two stage transportation problem,
bi-criteria transportation problems

Ahmad MURAD, Amier AL-ALI*, Esaid ELLAIMONY, Helal ABDELWALI
The Public Authority for Applied Education & Training
College of Technological Studies
Automotive & Marine Engineering Department
P.O.Box 24332, Safat, Kuwait - 13104
*****_Corresponding author_: E-mail: amieralali@hotmail.com

ON BI-CRITERIA TWO-STAGE TRANSPORTATION PROBLEM: A CASE STUDY

Summary. The study of the optimum distribution of goods between sources and destinations is one of the important topics in projects economics. This importance comes as a result of minimizing the transportation cost, deterioration, time, etc. The classical transportation problem constitutes one of the major areas of application for linear programming. The aim of this problem is to obtain the optimum distribution of goods from different sources to different destinations which minimizes the total transportation cost. From the practical point of view, the transportation problems may differ from the classical form. It may contain one or more objective function, one or more stage to transport, one or more type of commodity with one or more means of transport. The aim of this paper is to construct an optimization model for transportation problem for one of mill-stones companies. The model is formulated as a bi-criteria two-stage transportation problem with a special structure depending on the capacities of suppliers, warehouses and requirements of the destinations. A solution algorithm is introduced to solve this class of bi-criteria two-stage transportation problem to obtain the set of non-dominated extreme points and the efficient solutions accompanied with each one that enables the decision maker to choose the best one. The solution algorithm mainly based on the fruitful application of the methods for treating transportation problems, theory of duality of linear programming and the methods of solving bi-criteria linear programming problems.

O DWUKRYTERIALNYM, DWUETAPOWYM PROBLEMIE TRANSPORTOWYM: STUDIUM PRZYPADKU

Streszczenie. Analiza optymalnej dystrybucji towarów pomiędzy punktem początkowym a końcowym jest jednym z ważnych zagadnień w ekonomice projektów. Ma to znaczenie jako wynik minimalizacji kosztów transportu, rozkładu przewozów, czasu, etc. Klasyczny problem transportowy stanowi jedno z głównych zagadnień programowania liniowego. Rozwiązaniem tego problemu jest uzyskanie optymalnej dystrybucji towarów z różnych źródeł do różnych punktów przeznaczenia, co minimalizuje całkowity koszt transportu. Z praktycznego punktu widzenia problemy transportowe mogą się różnić od problemów w formie klasycznej. Mogą one zawierać jedną lub więcej funkcji celu, jedną lub więcej tras, jeden lub więcej rodzajów towarów przewożonych za pomocą jednego lub więcej środków transportu. Celem artykułu jest stworzenie modelu optymalizacyjnego, który rozwiązuje problem transportowy dla jednej z firm przewozowych, specjalizującej się w przewozie kruszyw. Model jest sformułowany jako dwukryterialny, dwuetapowy problem transportowy w celu pozyskania zbioru ekstremów lokalnych oraz skutecznych rozwiązań związanych z takimi, które pozwalają osobie podejmującej decyzję wybrać te najodpowiedniejsze. Algorytm rozwiązania bazuje głównie na efektywnej aplikacji metod rozwiązywania problemów transportowych, teorii dualizmu programowania liniowego oraz metodach poszukiwania rozwiązań dwukryterialnych problemów programowania liniowego.

1. INTRODUCTION

The classical transportation problem (TP) refers to a special class of linear programming problems. In a typical problem, a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ respectively. In addition there is a penalty c_{ij} associated with transporting a unit of product from source i to destination j. This penalty may be cost, delivery time, deterioration, or safety of delivery, etc… A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j.

In real life situations, the transportation problem usually involves multiple, conflicting, and in-commensurate objective functions. This type of problems is so called multi-objective transportation problem (MOTP). The solution of this problem is called a non-dominated solution (if we refer to the objective function) and an efficient solution (if we refer to the decision variables space) [1]. Each of them is defined as follows:

Definition 1.1 (Non-dominated solution; Ringust and Rinks [2]. A feasible vector $x^o \in s$ (s is a feasible region) yields a non-dominated solution of (MOTP), if and only if, there is no vector $x \in s$ such that $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}^o \ \forall k$ and $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} < \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}^o$ for some k).

Definition 1.2 (Efficient solution; Steuer [3]. A point $x^o \in s$ is efficient iff there does not exist another $x \in s$ such that $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \le \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}^o \ \forall k$ and $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij} \ne \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^k x_{ij}^o$ for some k. For more details, see [3] pp. 148-149.

