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Multi-sensor navigational measurements fusion

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ABSTRACT
The method of integrating navigational parameters obtained from non-simultaneous navigational measurements is presented. The proposed algorithm of position coordinates estimation is general and includes two modes of data processing – from simultaneous and non-simultaneous measurements. It can be used in hybrid receivers of radionavigation systems integrating non-homogeneous position lines or in integrated navigation systems, particularly in receivers combining the measurements of various satellite navigation systems.

KEYWORDS: navigational mathematics, GPS, sequential estimation, maritime navigation

1. Introduction

In navigation, position determination consists in the identification of coordinates in the adopted coordinate system and reference system. Position coordinates cannot be measured directly, therefore they are indirect measurements. The original measurements involve physical quantities, which are used to determine geometric relations between the observer's position and the positions of aids to navigation (lighthouses, radionavigational system stations, navigational satellites orbiting the Earth). Geometric quantities which express the relations between navigational mark coordinates and the measurement point (observer's position) are known as the navigational position parameter u, whereas the relation between the navigational parameter and the measurement position in the examined space (coordinate system) is termed the position navigational function f.

Traditionally, there has been a tacit assumption in navigational algorithms [5], [6], [7], [8] that measurements of navigational position parameters are made simultaneously, although in many cases this assumption is not justified. In practice, we always have to do with non-simultaneous measurements of navigational parameters. This is due to:

- movement of the ship (sensor, receiver),
- movement of an aid to navigation (e.g. satellite),
- technical conditions (operation of a radionavigational system station in a chain, single-channel measurement path of the receiver, sequential measurement cycle, asynchronous measurements from individual navigational devices).

Traditional navigational methods of position determination are based on an assumption that navigational parameters are measured simultaneously, or it is assumed that errors due to non-simultaneity are negligibly small. One exception to this is the position determination from non-simultaneous position lines in terrestrial and celestial navigation, where the time between measurement moments is considerable. However, in accurate automated or integrated navigation even small time intervals (several seconds, depending on the precision of dead reckoning navigation) translate into essential errors of coordinates position or estimator instability. Therefore, algorithms of navigational data processing should take account of non-simultaneity of measurements.

2. Determination of position coordinates from simultaneous measurements of navigational parameters

The simplest practical case of position coordinates determination is the calculation of coordinates from simultaneously measured navigational parameters. In the general case of position coordinate calculations we have a navigational vector function mapping navigational space elements into the space of measurements. This function can be written as the following vector mapping:

$$\mathbf{f} : R^m \supset N \to U \subset R^n, \quad n \geq m, \tag{1}$$

where:
R – real space,
N – navigational space,
U – measurement space,
m – dimension of navigational space,
n – dimension of measurement space.

The mapping will be put in a form of the system of equations:

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_m) - u_1 &= 0, \\ f_2(x_1, x_2, \ldots, x_m) - u_2 &= 0, \\ &\ldots \\ f_n(x_1, x_2, \ldots, x_m) - u_n &= 0, \end{aligned} \tag{2}$$

where:
x_i – i-th coordinate of position,
u_i – measured navigational parameter (bearing, range difference, pseudo-range).

The system of equations (2) in the vector notation will have this form:

$$\mathbf{f}(\mathbf{x}) - \mathbf{u} = \mathbf{0}, \tag{3}$$

where:
$x = [x_1, x_2, \ldots, x_m]^T$ – generalized vector of position coordinates (state vector), depending on the assumed coordinate system (X, Y, Z, Δt or φ,λ,h,Δt), $x \in N$,
$u = [u_1, u_2, \ldots, u_n]^T$ – vector of measured navigational parameters, $u \in U$.

