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FUZZY CELLULAR MODEL FOR TRAFFIC DATA FUSION

Summary. In this paper the requirements for road traffic models designed to collect complex data from vehicles detectors in a traffic control system are discussed. A cellular model is proposed that uses the theory of fuzzy sets to deal with these requirements and to enable traffic data fusion from various sources. In the introduced approach all parameters of vehicles are described by means of fuzzy numbers. Vehicles are modelled individually, thus various classes of them can be taken into consideration. The model was implemented in a simulation of traffic flow at a crossroad approach. A discharge time of vehicles queue was analysed in this experiment and compared to the results of Nagel-Schreckenberg model [4].

ROZMYTY MODEL KOMÓRKOWY DLA FUZJI DANYCH O RUCHU DROGOWYM

Streszczenie. Niniejszy artykuł zawiera dyskusję wymagań dla modeli ruchu, przeznaczonych do gromadzenia złożonych danych z detektorów pojazdów w systemie sterowania ruchem drogowym. Aby spełnić określone wymagania zaproponowano komórkowy model ruchu, w którym zastosowano teorię zbiorów rozmytych. Rozwiązanie to umożliwia fuzję danych o ruchu, pochodzących z różnych źródeł. Wszystkie parametry pojazdów są modelowane za pomocą liczb rozmytych, indywidualnie dla każdego pojazdu. Dzięki temu można uwzględnić zróżnicowane klasy pojazdów. Opracowany model został zaimplementowany w symulacji ruchu pojazdów na wlocie skrzyżowania. Przeprowadzono analizę czasu rozładowania kolejki pojazdów, uzyskane wyniki zostały porównane z rezultatami symulacji wykorzystującej model Nagela-Schreckenberga [4].

1. INTRODUCTION

The data fusion becomes crucial in case of actuated and synchronised traffic control systems that are usually equipped with a number of various vehicles detectors (inductive loops, vision-based sensors, etc.). The data gathered by many detectors have to be collected and combined in a single traffic model to determine current and actual traffic parameters as well as to forecast future state of the flow for traffic control purposes. An example can be given here of an on-line traffic model that uses counts of detected vehicles to calculate discharge time of a vehicles queue in a crossroad approach [1].

In this paper a fuzzy cellular traffic model is proposed. This model was intended for detectors data fusion in traffic control system. It enables utilisation of complex traffic data registered by many sensors of different types. The development of this model was motivated by the following requirements.:

- The traffic model has to provide interfaces for many data sources of different types (various detectors of vehicles).
- The uncertainty has to be described in the traffic model to take into account random nature of traffic processes as well as rough character of vehicles recognition (detection) results.
- A vehicle influence on traffic conditions depends on its individual features, thus various classes of vehicles have to be modelled.
- Computational complexity of the traffic model has to be appropriately low to allow for the on-line processing.

The available systems of adaptive traffic control (e.g. SCOOT [1], UTOPIA [2]) use macroscopic traffic models that describe basic parameters of traffic stream: density, intensity and mean velocity. Individual parameters of vehicles are not taken into consideration if the macroscopic model is used. However, these characteristics related to the class of a vehicle are very important from the traffic control point of view, as they have a significant influence on traffic conditions and capacity of road infrastructure.

Traffic control systems using macroscopic models are intended for cooperation with traffic detectors that recognises the presence or passing of vehicles and count them (e.g. inductive loops). Additional functionalities offered by vision based sensors cannot be fully utilised in systems of this kind [3]. Usually, the video-detection technology is simply adopted as a substitute for inductive loops, thus important data available for vision based sensors is discarded.

The properties of particular vehicles are considered when using microscopic traffic models. Computationally efficient and sufficiently accurate models have been developed based on cellular automata theory [4]. Extended cellular automata models that distinguish features of individual vehicles (classes) can be referred to as multi-agent systems [5]. In the literature there is a lack of information about the experiments on cellular models implementation in traffic control systems.