In the literature solution approaches for OTP are classified into four categories [1]: Interactive approach, non-interactive approach, goal programming approach and fuzzy programming approaches. Each category has its advantages and limitations. Aneja and Nair [4] studied a bi-criteria TP. Diaz [5,6], Iserman [7], and Kasana [8] developed different approaches to generate the set of efficient solutions. The solution procedure of these methods depends on determining the set of efficient solutions and finally the DM is responsible for selecting the preferred solution out of this set. Also in real life situations, the transportation processes may not operate always directly among suppliers and customers. It may done in two or multiple stages. Some approaches for solving such transportation problems are listed in [9], [10], and [11]. In [10], the formulation of different multistage transportation problems and an algorithm for solving a class of them are presented this class can be solved using the decomposition technique of large scale linear programming utilizing the special nature of the transportation problems. In [11], different formulations of two-stage transportation problem depend on the relation between capacities of suppliers at first stage, capacities of warehouses at first stage which are the suppliers of the second stage, and requirements of customers at second stage. The algorithm of solving such problems is also presented in [11]. This algorithm based on a duality theory.

In this paper, we introduce the mathematical formulation of the transportation for one of mill-stones companies which is a bi-criteria two-stage transportation problem. The relation between the summation of average capacities of main stores from wheat which must be transported to mill-stones ($\sum a_i$) and summation of average capacities of mill-stones itself ($\sum e_k$) for the first stage, and the summation of average requirements of sub-stores and customers ($\sum b_j$) make the mathematical formulation of the problem takes one of mathematical forms which are presented in [11] but with two objective functions. The company's DM needs to introduce a solution algorithm which gives the set of non-dominated extreme points and its related set of efficient solutions where the DM is responsible for selecting the preferred solution out of these sets. We introduce the mathematical formulation of the problem and the solution algorithm. The presented algorithm consists of two phases, phase I based on the algorithm presented in [4] by aneja and Nair to generate the set of non-dominated extreme points and its related efficient solutions for bi-criteria linear programming problems in general. Phase II is a modification of the algorithm presented in [11] to solve our class of two-stage transportation problem.

2. CASE STUDY

2.1. Problem description

The transportation problem of one of mill-stones companies is a bi-criteria two stage problem. There are 4 sources (main stores) and 6 warehouses (mill-stones), at the first stage. The second stage contains 6 sources (mill-stones) and 9 destinations include sub-stores and customers. The problem includes 2-objective functions; minimization of transportation cost and minimization of transportation deteriorations.

2.2. Problem formulation

The problem takes the following form:

Minimize $Z_1 = \sum_{i=1}^{4}\sum_{k=1}^{6} c_{ik}^1 x_{ik}^1 + \sum_{k=1}^{6}\sum_{j=1}^{9} c_{kj}^2 x_{kj}^2$, Minimize $Z_2 = \sum_{i=1}^{4}\sum_{k=1}^{6} d_{ik}^1 x_{ik}^1 + \sum_{k=1}^{6}\sum_{j=1}^{9} d_{kj}^2 x_{kj}^2$, Subj.to

$$\sum_{k=1}^{6} x_{ik}^1 = a_i, i = 1,2,3,4; \quad (1); \qquad \sum_{i=1}^{4} x_{ik}^1 = e_k, i = 1,2,...,6; \quad (2)$$

$$\sum_{j=1}^{9} x_{kj}^2 = e_k, k = 1,2,...,6; \quad (3); \qquad \sum_{k=1}^{6} x_{kj}^2 = b_j, j = 1,2,...,9; \quad (4)$$

$$x_{ik}^1, x_{kj}^2 \geq 0, \text{ i=1, 2, 3, 4; k=1,2,…,6; and j=1,2,…,9.} \quad (5)$$

Where:

c_{ik}^1, d_{ik}^1 : transportation cost and transportation deterioration for first stage, i=1, 2, 3, 4; k=1,2,., 6;

c_{kj}^2, d_{kj}^2 : transportation cost and transportation deterioration for second stage, k=1, 2, …, 6; j=1,2,…, 9; , e_k: capacities of warehouses k, k=1,2,.,6;

a_i: capacities of suppliers i, i=1,2, 3,4; , b_j: requirements of destinations j, j=1,2,.,9.