There are two cases of solving the equation (3). One is deterministic, where the number of position parameters measurements is equal to the number of estimated coordinates, i.e. n = m. In this case, we solve the equation (3) by Newton's method for non-linear equation systems [4]. In the (k+1)-th step the position coordinate vector will be expressed by this formula:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{G}^{-1}(\mathbf{x}^{(k)})\mathbf{z}^{(k)}, \tag{4}$$

where:
z – measurement vector, the difference between the measured navigational parameters

and the vector of projected (estimated) measurements; this vector is defined by the relation:

$$\mathbf{z}^{(k)} = \mathbf{u} - \mathbf{f}(\mathbf{x}^{(k)}) \tag{5}$$

G – Jacobian matrix of the mapping f (navigational position function).

The other case occurs when the number of position parameter measurements is greater than the number of coordinates to be determined (n > m); then the equation (3) is solved using the method of least squares. In this case in (k + 1)-th step we will obtain the following approximation:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \left[\mathbf{G}^T(\mathbf{x}^{(k)})\mathbf{P}_\mathbf{u}^{-1}\mathbf{G}(\mathbf{x}^{(k)})\right]^{-1}\mathbf{G}^T(\mathbf{x}^{(k)})\mathbf{P}_\mathbf{u}^{-1}\mathbf{z}^{(k)}, \tag{6}$$

where:
P_u – covariance matrix of the vector of measured position parameters u.

The vector z is defined by the formula (5). Formally, the matrix P_u is the covariance matrix of vector z. As this vector is the difference of vectors according to the relation (5), and the vector f(x) being the result of calculations is determined with any accuracy, then we can assume that the covariance matrix of the vector u is equal to the covariance matrix of the vector z. We continue calculations through subsequent iterations, until the assumed accuracy of coordinates is reached. If the iteration process (4) or (6) is convergent with the real solution x, then the estimation of accuracy of position coordinates calculations is approximately equal to the value of the second addend in (4) or (6) calculated in the final step. This fact is often used for the evaluation of iteration process stop. The estimated position or previous fix is usually adopted as the first approximation. In both cases, i.e. Newton's method or the method of least squares, the state vector covariance matrix (position coordinates) is calculated from the formula below

$$\mathbf{P_x} = \left(\mathbf{G}^{\mathrm{T}}(\mathbf{x}^{(k)})\mathbf{P_u}^{-1}\mathbf{G}(\mathbf{x}^{(k)})\right)^{-1} \quad (7)$$

3. Determination of position coordinates from non-simultaneous measurements of navigational parameters

Due to the length of measurement cycles or delay in data distribution and transmission, principally measurements are not performed simultaneously.

Let us choose a series of moments $t_1 < t_2 < \ldots < t_n$ (after bringing them down to a common time scale, if necessary), where ti denotes the moment of i-th measurement of a navigational parameter. For convenience of our considerations, let us adopt that the measurements will be brought down to the last moment of measurement (then we will obtain the coordinates of the current position). The projected vector of measured navigational position parameters up can be calculated from this relation:

$$\mathbf{u}_p = \mathbf{u} + \Delta\mathbf{u}, \quad (8)$$

where:

- vector of projected increments of navigational position parameters values:

$$\Delta\mathbf{u} = \sum_{i=1}^{n} \Delta\mathbf{u}_i \quad (9)$$

$$\Delta\mathbf{u}_i = \mathbf{e}_i^{\mathrm{T}} \cdot \mathrm{grad} f_i \cdot \Delta\mathbf{x}^{(i)}, \quad (10)$$

- vector of the canonical base of n-dimensional space of measurements (1 occurs in an i-th position, corresponding to a given coordinate in the space of measurements):

$$\mathbf{e}_i = \left[0,0,\ldots,0,\underset{i}{1},0,\ldots,0\right], \quad (11)$$

- gradient of i-th navigational function (position line, plane or hypersurface), it is an i-th row of the matrix G:

$$\mathrm{grad} f_i = \left[\frac{\partial f_i}{\partial x_1}, \frac{\partial f_i}{\partial x_2}, \ldots, \frac{\partial f_i}{\partial x_m}\right], \quad (12)$$