The model proposed here is based on cellular approach to traffic modelling; it uses fuzzy sets theory to describe uncertainty of both input data and simulation results. All parameters of vehicles are modelled individually by means of fuzzy numbers, thus various classes of vehicles can be taken into consideration. The application of fuzzy sets theory provides effective tools to create multiple interfaces for uncertain and partially incoherent data from vehicles detectors.

The paper is organised as follows: Section 2 presents the related works in the field of traffic modelling based on cellular automata theory. In section 3 the fuzzy cellular model is introduced. Traffic simulation results for the proposed model are discussed and compared with those of NaSch cellular automaton in section 4. Conclusions and directions for future researches are given in section 5.

2. RELATED WORKS

A cellular automaton model for freeway traffic was introduced by Nagel and Schreckenberg (NaSch) in 1992 [4]. In this model the traffic lane was divided into cells of 7,5 m. Each vehicle is characterised by its position (cell number) and velocity (a positive discrete value lower than fixed maximum). The velocity is expressed as a number of cells that vehicle advances in one time step. The movement of vehicles in this model is described by a simple rule that is executed in parallel for each vehicle. It has capability for the mapping of real traffic streams parameters (fundamental diagram) and enables simulation of phenomena that can be observed in reality (e.g. traffic jam formation).

Traffic models using cellular automata have high computational efficiency and allows the sufficiently accurate simulation of real traffic phenomena [6]. Due to their simplicity, cellular automata have become a frequently used tool for microscopic modelling of road traffic processes.

In the literature many traffic models can be found that are based on the Nagel-Schreckenberg concept. Numerous models have been introduced that uses so-called slow-to-start rules to reflect metastable state of traffic flow [7, 8, 9, 10]. According to the slow-to-start (s2s) rule the stopped vehicles accelerate with lower probability than the moving ones. Different rules of this kind take into account various factors: number of free cells in front of a vehicle (gap), present or previous state of a vehicle (velocity). In [11] a new set of rules has been proposed to better capture driver reactions to traffic that are intended to preserve safety on the highway. The NaSch model was also extended by introducing disordered acceleration and deceleration terms to simulate phase separated state, density waves and self organised criticality in the traffic flow [23].

On the basis of the NaSch cellular automaton the multi-lane traffic models have been formulated [12, 13, 14, 15]. The rules of these models consist of two steps: first, the additional step takes into account lanes changing behaviour and second, the basic step describes the forward movement of a vehicle.

Cellular automata have also been used for junctions modelling. The simplest cellular model of a crossroad [16] did not take into account the region inside the junction as well as the priority rules. Vehicles were just randomly selected to pass the crossroad. In [17] other simple model of cellular automaton with closed boundary conditions (ring of cells) for crossroads simulation has been proposed. All junctions in this case were modelled as roundabouts. For more realistic simulations the sophisticated models have been applied that include definitions of traffic regulations (priority rules, signs, signalisation) and allow for determination of actual junction capacity [18].

A limited research has been completed in the application of microscopic cellular models for traffic control in road networks. Certain methods of optimal route selection in urban networks have been proposed [19]. The cellular automata model of road network has been used in this approach to evaluate current traffic conditions for particular connections. In [20] the similar model was adopted to calculate basic parameters of coordination plan for signalised intersections network. However, the analysed cases were significantly simplified and far from practical solutions.

In the field of road traffic modelling several methods are known using cells defined as road segments for the macroscopic flow description. Although the term “cell” is used in these methods, they are not derived from cellular automata theory. One cell in this case can be occupied by many vehicles thus its state is described using parameters of traffic stream (density, intensity). A model of this type was implemented for traffic control purposes in the UTOPIA method [2], different models have been introduced for highway traffic analysis [21].

3. FUZZY CELULAR APPROACH TO TRAFFIC MODELLING

The use of fuzzy sets in cellular traffic model allows single simulation to take into account many scenarios (traffic state evolutions) along with their uncertainty. Moreover, the objective of fuzzy numbers application is to establish basis for design of traffic data interfaces and data fusion procedures. Operations on fuzzy sets can be used for this task as it was shown in [25] that this formal framework is suitable to describe imprecise data coming from vehicles detection.