2.3. Data collection

Tab. 1

Ilustrates transportation costs, transportation deteriorations (in unit cost per ton) inside table cells, capacities of suppliers (in ton) and capacities of warehouses (in ton) for the first stage

Mill stones Main Stores	W_1	W_2	W_3	W_4	W_5	W_6	Availabilities
S_1	(0, 0)	(9, 1)	(12, 3)	(15, 4)	(18, 5)	(21, 6)	4350
S_2	(12, 3)	(9, 2)	(0, 0)	(9, 2)	(12, 3)	(15, 4)	5340
S_3	(15, 4)	(13, 3)	(9, 2)	(0, 0)	(9, 3)	(12, 4)	5320
S_4	(22, 5)	(20, 5)	(17, 3)	(13, 2)	(11, 2)	(9, 2)	4017
Requirements	2900	2624	3560	4213	3729	4011	19027 21037

Tab. 2

Illustrates transportation costs, transportation deteriorations (in unit cost per ton) inside table cells, capacities of warehouses (in ton) and requirements of destinations (in ton) for the second stage

Mill stones / Main Stores	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	Availa-bilities
W_1	(9, 2)	(0,0)	(9,1)	(11,2)	(12,3)	(15, 4)	(18, 5)	(21, 6)	(22, 5)	2900
W_2	(9, 1)	(9,1)	(0,0)	(9,1)	(9,2)	(13, 3)	(15, 2)	(19, 3)	(20, 5)	2624
W_3	(13, 4)	(12,3)	(9,2)	(9,2)	(0,0)	(9,2)	(12, 3)	(15, 4)	(17, 3)	3560
W_4	(16,3)	(15,4)	(13,3)	(11,4)	(9,2)	(0,0)	(9,3)	(12, 4)	(13, 2)	4213
W_5	(19,5)	(18,5)	(15,2)	(14,3)	(12,3)	(9,3)	(0,0)	(9,2)	(11, 2)	3729
W_6	(22,5)	(21,6)	(19,3)	(17,4)	(15,4)	(12, 4)	(9,2)	(0,0)	(9,2)	4011
Requirements	910	2853	2594	1466	1759	2848	1389	2136	1424	21037 17379

From the collected data, the relation between capacities of suppliers a_i, i=1, 2, 3, 4; capacities of warehouses e_k, k=1, 2, …, 6; and requirements of destinations b_j, j=1, 2, ,9 takes the following relation:

$$\sum_{j=1}^{9} b_j\,(17379) < \sum_{i=1}^{4} a_i\,(19024) < \sum_{k=1}^{6} e_k\,(21037)$$

From the relation between $\sum_i a_i$, $\sum_k e_k$, $\sum_j b_j$, the problem cannot be solved as a two-separated bi-criteria single stage transportation problem so it could be solved as a bi-criteria linear programming problem or by using our presented algorithm. This algorithm is based on the algorithm presented in [11] to solve unbalanced cases of single-objective two-stage transportation problems and the algorithm presented in [4] to obtain the set of non-dominated extreme points in the objective space of bi-criteria single stage transportations.

3. A SOLUTION ALGORITHM

In the following, we describe an algorithm for solving a class of bi-criteria two-stage transportation problems with the relation between capacities of suppliers (a_i), capacities of warehouses (e_k) and requirements of destinations (b_j) is: $(\sum_{j=1}^{n} b_j < \sum_{i=1}^{m} a_i < \sum_{k=1}^{K} e_k)$. The solution algorithm is divided into two phases:

Phase (I): Determines the set of non-dominated extreme points in the objective space for bi-criteria transportation problems. Phase (II): Determines the optimal solution of a class of two-stage transportation problems. The proposed solution algorithm can be summarized in the following steps.

Phase (I):

Step (1):

Find two dual forms, one dual form for each single objective two-stage transportation problem. Use the variables (u_i , v_k) and (v'_k , w_j) for the dual problems of the first and the second stages respectively.

Step (2):

Go to Phase (II), find the optimum solution for the two-stage problem with the first objective function $Z_1^{(1)} = \min (Z_1 \mid x \in M)$. Then calculate $Z_2^{(1)} = \min (Z_2 \mid Z_1 = Z_1^{(1)}$ and $x \in M)$. Set h = 1. Similarly, go to Phase (II), find the optimum solution for the two-stage problem with the second objective function $Z_2^{(2)} = \min (Z_2 \mid x \in M)$. Then calculate $Z_1^{(2)} = \min (Z_1 \mid Z_2 = Z_2^{(2)}$ and $x \in M)$.
If $(Z_1^{(2)}, Z_2^{(2)}) = (Z_1^{(1)}, Z_2^{(1)})$ stop. Otherwise, record $(Z_1^{(2)}, Z_2^{(2)})$ and set h= h+1.
Define sets L = {(1,2)} and E = φ

Step (3):