- vector of value changes in position coordinates between the moment of navigational parameter measurement t_i and the common time t_n:

$$\Delta\mathbf{x}_i = \left[\Delta x_{1_i}, \Delta x_{2_i}, \ldots, \Delta x_{m_i}\right]^{\mathrm{T}}. \quad (13)$$

We can assume that in sufficiently short time intervals navigational position parameters change linearly. Putting (8) to (5) then to (4) we obtain the formula for (k+1)-e approximation of the position coordinates vector in the Newton's method:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \mathbf{G}^{-1}(\mathbf{x}^{(k)})\left[\mathbf{u} + \Delta\mathbf{u} - \mathbf{f}(\mathbf{x}^{(k)})\right]. \quad (14)$$

We will proceed similarly in the method of least squares. Having substituted (8) to (5) and the substitution result to (6) we get:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \left[\mathbf{G}^{\mathrm{T}}(\mathbf{x}^{(k)})\mathbf{P}_p^{-1}\mathbf{G}(\mathbf{x}^{(k)})\right]^{-1}\mathbf{G}^{\mathrm{T}}(\mathbf{x}^{(k)})\mathbf{P}_p^{-1}\left[\mathbf{u} + \Delta\mathbf{u} - \mathbf{f}(\mathbf{x}^{(k)})\right]. \quad (15)$$

From the theorem on the mean value and the covariance matrix of constant matrices multiplied by random vector [3] we will obtain the following formulas for mean vector value of position coordinates increments (initial approximation in iterations is adopted with any small errors):

$$\Delta\mathbf{x}_{śr}^{(k)} = \mathbf{G}^{-1}(\mathbf{x}^{(k)})\mathbf{z}_p^{(k)}, \quad (16)$$

$$\mathbf{P_x} = \left(\mathbf{G}^{\mathrm{T}}(\mathbf{x}^{(k)})\mathbf{P}_p^{-1}\mathbf{G}(\mathbf{x}^{(k)})\right)^{-1}, \quad (17)$$

where:

- z_p – projected vector of measurements:

$$\mathbf{z}_p^{(k)} = \mathbf{u}_p - \mathbf{f}(\mathbf{x}^{(k)}) \quad (18)$$

- P_p – covariance matrix of the projected vector of measured navigational position parameters, as per formulas (8), (9) and (10) is expressed as:

$$\mathbf{P}_p = \mathbf{P_u} + \sum_{i=1}^{n}\mathbf{P}_{\Delta\mathbf{u}_i} + \sum_{i=1}^{n}(\mathbf{P}_{\mathbf{u}\Delta\mathbf{u}_i} + \mathbf{P}_{\mathbf{u}\Delta\mathbf{u}_i}^{\mathrm{T}}) + \sum_{\substack{i=1 \\ i\neq j}}^{n}\sum_{j=1}^{n}\mathbf{P}_{\Delta\mathbf{u}_i\Delta\mathbf{u}_j} \quad (19)$$

- covariance matrix of increment values of navigational position parameters:

$$\sum_{i=1}^{n}\mathbf{P}_{\Delta\mathbf{u}_i} = \sum_{i=1}^{n}\mathbf{e}_i^{\mathrm{T}} \cdot \mathrm{grad} f_i \cdot \mathbf{P}_{\Delta\mathbf{x}_i}\left(\mathrm{grad} f_i\right)^{\mathrm{T}} \cdot \mathbf{e}_i, \quad (20)$$

