Kirill VOYNOV
Petersburg State Transport University,
Mechanical Faculty, Chair of Theory of mechanisms and robotics
RUSSIA, 190031, Saint-Petersburg, Moskovsky pr., 9
Corresponding author. E-mail: forstar@mail.ru

STABILITY OF THE LAWS FOR THE DISTRIBUTION OF THE CUMULATIVE FAILURES IN RAILWAY TRANSPORT

Summary. There are very many different laws of distribution (for example), bell-shaped (Gaussian) distribution, lognormal, Weibull distribution, exponential, uniform, Poisson's, Student's distributions and so on, which help to describe the real picture of failures with elements in various mechanical systems, in locomotives and carriages, too. To diminish the possibility of getting the rough error in the output of maths data treatment the new method is demonstrated in this article. The task is solved both to the discrete, and to the continuous distributions.

СТАБИЛЬНОСТЬ ЗАКОНОВ РАСПРЕДЕЛЕНИЯ ДЛЯ НАКОПЛЕННЫХ ОТКАЗОВ НА ЖЕЛЕЗНОДОРОЖНОМ ТРАНСПОРТЕ

Резюме. Имеется очень много различных законов распределения (например), нормальный (Гаусса), логарифмически нормальный, Вейбулла, экспоненциальный, равномерный, Пуассона, Стьюдента и др., которые помогают описать реальную картину отказов с элементами в разных механических системах, а также в локомотивах и вагонах.

1. INTRODUCTION

It is common knowledge that the mistake in a description and acceptance of the wrong distribution law leads to the great financial expenditure and to the infringement of the production cycle (Fig. 1).

The term of operation for the same system will be quite different if one or another law of distribution is taken into account using traditional criteria (for example, χ^2, $n\omega^2$, λ and others). For this case if the survival probability has the constant meaning 0.6, the first term of operation gives 0.243 only but if it is the second version, the term is corresponded to 24.0! It's the essential difference.

But could the received law be extrapolated for the longer term of operation for our system than it was accepted? Or maybe it's true only for definite limits of statistical data. This important task will be theoretically solved for two examples: to the discrete (1) and to the continuous (2) distributions for normal law. In both cases the main parameter (as a criterion will be the coefficient of variation /*f* or *v*/ which for the normal distribution must be less than 0.3(3)). Moreover $v = f = \sigma / M$, where: σ - the quadratic mean declination; M - is the mathematical expectation.

$i := 1..10 \quad j := 1..10 \quad T := 9 \quad \sigma := 12 \quad \lambda := 10^{-1.2}$

$t_i :=$ $k_j :=$

t_i	k_j
0	0
8	15
16	30
24	45
32	60
40	65
48	70
56	75
64	80
72	85

$$P1_i := e^{-\lambda \cdot t_i} \qquad P2_j := e^{\frac{-(k_j)^2}{8\cdot\sigma^2}}$$

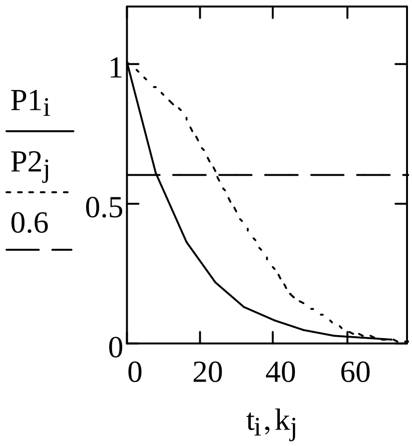

Fig. 1. The two laws of distribution with different survival probability $P1_i$ and $P2_j$
Рис. 1. Два закона распределения с различными вероятностями безотказной работы $P1_i$ и $P2_i$

2. THE ANALYSIS OF THE STABILITY FOR THE DISCRETE LAW OF DISTRIBUTION (1)

Let's N is the initial value of the unregulated statistical information.

