quad (12)$$

where,

$$\hat{n}_{\alpha\%} = \frac{\log b_{\alpha\%}(N, p) - \log N}{\log(1 - \frac{1}{N})}, \quad (13)$$

where:  $b_{\alpha\%}(N, p)$  denotes  $\alpha\%$  quantile from Bernoulli distribution  $B(N, p)$ . Confidence intervals are presented in Fig. 5.

It is needless to point out that allowing the collision detection is a crucial assumption. Let us compare expected number of SINGLES and NULLs within 150 time slots. As we have pointed out an early time is crucial. Therefore, let us decrease the length of the first phase from 150 to 75 time slots and again let us calculate the expected number of SINGLES and NULLs.

The key fact is that the expected number of NULLs remains a 1-1 function in the case of 75 time slots, whereas the expected number of SINGLES does not. This is shown

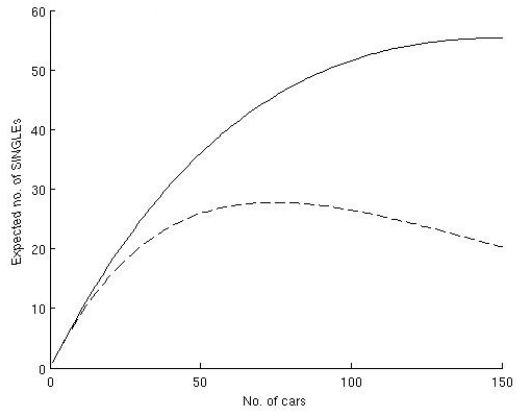


Fig. 2. Expected number of SINGLES in the case of N=150 (solid) and N=75 (dashed)  
Source: [own work]

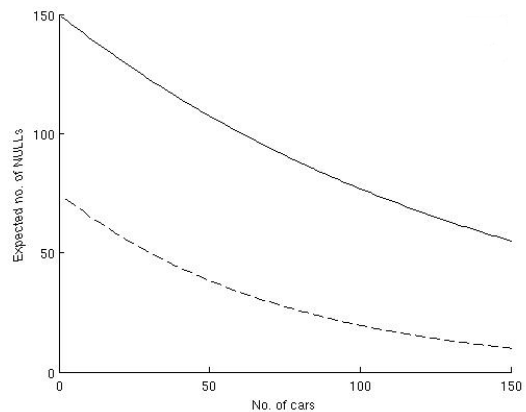


Fig. 3. Expected number of SINGLES in the case of N=150 (solid) and N=75 (dashed)  
Source: [own work]

in Fig. 2 and Fig. 3. Therefore, while counting NULLs we can still derive car's quantity estimates even in the case with 75 time slots although we lose a bit of accuracy, which is shown in Fig. 4.

In the case when  $N=75$ , the proposed estimate is unbiased for  $n < 50$ , but the bias for  $50 < n < 150$  does not exceed 2.5%. Comparing the length of confidence intervals, the ones for the case of  $N=75$  are slightly wider than in the case of  $N=150$ .

### 3. Leader selection

Having obtained the car's quantity estimate it is crucial to inform the incoming drivers how many cars there are in front of them. Therefore, a leader should be selected. We consider the leader selection algorithm working as follows. In the first round, every car within a given highway section sends a small information with the probability  $p=1/n$  (where  $n$  is equal to car's quantity estimates derived in the first phase). If there is a SINGLE case, the one who succeeded, becomes a leader. If the NULL appears, the round is repeated. In the case of COLLISION only those, who were sending in the first round enter the next phase. In this case, the second round participants take new probability constants equal to  $1/2$ . This round is repeated until the leader is chosen (in the case of COLLISION only those who were sending in a given round remain in the competition).

Once the leader is selected he can spread the information to the earlier sector.

