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Chosen problems of examination of car stability

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ABSTRACT
This paper gives an overview on some chosen aspects of examination of car stability. It especially stresses the situation of an after repair car, with disturbances in its body geometry, mass, inertia and deviation. Such disturbances can occur especially after the side crash, which is one of the most common types of accidents.

KEYWORDS: safety, stability ,car accidents, crashes

1. Introduction

The examination of road vehicles can be conducted by two methods: through analysis of results of real cars or through simulations made on car mathematical models with the use of software tools, which is more convenient and, above all, less expensive.

One of the problems of examination of road vehicle motion is the examination of its stability. In general, as well as in respect of means of transport, stability is described as the ability of considered object to return to the position of balance by itself, after disturbing this position with small disturbances.

During the examination of road vehicle dynamics different definitions of stability can be used. The first one was introduced by Lapunov. It is, however, like some of its derivatives, inconvenient in analysis of real objects in recpect of the limitations put on the solutions of its characteristic equations.

The notions, that were introduced to examine road vehicles as technical objects are the definitions according to the ISO 8855:1991. The closest to these is the definition of stochastic technical stability [2]. It refers to mathematical models of real objects. It is the definition deriving from the stability according to Lapunov.

2. Definitions of stability

In the ISO 8855 such definitions were created:

- non-periodic stability – stability characteristic at a prescribed steady-state equilibrium if, following any small temporary disturbance or control input, the vehicle will return to the steady-state equilibrium without oscillation;
- neutral stability – stability characteristics at a prescribed steady state equilibrium if, for any small temporary disturbance or control input, the resulting motion of the vehicle remains close to, but does not return to the motion defined by the steady-state equilibrium;
- oscillatory stability – oscillatory vehicle response of decreasing amplitude and a return to the original steady-state equilibrium;
- non-periodic instability – an ever-increasing response without oscillation;
- oscillatory instability – an oscillatory response of ever-increasing amplitude about the initial steady-state equilibrium.

There are also definitions of stability created for the needs of examination of mathematical models of technical objects, which motion is disturbed by the real disturbances. One of the definitions is the definition of technical stability, which refers to the conditions, where the occurring disturbances are constant. To present it, differential equations (1) and (2) were considered, along with the initial conditions.

$$\frac{dz}{dt} = F(t,z) + \Phi(t,z), \qquad z(t_0) = z_0 \qquad (1)$$

$$\frac{dx}{dt} = F(t,x), \qquad x(t_0) = x_0 = z_0 \qquad (2)$$

The equation (1) comprises constant disturbances, while (2) is the one without disturbances. It was assumed, that the functions F and Φ fulfill the conditions of existence and interchangeability of their solutions in the finite area $t \in [t_0, T]$, $\|x\| \le H, \|z\| \le H$. Moreover, it was assumed, that $F(t,0), \Phi(t_0, z_0) = 0$, and that the function $\Phi(t,z)$ can be unknown. It is necessary to know the estimation of this function as well as its initial values (3).

$$\|z_0\| \le z_0^*, \qquad \|\Phi(t,z)\| \le \Gamma(t), \qquad t \in [t_0, T] \qquad (3)$$

where $z_0^*, \Gamma(t)$ - limitations.

In particular event $\Gamma(t) = \gamma = const$. The definition of technical stability is as follows: let the solution $z = z(t, t_0, z_0)$ presents every motion described by (1). These fulfill the initial condition $z(t_0) = z_0$ and limitations (3). The solution of equation (2)

$$x = x(t, t_0, x_0) \qquad (4)$$

is called technically stable when

$$\|z(t, t_0, z_0)\| \le \Lambda(t), \qquad t \in [t_0, T] \qquad (5)$$

where $\Lambda(t)$ - limitation.

In particular event $\Lambda(t) = \gamma = const$.

Such motion is technically stable in relation to the assumed limitations. The solution (motion) is technically unstable in relation to limitations when whichever of the solutions $z = z(t, t_0, z_0)$ does not fulfill the limitations (5), in any while from the area $t \in [t_0, T]$.

The comparison between technical stability and stability by Lapunov is reasonable only with the assumption that for the Lapunov stability there occur constant disturbances.