Then $M = \sum n_i x_i / N$; dispersion $D = \sigma^2 = \dfrac{\sum n_i (x_i - M)^2}{N-1}$,

where: x_i – the meaning of the accidental observed magnitude; n_i – the number of cases and $\sum n_i = N$; moreover $\nu^2 = f^2 = \sigma^2 / M^2$.

Now let's throw off ΔN from N by accidental way. Then the residual part of the initial statistical data $N_1 = N - \Delta N$.

Indexes Δ and 1 have the link only with the parameters connected for ΔN and N_1.

The next dependence must be obtained:

$$f^2 = \frac{1}{N-1}\left(\sum n_i x_i^2 - 2M\sum n_i x_i + M^2 \sum n_i\right)\frac{1}{M^2} = \frac{1}{N-1}\left(\frac{\sum n_i x_i^2}{M^2} - N\right);$$

$$\sum n_i x_i^2 = \sigma^2 (N-1) + M^2 N;$$

$$\sum \Delta n_i x_i^2 = \sigma_\Delta^2 (\Delta N - 1) + M_\Delta^2 \Delta N;$$

$$\sum (n_i - \Delta n_i) x_i^2 = \sigma_1^2 (N - \Delta N - 1) + M_1^2 (N - \Delta N);$$

$$M_1 = \frac{1}{N - \Delta N}\left(NM - \Delta N M_\Delta\right) = \frac{1}{N - \Delta N}\sum (n_i - \Delta n_i) x_i .$$

Let's the sign $g = M/M_\Delta$. Then

$$f_1^2 = \frac{1}{N-\Delta N-1}\left[\frac{\sum n_i x_i^2}{M_1^2} - \frac{\sum \Delta n_i x_i^2}{M_1^2} - (N-\Delta N)\right] =$$

$$= \frac{1}{N-\Delta N-1}\left\{\frac{\left[\sigma^2(N-1)-\sigma_\Delta^2(\Delta N-1)+M^2N-M_\Delta^2\Delta N\right](N-\Delta N)^2}{(NM-\Delta NM_\Delta)^2} - (N-\Delta N)\right\} =$$

$$= \frac{N-\Delta N}{N-\Delta N-1}\left\{g^2\frac{\left[f^2(N-1)-f_\Delta^2 g^{-2}(\Delta N-1)+N-g^{-2}\Delta N\right](N-\Delta N)-(gN-\Delta N)^2}{(gN-\Delta N)^2}\right\} =$$

$$= \frac{N-\Delta N}{N-\Delta N-1}\,\frac{(N-\Delta N)\left[(N-1)f^2g^2-(\Delta N-1)f_\Delta^2\right]-N\Delta N(g-1)^2}{(gN-\Delta N)^2}.$$

Having taken the square root from f_1^2, the output will be found. But the all terms in the last formula don't have index 1. So we needn't work with the big part of statistical data to get the final result. If it will be less 0.3(3), the law of distribution is the same. So it's a stable law, and the observation for the system can be stopped.

Analogously the next results can be obtained to coefficient of asymmetry C_1 and of excess E_1. For all initial statistical data:

$$C = \frac{\sum n_i (x_i - M)^3}{N\sigma^3} = \frac{1}{N\sigma^3}\left[\sum n_i x_i^3 - 3M\sigma^2(N-1) - M^3N\right] = \frac{\sum n_i x_i^3}{N\sigma^3} - \frac{3(N-1)}{Nf} - \frac{1}{f^3};$$

$$\frac{\sum n_i x_i^3}{N\sigma^3} = C + \frac{3(N-1)}{Nf} + \frac{1}{f^3};$$

$$\frac{\sum \Delta n_i x_i^3}{\Delta N\sigma_\Delta^3} = C_\Delta + \frac{3(\Delta N-1)}{\Delta N f_\Delta} + \frac{1}{f_\Delta^3};$$