The fuzzy cellular model of road traffic flow assumes division of traffic lane into cells that correspond to road segments of equal length. The traffic state is described in discrete time steps. These basic assumptions are consistent with those of cellular automata models. Thus, the calibration methods proposed for NaSch model [4] are also applicable here for determination of the cells length and vehicles properties. A major feature differencing the presented approach from NaSch-like models is that vehicle position, its velocity and other parameters are modelled by fuzzy numbers defined on the set of integers. Moreover, the rule of model transition between consecutive time steps also uses fuzzy operations.

The state of a cell c in time step t is defined by fuzzy set of vehicles (n) that currently occupy this cell:

$$S_{c,t} = \{\mu_{S_{c,t}}(n)/n\}. \tag{1}$$

Thus, one cell can be occupied by more than one vehicle in the same time. Conventionally, $\mu_A(x)$ denotes value of the membership function of fuzzy set A for an element x. The position of vehicle n in a time step t is a fuzzy number defined on the set of cells indexes (c):

$$P_{n,t} = \{\mu_{P_{n,t}}(c)/c\}. \tag{2}$$

The interpretation of the membership functions used to express vehicle position (2) and cell state (1) is shown in fig. 1. In this example two vehicles are taken into consideration that are located in road segment comprised five cells. The positions of vehicles are not defined in a precise way; for each vehicle three cells indicate its three potential locations. The most probable location of vehicle n is determined by cell c, where membership function of fuzzy set (2) has the highest value ($\mu_{P_{n,t}}(c)/c = 1$).

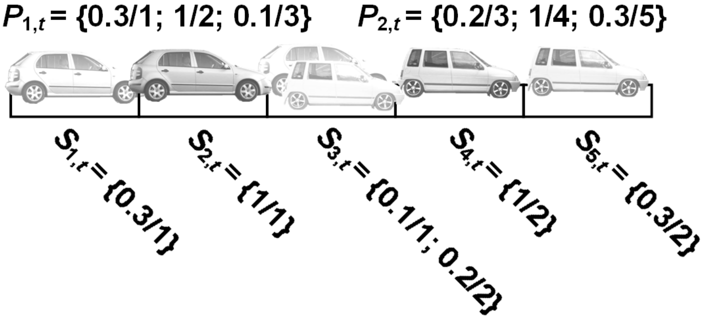

Fig. 1. Fuzzy cellular traffic model: vehicles positions (P) and cells states (S)
Rys. 1. Rozmyty komórkowy model ruchu drogowego: pozycje pojazdów (P) oraz stany komórek (S)

A vehicle n is described by its class and velocity $V_{n,t}$ (in cells per time step). The class determines properties of a vehicle n: length L_n (in cells), maximal velocity V_n^{max}, and acceleration A_n. All these quantities are expressed by fuzzy numbers. The velocity of the vehicle n in time step t is computed as follows:

$$V_{n,t} = \widetilde{\min}\{V_{n,t-1} \tilde{+} A_n, G_{n,t}, V_n^{\max}\}. \tag{3}$$

The tilde (~) symbol is used to distinguish operations on fuzzy numbers [22]. A gap $G_{n,t}$ is the number of free cells in front of a vehicle n:

$$G_{n,t} = \widetilde{\min_{m \neq n}}\{P_{m,t} \tilde{-} L_m \tilde{-} P_{n,t} : P_{m,t} \tilde{>} P_{n,t}\}. \tag{4}$$

If there is no vehicle m fulfilling the condition in (4), the gap $G_{n,t}$ is assumed to be equal to the maximal velocity.