- $\mathbf{P}_{\Delta\mathbf{x}_i}$ - covariance matrix of coordinates increments,
- $\sum_{i=1}^{n}\mathbf{P}_{\mathbf{u}\Delta\mathbf{u}_i}$ - covariance matrix between the vector of measured navigational position parameters and the vector of their increments (cross covariance matrix of two random vectors [3],
- $\sum_{\substack{i,j=1 \\ i\neq j}}^{n}\mathbf{P}_{\Delta\mathbf{u}_i\Delta\mathbf{u}_j}$ - covariance matrix between individual increments of measured navigational position parameters values (cross cova-

riance matrix of two random vectors):

$$\mathbf{P}_{\Delta\mathbf{u}_i\Delta\mathbf{u}_j} = \mathbf{e}_i^{\mathrm{T}} \cdot \mathrm{grad} f_i \cdot \mathbf{P}_{\Delta\mathbf{x}_i\Delta\mathbf{x}_j} \left(\mathrm{grad} f_j\right)^{\mathrm{T}} \cdot \mathbf{e}_j \quad (21)$$

- $\mathbf{P}_{\Delta\mathbf{x}_i\Delta\mathbf{x}_j}$ - cross covariance matrix of coordinates increments.

The equation (19) describes the covariance matrix of the projected measurement vector. If the total distribution of measurement vectors and navigational parameters increment projections is normal, then, with a natural assumption that navigational parameters measurement errors and estimation (projections are based on this assumption) are independent, we will see that the third addend on the right-hand side of the equation (19) is a zero vector. Finally, we will obtain the following formula for the covariance matrix of fix coordinates from non-simultaneous measurements of navigational position parameters:

$$\mathbf{P}_p = \mathbf{P}_{\mathbf{u}} + \sum_{i=1}^{n} \mathbf{P}_{\Delta\mathbf{u}_i} + \sum_{\substack{i=1 \\ i\neq j}}^{n} \sum_{j=1}^{n} \mathbf{P}_{\Delta\mathbf{u}_i\Delta\mathbf{u}_j} \quad (22)$$

4. Sequential integration of measurements from two positioning systems

Let us illustrate the above considerations with an example. For this purpose we will assume that two different positioning systems are being integrated:

- satellite GPS system,
- another satellite pseudo-range system.

Let us assume that all the measurements are made sequentially. Then the individual vectors and matrices take this form:

- state vector:

$$\mathbf{x} = \left[\varphi, \lambda, h, \Delta t_I, \Delta t_{II}\right]^T \quad (23)$$

where: φ – geodetic latitude,
λ – geodetic longitude,
h – geodetic (ellipsoidal) height,

Δt_I – GPS receiver clock error,

Δt_{II} – clock error of another pseudo-range system,

- vector of measurements (k pseudo-range measurements of the first system and m measurement of the second system):

$$\mathbf{z}_p = \begin{bmatrix} d_1 + \frac{\partial d_1}{\partial\varphi}\Delta\varphi_1 + \frac{\partial d_1}{\partial\lambda}\Delta\lambda_1 + \frac{\partial d_1}{\partial h}\Delta h_1 - d_{z_1} \\ d_2 + \frac{\partial d_2}{\partial\varphi}\Delta\varphi_2 + \frac{\partial d_2}{\partial\lambda}\Delta\lambda_2 + \frac{\partial d_2}{\partial h}\Delta h_2 - d_{z_2} \\ \cdots \\ d_k + \frac{\partial d_k}{\partial\varphi}\Delta\varphi_k + \frac{\partial d_k}{\partial\lambda}\Delta\lambda_k + \frac{\partial d_k}{\partial h}\Delta h_k - d_{z_k} \\ d_{k+1} + \frac{\partial d_{k+1}}{\partial\varphi}\Delta\varphi_{k+1} + \frac{\partial d_{k+1}}{\partial\lambda}\Delta\lambda_{k+1} + \frac{\partial d_{k+1}}{\partial h}\Delta h_{k+1} - d_{z_{k+1}} \\ d_{k+2} + \frac{\partial d_{k+2}}{\partial\varphi}\Delta\varphi_{k+2} + \frac{\partial d_{k+2}}{\partial\lambda}\Delta\lambda_{k+2} + \frac{\partial d_{k+2}}{\partial h}\Delta h_{k+2} - d_{z_{k+2}} \\ \cdots \\ d_{k+m} + \frac{\partial d_{k+m}}{\partial\varphi}\Delta\varphi_{k+m} + \frac{\partial d_{k+m}}{\partial\lambda}\Delta\lambda_{k+m} + \frac{\partial d_{k+m}}{\partial h}\Delta h_{k+m} - d_{z_{k+m}} \end{bmatrix} \quad (24)$$