The examination of stability of road vehicle motion is strictly related to the problems of road traffic safety. A question of stable car motion is important, particularly in the event of occurrence of random disturbances from the uneven road surface. The definition closest in its assumptions to examination of mathematical models is stochastic technical stability. It allows including the random disturbances, and the obtained results can be related to the results of analysis of real technical objects.

2.1. Definition of stochastic technical stability

Assumptions. There is the given set of stochastic equations

$$x(t) = f[x, t, \xi(t)] \qquad (6)$$

where and and are vectors, whereas , is stochastic process describing the random disturbances. For the function assumptions were established, that for every x, $y = (y_1, ..., y_n)$ i $t \ge 0$, and for the stochastic process $f[0, t, \xi(t)]$ there is

$$P\{\int_0^T |f(0,t,\xi(t))| dt < \infty\} = 1, \quad \forall T > 0 \qquad (7)$$

The existence of stochastic process was also assumed, which fulfills the Lipschitz criteria in [0,T] (8) for another process *η(t)*, also absolutely integrable in the given area.

$$|f(x_2, t, \xi(t)) - f(x_1, t, \xi(t))| \le \eta(t)|x_2 - x_1| \qquad (8)$$

The result of the assumptions above is the existence of only one solution with the initial conditions $t = t_0$ and $x(t_0) = x_0$, which is the stochastic process, absolutely constant, with the probability *1* for $t \ge t_0$. It was also assumed that there are two areas within the Euclidean space

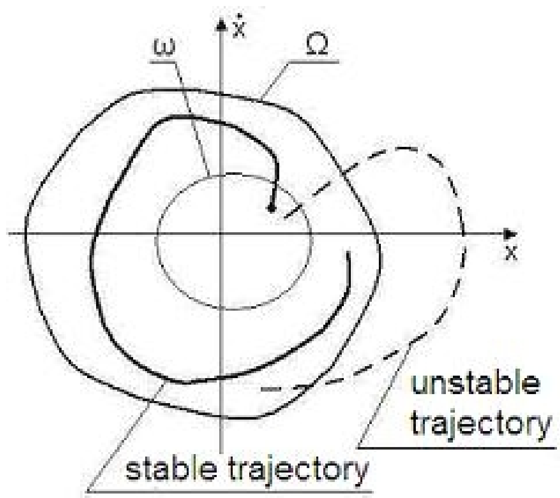

Fig.1. Graphic illustration of stochastic technical stability
Source: [4]

En: ω – finite and open, and Ω – finite and closed, where $\omega \subset \Omega$. There is also a positive number ε, where $0<\varepsilon<1$ and stochastic process $X(t)$ defined for $t \geq t_0$. The initial conditions of the solution are marked as $t = t_0$, $x(t_0) = x_0$ and the solution itself is (t, t_0, x_0). The definition of stochastic technical stability is as follows: if every solution of (6), having the initial conditions within ω, lies within Ω with probability $1\text{-}\varepsilon$ then the structure (6) is technically stochastically stable in respect of ω, Ω and $\xi(t)$ with the probability of $1\text{-}\varepsilon$ (fig.1).

$$P\{(t, t_0, x_0) \in \Omega\} > 1 - \varepsilon \quad for \quad \bar{x}_0 \in \omega \tag{9}$$

The notion of stochastic technical stability was previously used in examination of the rail wheel set motion [3].

3. Road accidents analysis – side crashes

Car body deformations resulting from collision can be the reason of disturbance of its geometry. Important aspect is to define the form, which these disturbances can adopt and the consequences of such situation. The areas especially exposed to damage and simultaneously responsible for energy absorption and passive safety are the longitudinal and lateral elements of the road vehicle frame or body. They create the carrying structure of a car. The matter of negative influence of disturbances, which can occur in three directions (longitudinal, horizontal and vertical) bases mainly on deformation of these responsible elements. If not repaired properly, in the process of further exploitation they can cause excessive bending and twisting of car body as a result of different normal road reactions on wheels (fig.2). In such manner the car frame or floorboard, which is bended, twisted or both, is the main reason of stability disturbances. Within the scope of this paper the authors focused on the deformations caused by side impact accidents, when a car is hit in the side, because it is one of the most dangerous, most frequent (tab.1) and difficult in the process of damage elimination.