The above formulas are consistent with assumptions that have been used for the rule definition of the NaSch probabilistic cellular automaton. This definition takes into account the simple principles describing movement of a single vehicle:

- vehicle accelerates if its velocity is lower than maximal velocity and if distance to the next car ahead (gap) is sufficiently large,
- vehicle reduces its speed if it approaches the car ahead and the gap becomes small.

After velocities determination for all vehicles, their positions are updated. The position of the vehicle n in the next time step (t+1) is computed on the basis of the model state in time t:

$$P_{n,t+1} = \widetilde{\mathrm{dil}}(P_{n,t} \tilde{+} V_{n,t}), \tag{5}$$

$\widetilde{\mathrm{dil}}$ denotes fuzzy set dilation:

$$\mu_{\widetilde{\mathrm{dil}}(P_{n,t} \tilde{+} V_{n,t})}(x) = [\mu_{(P_{n,t} \tilde{+} V_{n,t})}(x)]^{e}, \tag{6}$$

where $0 < e \leq 1$.

Dilating the fuzzy set increases the fuzziness (uncertainty) of the vehicles position. This operation corresponds to the randomization step of traffic models based on the NaSch cellular automaton. In the models that use s2s rules the randomization level decreases with increasing velocity of a vehicle as the random driver behaviours are more intense at the low velocity range. To achieve similar effect for the presented model the exponent *e* in (6) was defined as an increasing function of velocity. It was also assumed that when the maximal velocity is reached the vehicle position is not further dilated (*e* = 1). A simple linear dependency was used to control the dilation:

$$e = \alpha + \frac{1-\alpha}{\hat{v}_n^{\max}} \hat{v}_{n,t}, \tag{7}$$

where $0 \leq \alpha \leq 1$ and $\hat{v}$ denotes defuzzified (numeric) value of velocity:

$$\hat{v}_{n,t} = \arg\max_y \mu_{V_{n,t}}(y). \tag{8}$$

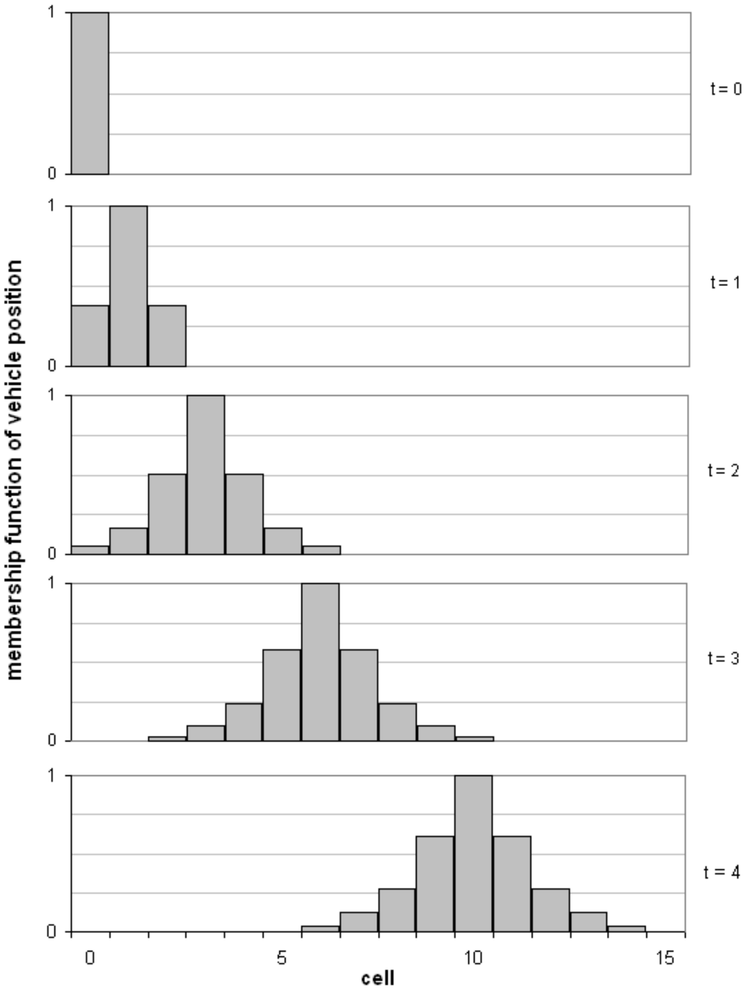

Fig. 2. Fuzzy cellular traffic model: position of an accelerating vehicle in sequence of time steps
Rys. 2. Rozmyty komórkowy model ruchu drogowego: pozycja pojazdu przyspieszającego w kolejnych krokach czasu