Choose an element $(r,s) \in L$ and set $\alpha_1^{(r,s)} = \mid Z_2^{(s)} - Z_2^{(r)} \mid$ and $\alpha_2^{(r,s)} = \mid Z_1^{(s)} - Z_1^{(r)} \mid$.
Prepare the problem using the new objective function as follows:

$$\min \left(\sum_{i=1}^{m}\sum_{k=1}^{k}\bar{c}_{ik}^{1}x_{ik}^{1} + \sum_{k=1}^{K}\sum_{j=1}^{n}\bar{c}_{kj}^{2}x_{kj}^{2}\right) \quad , \quad \text{S.T.} \quad x \in M, \quad x \geq 0,$$

Where: $\bar{c}_{ik}^{1} = (\alpha_1^{(r,s)}c_{ik}^{1} + \alpha_2^{(r,s)}d_{ik}^{1}),\qquad \bar{c}_{kj}^{2} = (\alpha_1^{(r,s)}c_{kj}^{2} + \alpha_2^{(r,s)}d_{kj}^{2})$

Find the dual form of the new problem.

Go to Phase (II), obtain the optimum solution of the new problem ($\bar{x}_{ik}^{1}$ and $\bar{x}_{kj}^{2}$).
If there are alternative optima, choose the optimum solution $\bar{x}_{ik}^{1}$ and $\bar{x}_{kj}^{2}$ for which:

$$\left(\sum_{i=1}^{m}\sum_{k=1}^{K}c_{ik}^{1}\bar{x}_{ik}^{1} + \sum_{k=1}^{K}\sum_{j=1}^{n}c_{kj}^{2}\bar{x}_{kj}^{2} \text{ minimum}\right).$$

Let: $\bar{Z}_1 = \sum_{i=1}^{m}\sum_{k=1}^{K}c_{ik}^{1}\bar{x}_{ik}^{1} + \sum_{k=1}^{K}\sum_{j=1}^{n}c_{kj}^{2}\bar{x}_{kj}^{2}$; $\qquad \bar{Z}_2 = \sum_{i=1}^{m}\sum_{k=1}^{K}d_{ik}^{1}\bar{x}_{ik}^{1} + \sum_{k=1}^{K}\sum_{j=1}^{n}d_{kj}^{2}\bar{x}_{kj}^{2}$

If $(\bar{Z}_1, \bar{Z}_2)$ equal $(Z_1^{(r)}, Z_2^{(r)})$ or $(Z_1^{(s)}, Z_2^{(s)})$, set $E = E \cup \{(r,s)\}$ and go to step (4).
Otherwise, set h = h + 1, record $(Z_1^{(h)}, Z_2^{(h)})$ such that $Z_1^{(h)} = \bar{Z}_1$ and $Z_2^{(h)} = \bar{Z}_2$ and set $L = L \cup \{(r,h), (h,s)\}$.

Step (4):

Set L = L – {(r,s)}. If L = φ, stop. Otherwise, go to step (3).

Phase (II):
Step (1):

Find the initial basic feasible solution for the second stage of the problem using anyone of the methods which are used in the classical transportation problems. Add a fictitious customer O_{fict}… due to unbalancing between $\sum_k e_k$ and $\sum_j b_j$ for this stage. Also, find initial basic solution for the first stage; add a fictitious source M_{fict} due to unbalancing between $\sum_k e_k$ and $\sum_j a_i$.

Step (2):

Based on step (1), construct a generalized table (Tab. 3) includes the two-stage problem, where the rows in the upper part represent constraints of suppliers at first stage (constraints (1)). Rows in the lower part represent constraints of destinations at second stage (constraints (4)). Every column represents two types of constraints, in the upper part constraints of warehouses at first stage (constraints (2)) and in the lower part constraints of warehouses at second stage (constraints (3)).

Step (3):

For the upper part of the table, check all cells in the M_{fict} column. If any of the cells has a value greater than zero (i.e. if $x_i^1 > 0$), set appropriate $u_i=0$. Also, for any cell in the upper part of the table with a value greater than zero $(x_{ik}^1 > 0)$, set relevant $u_i + v_k = c_{ik}^1$ for the problem with the first objective or $u_i + v_k = d_{ik}^1$ for the problem with the second objective or $\bar{c}_{ik}^1$ for the problem with the new objective.

For any cell in the O_{fict} row under k column has a value greater than zero, put relevant $v_k + v_k^' = 0$, (i.e. $v_k = -v_k^'$).

For any cell in the lower part of the table with a value greater than zero $(x_{kj}^2 > 0)$, put $v_k^' + w_j = c_{kj}^2$ for the problem with the first objective or d_{kj}^2 for the problem with the second objective or $\bar{c}_{kj}^2$ for the problem with the new objective. In case that the problem is not degenerated, we fill (m+2K+n) cells (where (m) is the number of suppliers, (K) is the number of warehouses and (n) is the number of destinations). Calculate values of u_i, v_k, $v_k^'$, and w_j.