then

$$C_1 = \frac{\sum (n_i - \Delta n_i)(x_i - M_1)^3}{\sigma_1^3(N-\Delta N)} =$$

$$= \frac{(N-\Delta N)^2}{f_1^3(gN-\Delta N)^3}\left\{g^3\left[CNf^3+3f^2(N-1)+N\right]-\left[C_\Delta f_\Delta^3\Delta N+3(\Delta N-1)f_\Delta^2+\Delta N\right]\right\} -$$

$$- \frac{3(N-\Delta N-1)}{(N-\Delta N)f_1} - \frac{1}{f_1^3},$$

and for excess:

$$(E_1+3)(N-\Delta N) = \frac{1}{(gN-\Delta N)^4 f_1^4}\left\{g^4\left[(E+3)Nf^4+4CNf^3+6(N-1)f^2+N\right] -\right.$$

$$\left. -\left[(E_\Delta+3)\Delta N f_\Delta^4 + 4C_\Delta \Delta N f_\Delta^3 + 6(\Delta N-1)f_\Delta^2 + \Delta N\right]\right\} - \frac{4C_1}{f_1}(N-\Delta N) - \frac{6(N-\Delta N-1)}{f_1^2} - \frac{N-\Delta N}{f_1^4}.$$

3. THE ANALYSIS OF THE STABILITY FOR THE CONTINUOUS LAW OF DISTRIBUTION (2)

Now let's get the same formulas as it was for position (**1**).

Namely: MX - is the mathematical expectation; $\sigma^2 = DX$ is dispersion. Then

$$MX = \int_a^b x f(x)dx; \qquad \sigma^2 = DX = \int_a^b (x - MX)^2 f(x)dx;$$

$$\nu^2 = f^2 = \frac{1}{(MX)^2}\int_a^b (x - MX)^2 f(x)dx = \frac{1}{(MX)^2}\int_a^b f(x)x^2 dx - 1 = \frac{DX}{(MX)^2};$$

$$\int_a^b f(x)x^2 dx = DX + (MX)^2; \qquad \int_a^b f_1(x)x^2 dx = D_1 X + (M_1 X)^2;$$

$$\int_a^b f_\Delta(x)x^2 dx = D_\Delta X + (M_\Delta X)^2; \qquad M_1 X = \int_a^b f(x)x dx - \int_a^b f_\Delta(x)x dx;$$

then

$$\nu_1^2 = f_1^2 = \frac{1}{(M_1 X)^2}\int_a^b f_1(x)x^2 dx - 1 = \frac{DX + (MX)^2 - D_\Delta X - (M_\Delta X)^2 - (MX - M_\Delta X)^2}{(MX - M_\Delta X)^2} =$$

$$= \frac{(MX)^2 f^2 - (M_\Delta X)^2 f_\Delta^2 - 2(M_\Delta X)^2 + 2MX \cdot M_\Delta X}{(MX - M_\Delta X)^2} =$$

$$= \frac{(M_\Delta X)^2}{(g-1)^2}\cdot\frac{g^2 f^2 - f_\Delta^2 - 2 + 2g}{(M_\Delta X)^2} = \frac{g^2 f^2 - f_\Delta^2}{(g-1)^2} + \frac{2}{g-1}.$$

So the final formula is mighty easy to work with it. And else: we needn't calculate the big residual data N_1.

Now, as in the first example above, analogously the next results can be obtained to coefficients of asymmetry C_1 and of excess E_1.