Fig. 2 presents the position of an accelerating vehicle in the fuzzy cellular model for five time steps. The simulation was started with a single vehicle (n = 0) stopped in the first cell (c = 0): $P_{0,0}$ = {1/0}, $V_{0,0}$ = {1/0}, the vehicle properties was set: L_0 = {1/0}, $V_0^{\max}$ = {0,2/4; 1/5; 0,2/6}, A_0 = {0,2/0; 1/1; 0,2/2}. Results of the vehicle movement simulation are depicted for the single value of parameter $\alpha = 0{,}5$ that was used for controlling the dilation (eq. 7). It was observed in this simulation that decreasing value of α causes increase in fuzziness of the vehicle position (higher level of model uncertainty).

4. EXPERIMENTAL RESULTS

The fuzzy cellular model presented in previous section was applied for simulations of vehicles queue discharge process that correspond to real-life situations observed at approaches of signalised crossroads when green signal is given. During these experiments length of vehicles queue was analysed in sequence of time steps and discharge time of the queue was evaluated. This section includes discussion of the simulation results as well as their comparison with experimental data obtained from NaSch traffic model.

All experiments discussed in this section were executed for the queue model of twenty vehicles. In the first time step of each simulation procedure (t = 0) all twenty vehicles (n = 0...19) were stopped in a queue: $P_{n,0}$ = {1/n}, $V_{n,0}$ = {1/0}, it means that at the beginning length of the queue was equal to the number of vehicles. Only one class of vehicles was considered in these experiments, thus all vehicles had the same parameters. The traffic process of queue discharge was also simulated using NaSch model with assumptions similar to those formulated above.

The first part of experimental data presented here concerns position of the last vehicle in a queue (n = 0), which was analysed for fifty time steps of traffic simulation. Fig. 3 a) depicts the space-time diagram obtained from fuzzy cellular model – values of membership functions of sets $P_{0,t}$ are presented using gray scale (black colour corresponds to full membership). Properties of vehicles were set similarly to those of single-vehicle experiment reported in section 2, maximal velocity was defined by fuzzy set $V^{\max}$ = {0,2/1; 1/2; 0,2/3} and dilation operation was applied with $\alpha = 0{,}90$. Experimental probability of the vehicle position was evaluated taking into account the results of NaSch traffic simulator (fig. 3 b). The simulation with NaSch cellular automaton was executed 10 000 times to collect these data. The black colour in fig. 3 b) indicates cases, where the probability of vehicle position is one. Parameters of the NaSch model were defined as follows: maximal velocity v_{max} = 3 and probabilistic parameter p = 0,1.

The next part of the experimental results includes data on discharge time and length of vehicles queue. In case of the fuzzy cellular model these parameters were evaluated using fuzzy reasoning system. The following fuzzy rules were implemented for this task:

$$\begin{array}{l}\textbf{if } veh\_0 \textbf{ is } in\_queue \textbf{ and } veh\_1 \textbf{ is } in\_queue \textbf{ and} \ldots \textbf{and } veh\_x-1 \textbf{ is } in\_queue \\ \textbf{and } veh\_x \textbf{ is not } in\_queue \textbf{ and} \ldots \textbf{and } veh\_m \textbf{ is not } in\_queue \textbf{ then } Q_t \textbf{ is } x\end{array}, \tag{9}$$

where: Q_t is queue length in time step t, veh_n stands for "vehicle n", m = 19 and variable in_queue is determined by another fuzzy rule taking into account a position and velocity of a vehicle:

$$\textbf{if } P_{n,t} \textbf{ is } n \textbf{ and } V_{n,t} \textbf{ is } 0 \textbf{ then } veh\_n \textbf{ is } in\_queue\,. \tag{10}$$