Step (4):

If all calculated values u_i, v_k, $v_k^'$, and w_j satisfy the constraints of dual problem, the calculated solution in the generalized table is the optimum solution of the problem, return to Phase (I). If there is at least one constraint of the dual problem is not satisfied, the calculated solution in the generalized table is not optimum.

Step (5):

In case that the calculated solution is not optimum, like in the classical transportation problem, fill a cell in which is the optimum criterion mostly disturbed and find a closed loop of changes associated to this cell by the following process:

- If the conditions $u_i + v_k \le c_{ik}^1$ for the problem with the first objective or $u_i + v_k = d_{ik}^1$ for the problem with the second objective or $\bar{c}_{ik}^1$ for the problem with the new objective; or $v_k^' + w_j \le c_{kj}^2$ for the problem with the first objective or d_{kj}^2 for the problem with the second objective or $\bar{c}_{kj}^2$ for the problem with the new objective, are disturbed, fill relevant cell of the upper or lower table.
- If any of conditions $v_k + v_k^' \le 0$ is disturbed, fill relevant cell in the row O_{fict}.
- If any of conditions $u_i \le 0$ is disturbed, fill relevant cell in the column M_{fict}.

Step (6):

At a closed loop passing through the middle row of O_{fict}, this cell belongs also to vertexes of the closed loop, and it has an opposite sign as the vertexes in upper and lower part of the table. Separate the found loop into two semi-loops and from the values at negative signs choose the minimal one and add it to all values at positive signs and subtract all values at negative signs of the closed semi-loop.

Step (7):

Return to step (3) and after a finite number of iterations, we find the optimum solution. The solution algorithm is illustrated in the flow-chart Fig. 1.