Table 1. Proportional participation of different types of road accidents in the total amount of accidents in Poland between 1995 and 2004

frontal crash	side crash	rear crash	pedestrian crash	crash with barrier	rollover	total
15.31%	32.23%	14%	26.07%	8.62%	3.77%	100%

In fig.3 an exemplary distribution of tension during the side crash is shown. Dark areas indicate in what direction and range can spread the body deformation. It is worth noticing that they can reach the opposite door wall resulting in e.g. bending the body into a bow-like shape. It is obviously not literally the shape of a bow. It can not be noticed without relevant measures regarding of the body structure attributes. Taking under consideration all the parameters specifying wheels and suspension geometry it can be assumed that car body deformation influences their settings to a great extent. It results in direct fixing of those elements to the structure of car body.

The disturbance in wheels and suspension parameters can then cause the loss of car motion stability. Introduction of additional disturbances in the form of road roughness can greatly hamper the driving and, in the longer period, result in further damage, such as excessive use of tyres or suspension elements. An after-accident car in which, despite the repair process, there still exist disturbances in body geometry can pose a real threat to road traffic safety.

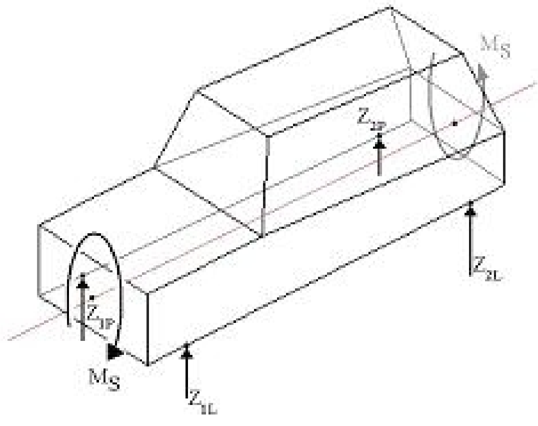

Fig.2. The influence of bending moment on the deformed car body
Source: own research

Fig.3. Tension and deformation directions as a result of side crash
Source: www.vis.uni.stuttgart.de

4. Calculation of location of the car body mass center

Location of the center of mass in car body is difficult to mark by calculation method. If a car is described as the so called digital surface model [4], then it is possible to use the relevant CAD software to define the location of the center of car body mass. A simpler solution is weighing the car without load, or with two and four passengers about 1.7 m tall and 68 kg in weight.

In general, for a car described by n blocks, the location of the center of mass in x, y, z coordinates can be calculated from (10) [4].

$$X_c = \frac{\sum_{i=1}^{n} m_i \cdot x_i}{m_S},\ Y_c = \frac{\sum_{i=1}^{n} m_i \cdot y_i}{m_S},\ Z_c = \frac{\sum_{i=1}^{n} m_i \cdot z_i}{m_S}, \qquad (10)$$

where:

m_S – car mass,
m_i – concentrated block masses representing separate car parts,
x_i, y_i, z_i – coordinates defining location of separate mass blocks,
X_c, Y_c, Z_c – coordinates of the center of mass of the car.

It is possible to define other parameters allowing to calculate the center of mass more accurately (fig.4).

The distance of the center of mass from the axles can be obtained from the equation of moments equilibrium for the given axles placement L (11). The masses burdening both axles are received during weighing of a car on a flat surface (fig.5). Each axle should then lie on a separate scale.

$$\sum M_c = 0:\ -Z_1 a + Z_2 b = 0 \qquad (11)$$

Hence, for $Z_1 = m_1 g$, $Z_2 = m_2 g$, $a + b = L$ and $m_S = m_1 + m_2$ the distance between the center of mass and front and rear axle is obtained.

$$a = \frac{m_2}{m_S} \cdot L\ \text{[m]},\ b = \frac{m_1}{m_S} \cdot L \qquad (12)$$

where:

m_1, m_2 – masses burdening front and rear axle respectfully,
L – placement of axles,
m_S – whole car mass.