The last fuzzy rule was defined for discharge time computations:

$$\textbf{if } Q_{t-1} \textbf{ is not } 0 \textbf{ and } Q_t \textbf{ is } 0 \textbf{ then } discharge\_time \textbf{ is } t\,. \tag{11}$$

The queue length is defined as a number of vehicles x staying in a row in cells 0, 1, …, x – 1 (eq. 9). Note that every single vehicle occupies one cell in this experiment. A vehicle is in queue if it is stopped in original position determined by starting conditions of the simulation (eq. 10). The discharge of vehicles queue is recognised in a given time step if length of the queue drops to zero: current queue length is zero and the length evaluated for previous time step is higher than zero (eq. 11).

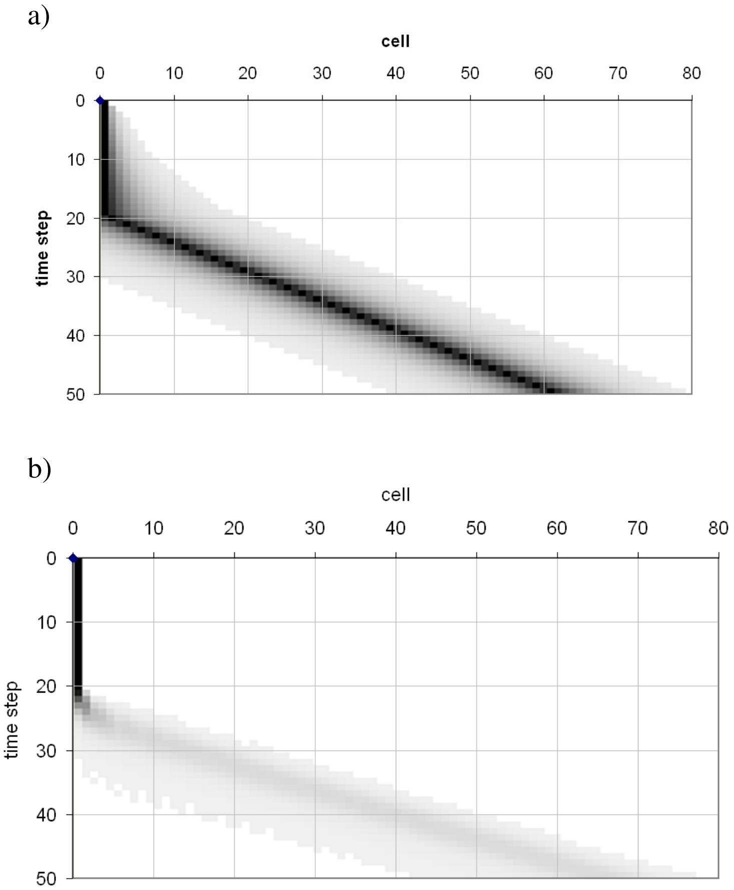

Fig. 3. Position of the last vehicle ($n = 0$) during queue discharge simulation: a) fuzzy cellular model - membership functions, b) NaSch model – experimental probability

Rys. 3. Pozycja ostatniego pojazdu ($n = 0$) podczas symulowanego rozładowania kolejki: a) rozmyty model komórkowy – funkcje przynależności, b) model NaSch – prawdopodobieństwo eksperymentalne

The results of vehicles queue length computations based on fuzzy cellular simulation are presented in fig 4 a), the membership function of zero queue length was evaluated for three different values of parameter α. In the experiment the maximal velocity was defined by fuzzy set $V^{\max} = \{0,2/2; 1/3; 0,2/4\}$ and remaining parameters were set as mentioned in the previous part of this section. Dilation operation (eq. 6) was applied with three different values of α (0,90; 0,87 and 0,85). The results are compared with experimental data on vehicles queue discharge that was collected from 10 000 executions of traffic simulation driven by the NaSch cellular automaton (fig. 4 b). Fig. 3 includes a similar comparison for the discharge time evaluated using both NaSch (fig. 5 b) as well as fuzzy cellular model (fig. 5 a).