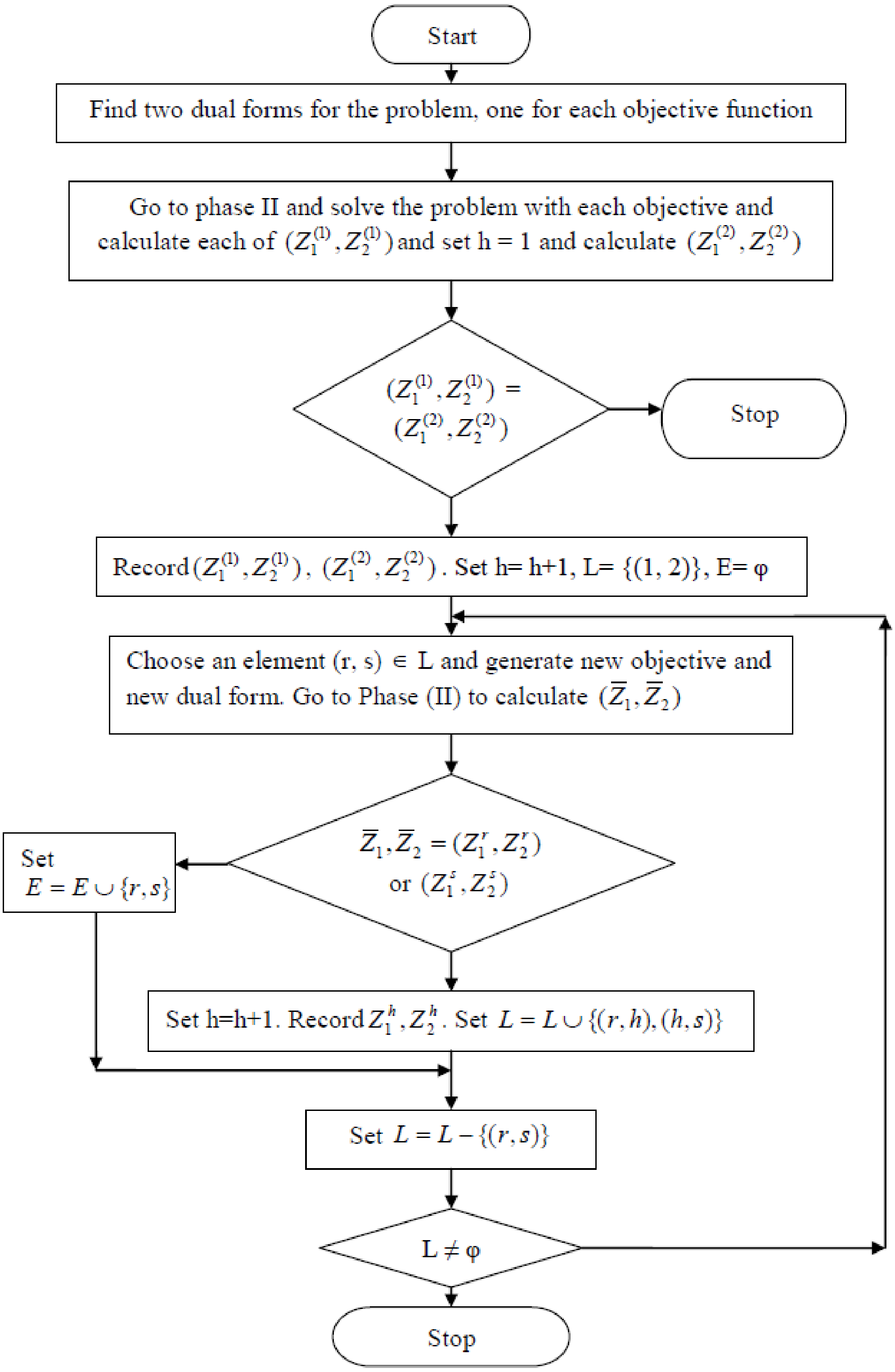

Fig. 1. The algorithm of solution
Rys. 1. Algorytm rozwiązania problemu

4. SOLUTION OF THE CASE STUDY

4.1. Primal form of the problem

Based on the collected data, the transportation problem of the Mill-Stones Company takes the following mathematical formulation:

Minimize $Z_1 = 0x_{11}^1 + 9x_{12}^1 + 12x_{13}^1 + 15x_{14}^1 + 18x_{15}^1 + 21x_{16}^1 +$
$12x_{21}^1 + 9x_{22}^1 + 0x_{23}^1 + 9x_{24}^1 + 12x_{25}^1 + 15x_{26}^1 +$
$15x_{31}^1 + 13x_{32}^1 + 9x_{33}^1 + 0x_{34}^1 + 9x_{35}^1 + 12x_{36}^1 +$
$22x_{41}^1 + 20x_{42}^1 + 17x_{43}^1 + 13x_{44}^1 + 11x_{45}^1 + 9x_{46}^1 +$
$9x_{11}^2 + 0x_{12}^2 + 9x_{13}^2 + 11x_{14}^2 + 12x_{15}^2 + 15x_{16}^2 + 18x_{17}^2 + 21x_{18}^2 + 22x_{19}^2 +$
$9x_{21}^2 + 9x_{22}^2 + 0x_{23}^2 + 9x_{24}^2 + 9x_{25}^2 + 13x_{26}^2 + 15x_{27}^2 + 19x_{28}^2 + 20x_{29}^2 +$
$13x_{31}^2 + 12x_{32}^2 + 9x_{33}^2 + 9x_{34}^2 + 0x_{35}^2 + 9x_{36}^2 + 12x_{37}^2 + 15x_{38}^2 + 17x_{39}^2 +$
$16x_{41}^2 + 15x_{42}^2 + 13x_{43}^2 + 11x_{44}^2 + 9x_{45}^2 + 0x_{46}^2 + 9x_{47}^2 + 12x_{48}^2 + 13x_{49}^2 +$
$19x_{51}^2 + 18x_{52}^2 + 15x_{53}^2 + 14x_{54}^2 + 12x_{55}^2 + 9x_{56}^2 + 0x_{57}^2 + 9x_{58}^2 + 11x_{59}^2 +$
$22x_{61}^2 + 21x_{62}^2 + 19x_{63}^2 + 17x_{64}^2 + 15x_{65}^2 + 12x_{66}^2 + 9x_{67}^2 + 0x_{68}^2 + 9x_{69}^2$