The height at which the center of mass is located also relates to the process of weighing of both axles in a car. They are lifted in sequence on the scales, possibly highest (first front then rear). When one axle is lifted, the other lies in the middle of the scale on ground level. The equation to calculate the height of the center of mass is as follows:

$$h_c = \frac{L}{m_S} \cdot \frac{\Delta m}{tg\alpha} \sqrt{L^2 - h^2} + r_d \qquad (13)$$

where:

Δm – the change of load resulting from lifting a car on scale,
α – the angle, at which a car is lifted above the ground level,
h – the height of lifting (between the lifted axle and the axle on ground),
r_d – dynamic radius of a wheel.

It is also possible to calculate the center of mass for each axle and for car body separately. The center of mass of the whole car can be defined through approximated methods. Considering the aspects of road accidents, the authors use the methods treating the whole car, with axles, as a one body.

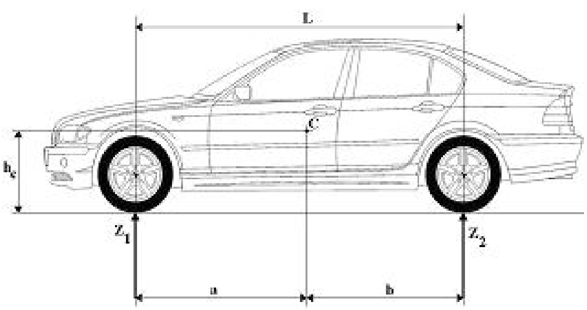

Fig.4. Location of the center of mass for the known axles' placement
Source: own research

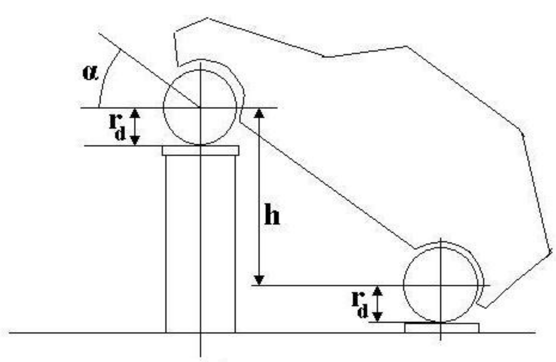

Fig.5. Car weighing in order to define the location of its center of mass
Source: own research

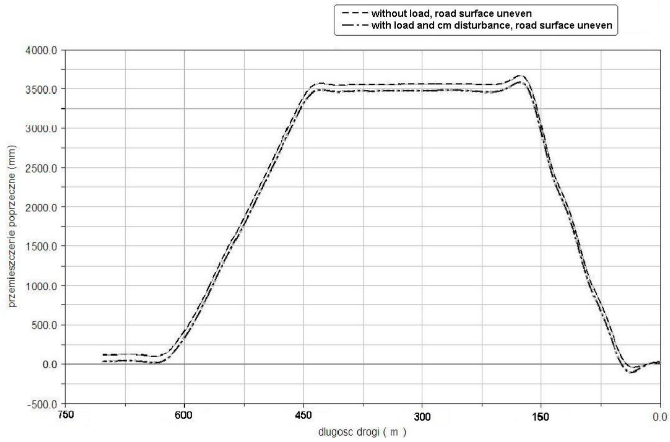

Fig.6. Results of the given simulation in MSC Adams/Car software
Source: own research

5. Stochastic technical stability – research

At present the simulations concerning stability of car mathematical model are lead by the authors. Simulations are executed with the use of MSC Adams/Car software, which allows analysis in real conditions of motion. Research is done for the velocity 100 km/h, for the car unloaded and loaded with driver plus the parameters of the center of mass disturbed by 60 mm each, as if the car was not controlled on the repairing frame in the process of repair. The car rides for about 700 m, realizing the double lane change maneuver as well. The results for two cases – no load and driver load with disturbed parameters of the center of mass both on uneven road are shown in fig.6.

6. Conclusions

First results of simulations show, that both uneven road and car body deformation (change of the center of mass parameters) result in the disturbances of car motion. These results have qualitative connections to the notion of stochastic technical stability. Further research will contain simulations of car mathematical model and definition of the probability of stable motion. The analysis will be made to define quantitative connections relating obtained results to stochastic technical stability and to real objects.

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