$$C = \frac{1}{\sigma^3}\int_a^b (x - MX)^3 f(x)dx =$$

$$= \frac{1}{\sigma^3}\left[\int_a^b x^3 f(x)dx - 3MX\int_a^b x^2 f(x)dx + 3(MX)^2\int_a^b x f(x)dx - (MX)^3\int_a^b f(x)dx\right] =$$

$$= \frac{1}{\sigma^3}\int_a^b x^3 f(x)dx - \frac{3}{f} - \frac{1}{f^3};$$

$$\int_a^b x^3 f(x)dx = \left(C + \frac{3}{f} + \frac{1}{f^3}\right)\sigma^3;$$

$$C_1 = \frac{1}{\sigma_1^3}\int_a^b (x - M_1 X)^3 f_1(x)dx =$$

$$= \frac{1}{\sigma_1^3}\int_a^b [f(x) - f_\Delta(x)]x^3 dx - \frac{3}{f_1} - \frac{1}{f_1^3} = \left(C + \frac{3}{f} + \frac{1}{f^3}\right)\frac{\sigma^3}{\sigma_1^3} - \left(C_\Delta + \frac{3}{f_\Delta} + \frac{1}{f_\Delta^3}\right)\frac{\sigma_\Delta^3}{\sigma_1^3} - \frac{3}{f_1} - \frac{1}{f_1^3} = (*)$$

$$\frac{\sigma^3}{\sigma_1^3} = \frac{f^3}{f_1^3}\left(\frac{MX}{M_1 X}\right)^3 = \frac{f^3}{f_1^3}\left(\frac{MX}{MX - M_\Delta X}\right)^3 = \frac{f^3 g^3}{f_1^3 (g-1)^3};$$

$$\frac{\sigma_\Delta^3}{\sigma_1^3} = \frac{f_\Delta^3}{f_1^3}\left(\frac{M_\Delta X}{MX - M_\Delta X}\right)^3 = \frac{f_\Delta^3}{f_1^3 (g-1)^3};$$

$$\frac{f^3}{f_1^3}\cdot\frac{(MX)^3}{(MX-M_\Delta X)^3}=\frac{f^3}{f_1^3}\cdot\frac{g^3}{(g-1)^3};$$

$$\frac{f_\Delta^3}{f_1^3}\cdot\frac{(M_\Delta X)^3}{(MX-M_\Delta X)^3}=\frac{f_\Delta^3}{f_1^3(g-1)^3}.$$

$$(*)=\frac{\left(C+\frac{3}{f}+\frac{1}{f^3}\right)f^3g^3}{f_1^3(g-1)^3}-\frac{\left(C_\Delta+\frac{3}{f_\Delta}+\frac{1}{f_\Delta^3}\right)f_\Delta^3}{f_1^3(g-1)^3}-\frac{3}{f_1}-\frac{1}{f_1^3}=$$

$$=\frac{1}{f_1^3(g-1)^3}\left[g^3\left(Cf^3+3f^2+1\right)-\left(C_\Delta f_\Delta^3+3f_\Delta^3+1\right)\right]-\frac{3}{f_1}-\frac{1}{f_1^3}.$$

Because the meaning for f_1 is determined above, so the task is solved too.
At last let's get the formula for excess:

$$3+E=\frac{\mu_4}{\sigma^4};$$

$$\mu_4=\int_a^b (X-MX)^4 f(x)dx;$$

$$3+E=\frac{1}{\sigma^4}\left\{\int_a^b x^4 f(x)dx-4MX\left(C+\frac{3}{f}+\frac{1}{f^3}\right)\sigma^3+6(MX)^2\left[DX+(MX)^2\right]-4(MX)^4+(MX)^4\right\}=$$

$$=\frac{1}{\sigma^4}\int_a^b x^4 f(x)dx-\frac{4}{f}\left(C+\frac{3}{f}+\frac{1}{f^3}\right)+\frac{6}{f^2}+\frac{3}{f^4};$$

$$\int_a^b x^4 f(x)dx=\left[3+E+\frac{4}{f}\left(C+\frac{3}{f}+\frac{1}{f^3}\right)-\frac{6}{f^2}-\frac{3}{f^4}\right]\sigma^4;$$

$$3+E_1=\frac{1}{\sigma_1^4}\int_a^b x^4 f_1(x)dx-\frac{4}{f_1}\left(C_1+\frac{3}{f_1}+\frac{1}{f_1^3}\right)+\frac{6}{f_1^2}+\frac{3}{f_1^4}=$$