where:

d_i - measured i-th pseudo-range,

d_{z_i} - calculated i-th pseudo-range,

$\Delta\varphi_i$, $\Delta\lambda_i$, Δh_i - increments of geodetic coordinates between i-th and n-th moment (one to which all measurements are brought down), obtained from estimation,

$\frac{\partial d_i}{\partial\varphi}$, $\frac{\partial d_i}{\partial\lambda}$, $\frac{\partial d_i}{\partial h}$ - partial derivatives of i-th pseudo-range (navigational function) relative to geodetic coordinates,

k – number of measurements from the of GPS system (GPS),

m – number of measurements from another pseudo-range navigational system,

- Jacobian matrix of navigational position function:

$$\mathbf{G} = \begin{bmatrix} \frac{\partial d_1}{\partial\varphi} & \frac{\partial d_1}{\partial\lambda} & \frac{\partial d_1}{\partial h} & \frac{\partial d_1}{\partial\Delta t_I} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial d_k}{\partial\varphi} & \frac{\partial d_k}{\partial\lambda} & \frac{\partial d_k}{\partial h} & \frac{\partial d_k}{\partial\Delta t_I} & \\ \frac{\partial d_{k+1}}{\partial\varphi} & \frac{\partial d_{k+1}}{\partial\lambda} & \frac{\partial d_{k+1}}{\partial h} & 0 & \frac{\partial d_{k+1}}{\partial\Delta t_{II}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial d_{k+m}}{\partial\varphi} & \frac{\partial d_{k+m}}{\partial\lambda} & \frac{\partial d_{k+m}}{\partial h} & 0 & \frac{\partial d_{k+m}}{\partial\Delta t_{II}} \end{bmatrix} \quad (25)$$

- covariance matrix of measurement vector (assuming the both systems are independent):

$$\mathbf{P}_{\mathbf{u}} = \begin{bmatrix} \sigma_{d_1}^2 & \cdots & \sigma_{d_1 d_k} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \sigma_{d_1 d_k} & \cdots & \sigma_{d_k}^2 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \sigma_{d_{k+1}}^2 & \cdots & \sigma_{d_{k+1} d_{k+m}} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \sigma_{d_{k+1} d_{k+m}} & \cdots & \sigma_{d_{k+m}}^2 \end{bmatrix} \quad (26)$$

where:

$\sigma_{d_i}^2$ - variance of pseudo-range d_i,

$\sigma_{d_i d_j}$ - covariance between pseudo-ranges di and dj.

- covariance matrix of coordinates increments (practically it is a covariance matrix of estimated position coordinates increments):