Minimize $Z_2 = 0x_{11}^1 + 1x_{12}^1 + 3x_{13}^1 + 4x_{14}^1 + 5x_{15}^1 + 6x_{16}^1 +$
$3x_{21}^1 + 2x_{22}^1 + 0x_{23}^1 + 2x_{24}^1 + 3x_{25}^1 + 4x_{26}^1 +$
$4x_{31}^1 + 3x_{32}^1 + 2x_{33}^1 + 0x_{34}^1 + 3x_{35}^1 + 4x_{36}^1 +$
$5x_{41}^1 + 5x_{42}^1 + 3x_{43}^1 + 2x_{44}^1 + 2x_{45}^1 + 2x_{46}^1 +$
$2x_{11}^2 + 0x_{12}^2 + 1x_{13}^2 + 2x_{14}^2 + 3x_{15}^2 + 4x_{16}^2 + 5x_{17}^2 + 6x_{18}^2 + 5x_{19}^2 +$
$1x_{21}^2 + 1x_{22}^2 + 0x_{23}^2 + 1x_{24}^2 + 2x_{25}^2 + 3x_{26}^2 + 2x_{27}^2 + 3x_{28}^2 + 5x_{29}^2 +$
$4x_{31}^2 + 3x_{32}^2 + 2x_{33}^2 + 2x_{34}^2 + 0x_{35}^2 + 2x_{36}^2 + 3x_{37}^2 + 4x_{38}^2 + 3x_{39}^2 +$
$3x_{41}^2 + 4x_{42}^2 + 3x_{43}^2 + 4x_{44}^2 + 2x_{45}^2 + 0x_{46}^2 + 3x_{47}^2 + 4x_{48}^2 + 2x_{49}^2 +$
$5x_{51}^2 + 5x_{52}^2 + 2x_{53}^2 + 3x_{54}^2 + 3x_{55}^2 + 3x_{56}^2 + 0x_{57}^2 + 2x_{58}^2 + 2x_{59}^2 +$
$5x_{61}^2 + 6x_{62}^2 + 3x_{63}^2 + 4x_{64}^2 + 4x_{65}^2 + 4x_{66}^2 + 2x_{67}^2 + 0x_{68}^2 + 2x_{69}^2$

Subject to: $x_{11}^1 + x_{12}^1 + x_{13}^1 + x_{14}^1 + x_{15}^1 + x_{16}^1 = 4350$,
$x_{21}^1 + x_{22}^1 + x_{23}^1 + x_{24}^1 + x_{25}^1 + x_{26}^1 = 5340$,
$x_{31}^1 + x_{32}^1 + x_{33}^1 + x_{34}^1 + x_{35}^1 + x_{36}^1 = 5320$,
$x_{41}^1 + x_{42}^1 + x_{43}^1 + x_{44}^1 + x_{45}^1 + x_{46}^1 = 4017$,
$x_{11}^1 + x_{21}^1 + x_{31}^1 + x_{41}^1 = 2900$,
$x_{12}^1 + x_{22}^1 + x_{32}^1 + x_{42}^1 = 2624$,
$x_{13}^1 + x_{23}^1 + x_{33}^1 + x_{43}^1 = 3560$,
$x_{14}^1 + x_{24}^1 + x_{34}^1 + x_{44}^1 = 4213$,
$x_{15}^1 + x_{25}^1 + x_{35}^1 + x_{45}^1 = 3729$,
$x_{16}^1 + x_{26}^1 + x_{36}^1 + x_{46}^1 = 4011$,

$$x_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2 + x_{15}^2 + x_{16}^2 + x_{17}^2 + x_{18}^2 + x_{19}^2 = 2900,$$
$$x_{21}^2 + x_{22}^2 + x_{23}^2 + x_{24}^2 + x_{25}^2 + x_{26}^2 + x_{27}^2 + x_{28}^2 + x_{29}^2 = 2624,$$
$$x_{31}^2 + x_{32}^2 + x_{33}^2 + x_{34}^2 + x_{35}^2 + x_{36}^2 + x_{37}^2 + x_{38}^2 + x_{39}^2 = 3560,$$
$$x_{41}^2 + x_{42}^2 + x_{43}^2 + x_{44}^2 + x_{45}^2 + x_{46}^2 + x_{47}^2 + x_{48}^2 + x_{49}^2 = 4213,$$
$$x_{51}^2 + x_{52}^2 + x_{53}^2 + x_{54}^2 + x_{55}^2 + x_{56}^2 + x_{57}^2 + x_{58}^2 + x_{59}^2 = 3729,$$
$$x_{61}^2 + x_{62}^2 + x_{63}^2 + x_{64}^2 + x_{65}^2 + x_{66}^2 + x_{67}^2 + x_{68}^2 + x_{69}^2 = 4011,$$
$$x_{11}^2 + x_{21}^2 + x_{31}^2 + x_{41}^2 + x_{51}^2 + x_{61}^2 = 910,$$
$$x_{12}^2 + x_{22}^2 + x_{32}^2 + x_{42}^2 + x_{52}^2 + x_{62}^2 = 2853,$$
$$x_{13}^2 + x_{23}^2 + x_{33}^2 + x_{43}^2 + x_{53}^2 + x_{63}^2 = 2594,$$
$$x_{14}^2 + x_{24}^2 + x_{34}^2 + x_{44}^2 + x_{54}^2 + x_{64}^2 = 1466,$$
$$x_{15}^2 + x_{25}^2 + x_{35}^2 + x_{45}^2 + x_{55}^2 + x_{65}^2 = 1759,$$
$$x_{16}^2 + x_{26}^2 + x_{36}^2 + x_{46}^2 + x_{56}^2 + x_{66}^2 = 2848,$$
$$x_{17}^2 + x_{27}^2 + x_{37}^2 + x_{47}^2 + x_{57}^2 + x_{67}^2 = 1389,$$
$$x_{18}^2 + x_{28}^2 + x_{38}^2 + x_{48}^2 + x_{58}^2 + x_{68}^2 = 2136,$$
$$x_{19}^2 + x_{29}^2 + x_{39}^2 + x_{49}^2 + x_{59}^2 + x_{69}^2 = 1424,$$
$x_{ik}^1, x_{kj}^2 \geq 0$, i=1,2,3,4; k=1,2,…,6; and j=1,2,…,9.