(*note*: let's { * } = *A*) then

$$=\frac{1}{\sigma_1^4}\int_a^b x^4 f(x)dx-\frac{1}{\sigma_1^4}\int_a^b x^4 f_\Delta(x)dx+A=\left[3+E+\frac{4}{f}\left(C+\frac{3}{f}+\frac{1}{f^3}\right)-\frac{6}{f^2}-\frac{3}{f^4}\right]\frac{\sigma^4}{\sigma_1^4}-$$

$$-\left[3+E_\Delta+\frac{4}{f_\Delta}\left(C_\Delta+\frac{3}{f_\Delta}+\frac{1}{f_\Delta^3}\right)-\frac{6}{f_\Delta^2}-\frac{3}{f_\Delta^4}\right]\frac{\sigma_\Delta^4}{\sigma_1^4}+A=\{*\}$$

$$\frac{\sigma^4}{\sigma_1^4}=\frac{f^4g^4}{f_1^4(g-1)^4};\qquad\qquad \frac{\sigma_\Delta^4}{\sigma_1^4}=\frac{f_\Delta^4}{f_1^4(g-1)^4}$$

$$\{*\}=\left[3+E+\frac{4}{f}\left(C+\frac{3}{f}+\frac{1}{f^3}\right)-\frac{6}{f^2}-\frac{3}{f^4}\right]\frac{f^4g^4}{f_1^4(g-1)^4}-$$

$$-\left[3+E_\Delta+\frac{4}{f_\Delta}\left(C_\Delta+\frac{3}{f_\Delta}+\frac{1}{f_\Delta^3}\right)-\frac{6}{f_\Delta^2}-\frac{3}{f_\Delta^4}\right]\frac{f_\Delta^4}{f_1^4(g-1)^4}-\frac{4}{f_1}\left(C_1+\frac{3}{f_1}+\frac{1}{f_1^3}\right)+\frac{6}{f_1^2}+\frac{3}{f_1^4}=$$

$$=\frac{1}{f_1^4(g-1)^4}\left\{\left[3+E+\frac{4}{f}\left(C+\frac{3}{f}+\frac{1}{f^3}\right)-\frac{6}{f^2}-\frac{3}{f^4}\right]f^4g^4-\right.$$

$$-\left[3+E_{\Delta}+\frac{4}{f_{\Delta}}\left(C_{\Delta}+\frac{3}{f_{\Delta}}+\frac{1}{f_{\Delta}^{3}}\right)-\frac{6}{f_{\Delta}^{2}}-\frac{3}{f_{\Delta}^{4}}\right]f_{\Delta}^{4}\Bigg\}-\frac{4}{f_{1}}\left(C_{1}+\frac{3}{f_{1}}+\frac{1}{f_{1}^{3}}\right)+\frac{6}{f_{1}^{2}}+\frac{3}{f_{1}^{4}}.$$

Because the meaning for f_1 is determined above, so this task is solved too.

3. CONCLUSION

1. By means of artificial elimination some amount of statistical data from initial information and using traditional criteria to check the model of distribution law the researcher can not only find is this law a stable one, but can get the response about possibility to do the extrapolation conclusions on the further term of operation for elements in locomotives, cars and others systems.

2. Another recommendation is next: the share of consistent elimination must be approximately 20, 40 and 60 % from the initial value of the statistical data. If in the first case the law of distribution will be the same (as for the all initial information), it means that the stability is not high and extrapolation of conclusions for reliability of railway parts mustn't be made for the future period of operation for the system. In the second case the decision for the extrapolation of conclusion has a chance about 50:50.And at last in the third case we can stop our observation for the system and calculation, because the law of distribution has a stability, and do the extrapolation our conclusions about the reliability of locomotives and cars for the future.

3. Suggested procedure works good if *N* is usually more than 40 or 50.

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