$$\mathbf{P}_{\Delta \mathbf{x}_i} = \begin{bmatrix} \sigma^2_{\Delta\varphi_i} & \sigma_{\Delta\varphi_i\Delta\lambda_i} & \sigma_{\Delta\varphi_i\Delta h_i} & \sigma_{\Delta\varphi_i\Delta t_i} & \sigma_{\Delta\varphi_i\Delta t_{II_i}} \\ \sigma_{\Delta\varphi_i\Delta\lambda_i} & \sigma^2_{\Delta\lambda_i} & \sigma_{\Delta\lambda_i\Delta h_i} & \sigma_{\Delta\lambda_i\Delta t_i} & \sigma_{\Delta\varphi_i\Delta t_{I_i}} \\ \sigma_{\Delta\varphi_i\Delta h_i} & \sigma_{\Delta\lambda_i\Delta h_i} & \sigma^2_{\Delta h_i} & \sigma_{\Delta h_i\Delta t_{I_i}} & \sigma_{\Delta h_i\Delta t_{II_i}} \\ \sigma_{\Delta\varphi_i\Delta t_{I_i}} & \sigma_{\Delta\lambda_i\Delta t_{I_i}} & \sigma_{\Delta h_i\Delta t_{I_i}} & \sigma^2_{\Delta t_{I_i}} & \sigma_{\Delta t_{I_i}\Delta t_{II_i}} \\ \sigma_{\Delta\varphi_i\Delta t_{II_i}} & \sigma_{\Delta\lambda_i\Delta t_{II_i}} & \sigma_{\Delta h_i\Delta t_{II_i}} & \sigma_{\Delta t_{I_i}\Delta t_{II_i}} & \sigma^2_{\Delta t_{II_i}} \end{bmatrix} \quad (27)$$

- covariance matrix between individual value increments of measured navigational position parameters:

$$\mathbf{P}_{\Delta \mathbf{x}_i \Delta \mathbf{x}_j} = \begin{bmatrix} \sigma_{\Delta\varphi_i\Delta\varphi_j} & \sigma_{\Delta\varphi_i\Delta\lambda_j} & \sigma_{\Delta\varphi_i\Delta h_j} & \sigma_{\Delta\varphi_i\Delta t_{I_j}} & \sigma_{\Delta\varphi_i\Delta t_{II_j}} \\ \sigma_{\Delta\lambda_i\Delta\varphi_j} & \sigma_{\Delta\lambda_i\Delta\lambda_j} & \sigma_{\Delta\lambda_i\Delta h_j} & \sigma_{\Delta\lambda_i\Delta t_{I_j}} & \sigma_{\Delta\lambda_i\Delta t_{II_j}} \\ \sigma_{\Delta h_i\Delta\varphi_j} & \sigma_{\Delta h_i\Delta\lambda_j} & \sigma_{\Delta h_i\Delta h_j} & \sigma_{\Delta h_i\Delta t_{I_j}} & \sigma_{\Delta h_i\Delta t_{II_j}} \\ \sigma_{\Delta t_{I_i}\Delta\varphi_j} & \sigma_{\Delta t_{I_i}\Delta\lambda_j} & \sigma_{\Delta t_{I_i}\Delta h_j} & \sigma_{\Delta t_{I_i}\Delta t_{I_j}} & \sigma_{\Delta t_{I_i}\Delta t_{II_j}} \\ \sigma_{\Delta t_{II_i}\Delta\varphi_j} & \sigma_{\Delta t_{II_i}\Delta\lambda_j} & \sigma_{\Delta t_{II_i}\Delta h_j} & \sigma_{\Delta t_{II_i}\Delta t_{I_j}} & \sigma_{\Delta t_{II_i}\Delta t_{II_j}} \end{bmatrix} \quad (28)$$

5. Conclusions

The presented method of position coordinates calculations based on data from non-simultaneous measurements of navigational position parameters can be used in integrated navigational systems or hybrid receivers of radionavigation systems (e.g. GPS/GALILEO, GPS/GLONASS, GPS/rho-rho system or another radionavigation system).

By extending the concept of navigational parameter measurement vector to the projected measurement vector we can standardize algorithms for calculations of a fix coordinates. The application of projected values of navigational parameters, in turn, enables determining a position using any set of position parameters – homogenous or non-homogenous. This is essential to automated navigational systems where measurements of navigational parameters are integrated. Projected values in non-simultaneous measurements are burdened with greater errors than simultaneous measurements. This, however, is accounted for in the resultant position covariance matrix. Then we obtain a non-biased assessment of position coordinates, correct from the viewpoint of the system measurement model [1], [2], [3]. The algorithm of position coordinates calculation from non-simultaneous measurements is more general, and when simultaneous measurements are made it is simplified to the first algorithm step.

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