### 4. Simulations

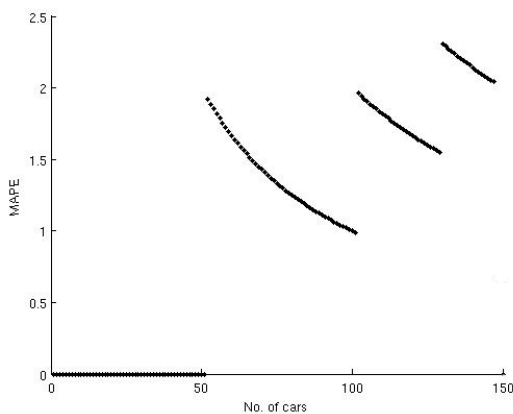


Fig. 4. Mean Absolute Percentage Error in the case of  $N=75$   
Source: [own work]

Table 1. Simulation results

	$\bar{S}$ (std)	$P(s \leq 2)$	$q_{95\%}$	$q_{99\%}$
$U_{dr}, N=150$	2.435 (1.695)	0.633	6	8
$U_{dr}, N=75$	2.442 (1.699)	0.630	6	8
$N_{dr}, N=150$	2.453 (1.701)	0.628	6	8
$N_{dr}, N=75$	2.453 (1.705)	0.627	6	8

Above-mentioned procedures were tested in the simulations. Two different car's quantity distributions were considered:

- Discrete uniform  $U_d \sim [1, 150]$ ,
- Discrete normal  $N_d \sim \{\max(\min(150, N(75, 15^2)), 1)\}$ .

Based on 1 000 000 simulations, a mean number of steps needed for a leader selection was calculated. Moreover, 95% and 99% quantiles were also obtained. The results, including 150 and 75 time slots scenarios, are presented in Table 1.

### 5. Conclusions

Simulations have revealed that the leader selection procedure performed very well, even if the car's quantity estimate is less accurate – the difference in mean time is around 0.01 time slot. Therefore, we must take into account that time is crucial and approximation algorithm based on 75 time slots should be used. Furthermore, the car's quantity distribution does not play a significant role according to the results, which means that the proposed algorithm will give similar results in different traffic conditions. The mean time for a leader selection is around 2.5 time slots and with the probability of 0.99 the leader will

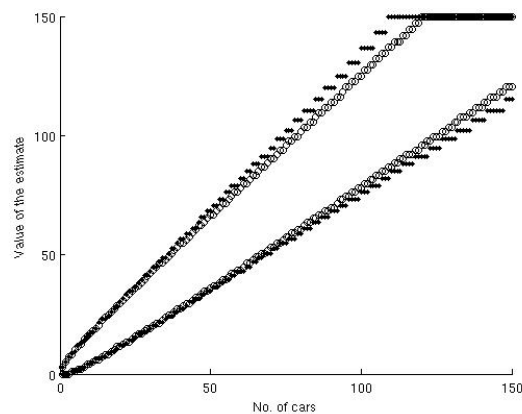


Fig. 5. 95% Confidence intervals for car's quantity estimate ( $N=150$  – circle,  $N=75$  – dotted)  
Source: [own work]

be chosen within 8 time slots. Therefore, we may assume that the whole procedure (calculating the car's quantity estimates, choosing the leader and spreading the information) lasts no longer than 85 time slots (75 time slots for the first phase, 8 time slots for the leader selection, 1 time slot for broadcasting, 1 time slot for receiving the information from the earlier sector) that is less than 2 seconds.

On the other hand, it is needless to point out that such algorithm will not give good estimates in a hostile environment that is in the case, when someone would cause interference and disable the communication. For adversary immune algorithm, please refer to [3].

## 6. Final Remarks

The proposed algorithm has many advantages when compared to the existing ones. First of all it enables instant communication between cars. In the case of an accident or other traffic difficulties the information may be spread very quickly (less than 2 seconds per 500 meters sector that is around 900 km/h) and incoming drivers will receive it on time and they will get the opportunity to react to the danger. The second main advantage of such a solution is that it may be easily used to identify particular cars. Every car has a unique transmitter and receiver. Therefore, there is no problem to recognize the car and collect the

highway fee or send a speeding ticket. Similar solutions are already implemented on highways in Austria where buses and trucks have a special transmitter attached to the windshield and every time the car is passing by a gate (there are gates every 20-30 km) an impulse is sent.

Such an ad hoc system may be connected to the GPS system and new routes may be calculated more efficiently based on the traffic information.

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