Vehicles characteristics and starting conditions for the simulation using the NaSch model were similar to those defined for fuzzy cellular simulation. The probabilistic parameter p was set to three different values: 0,1; 0,2; 0,3 and v_{max} was 3 cells per time step. The simulation was executed 10 000 times to gather the data presented in fig. 4 b) and fig. 5 b).

The comparison of simulation results allows the conclusion that the proposed fuzzy cellular model adequately describes the process of vehicles queue discharge. The model was used to determine trajectory of a vehicle in space-time diagram (fig. 3) and to evaluate time of queue discharge (fig. 5). These quantities were analysed along with their uncertainty, the results approximate those of the NaSch model. The shapes of characteristics presented in above plots and their interpretations are similar for both models. One should note that the results were achieved using different formal frameworks (fuzzy logic and statistical analysis of experimental data) and therefore they cannot be directly compared in a numerical domain. Scaling of the fuzzy cellular model can be obtained by setting parameter α and defining membership functions of maximal velocity V^{max} and acceleration A. It was observed that the α parameter has similar influence on results of simulation as probabilistic parameter p in case of the NaSch model. An apparent advantage of the introduced approach is connected with the fact that a high number of executions of NaSch simulation were necessary to get data that is available after single simulation using fuzzy cellular model. The proposed model allows describing many possible traffic states including their uncertainty while conventional cellular automaton represents only one of those states.

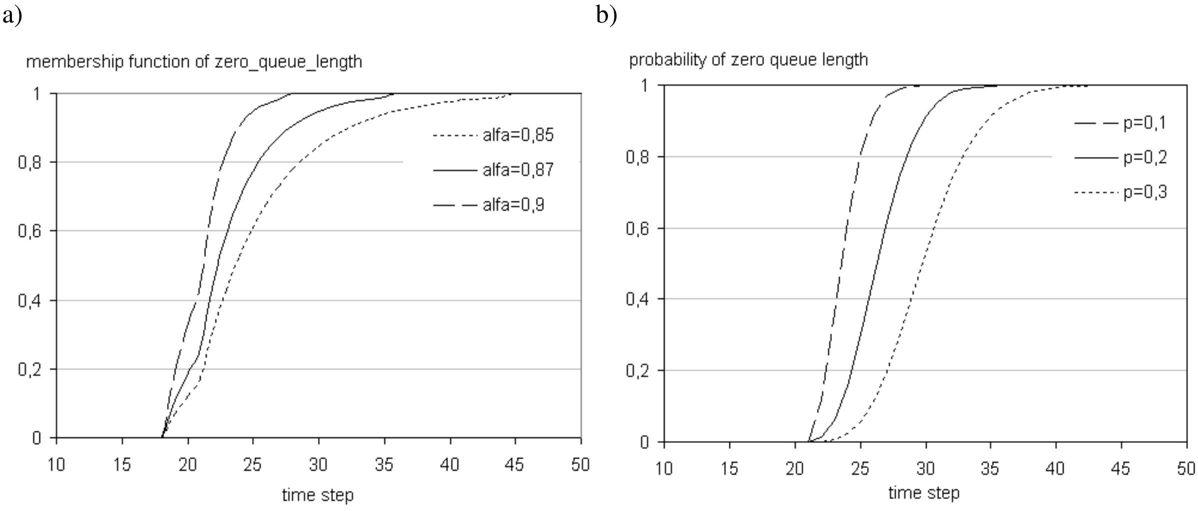
a)
b)

Fig. 4. Results of vehicles queue discharge simulation: a) fuzzy cellular model - membership function of zero queue length, b) NaSch model – experimental probability of zero queue length

Rys. 4. Wyniki symulacji rozładowania kolejki pojazdów: a) rozmyty model komórkowy – funkcja przynależności dla zerowej długości kolejki, b) model NaSch – eksperymentalne prawdopodobieństwo wystąpienia zerowej długości kolejki pojazdów

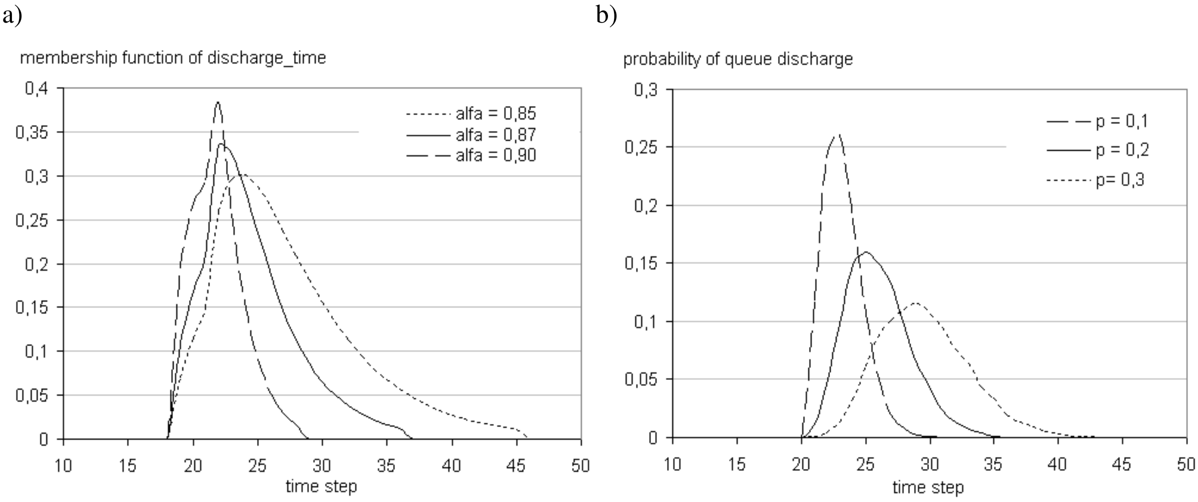
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Fig. 5. Discharge time of vehicles queue: a) fuzzy cellular model - membership function, b) NaSch model – experimental probability

Rys. 5. Czas rozładowania kolejki pojazdów: a) rozmyty model komórkowy – funkcja przynależności, b) model NaSch – prawdopodobieństwo eksperymentalne

5. CONCLUSIONS AND FUTURE WORK

The fuzzy cellular model of road traffic was formulated on the basis of Nagel-Schreckenberg approach. Parameters of vehicles are individually described by means of fuzzy numbers, thus various classes of vehicles can be modelled. The application of fuzzy sets theory allows to represent model uncertainty and to describe many possible traffic situations using the single instance of the model.

The uncertainty of traffic state was described applying fuzzy definitions of vehicles parameters and introducing fuzzy operations in the procedure of model update. These new elements take into account random nature of traffic processes and rough character of vehicles detection. All aforementioned features make the proposed model suitable for imprecise data fusion from multiple detectors in a traffic control system.

The experimental results of a vehicles queue discharge simulation reported in this contribution show that the fuzzy cellular model adequately describes the process of traffic flow. Space-time trajectories of a vehicle, the queue length as well as the queue discharge time were analysed in this experiment and compared to the results of the NaSch cellular automata model.

Further tests are necessary to evaluate the model applicability for other real-traffic situations. The planned research will also involve the design of interfaces that are needed to input data from traffic detectors into the model. These methods of traffic data fusion will be defined using fuzzy and rough sets operations. Future research directions will consider data interchange between modules of traffic control systems in terms of granular worlds communication [24].

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