4.2. Dual form of the problem with the first objective

By using the variables u_i, v_k, $v_k^{'}$, and w_j, the dual form of the primal problem with the first objective function takes the following form:

$$Max.Q_1 = 4350u_1 + 5340u_2 + 5320u_3 + 4017u_4 + 2900v_1 + 2624v_2 + 3560v_3 + 4213v_4 + 3729v_5 + 4011v_6 + 2900v_1^{'} + 2624v_2^{'} + 3560v_3^{'} + 4213v_4^{'} + 3729v_5^{'} + 4011v_6^{'} + 910w_1 + 2853w_2 + 2594w_3 + 1466w_4 + 1759w_5 + 2848w_6 + 1389w_7 + 2136w_8 + 1424w_9$$

Subject To:

$u_1 + v_1 \leq 0$, $u_1 + v_2 \leq 9$, $u_1 + v_3 \leq 12$, $u_1 + v_4 \leq 15$, $u_1 + v_5 \leq 18$, $u_1 + v_6 \leq 21$, $u_2 + v_1 \leq 12$,
$u_2 + v_2 \leq 9$, $u_2 + v_3 \leq 0$, $u_2 + v_4 \leq 9$, $u_2 + v_5 \leq 12$, $u_2 + v_6 \leq 15$, $u_3 + v_1 \leq 15$, $u_3 + v_2 \leq 13$,
$u_3 + v_3 \leq 9$, $u_3 + v_4 \leq 0$, $u_3 + v_5 \leq 9$, $u_3 + v_6 \leq 12$, $u_4 + v_1 \leq 22$, $u_4 + v_2 \leq 20$, $u_4 + v_3 \leq 17$,
$u_4 + v_4 \leq 13$, $u_4 + v_5 \leq 11$, $u_4 + v_6 \leq 9$, $v_1^{'} + w_1 \leq 9$, $v_1^{'} + w_2 \leq 0$, $v_1^{'} + w_3 \leq 9$, $v_1^{'} + w_4 \leq 11$,
$v_1^{'} + w_5 \leq 12$, $v_1^{'} + w_6 \leq 15$, $v_1^{'} + w_7 \leq 18$, $v_1^{'} + w_8 \leq 21$, $v_1^{'} + w_9 \leq 22$, $v_2^{'} + w_1 \leq 9$,
$v_2^{'} + w_2 \leq 9$, $v_2^{'} + w_3 \leq 0$, $v_2^{'} + w_4 \leq 9$, $v_2^{'} + w_5 \leq 9$, $v_2^{'} + w_6 \leq 13$, $v_2^{'} + w_7 \leq 15$, $v_2^{'} + w_8 \leq 19$,
$v_2^{'} + w_9 \leq 20$, $v_3^{'} + w_1 \leq 13$, $v_3^{'} + w_2 \leq 12$, $v_3^{'} + w_3 \leq 9$, $v_3^{'} + w_4 \leq 9$, $v_3^{'} + w_5 \leq 0$, $v_3^{'} + w_6 \leq 9$,
$v_3^{'} + w_7 \leq 12$, $v_3^{'} + w_8 \leq 15$, $v_3^{'} + w_9 \leq 17$, $v_4^{'} + w_1 \leq 16$, $v_4^{'} + w_2 \leq 5$, $v_4^{'} + w_3 \leq 13$,
$v_4^{'} + w_4 \leq 11$, $v_4^{'} + w_5 \leq 9$, $v_4^{'} + w_6 \leq 0$, $v_4^{'} + w_7 \leq 9$, $v_4^{'} + w_8 \leq 12$, $v_4^{'} + w_9 \leq 13$,
$v_5^{'} + w_1 \leq 19$, $v_5^{'} + w_2 \leq 18$, $v_5^{'} + w_3 \leq 15$, $v_5^{'} + w_4 \leq 14$, $v_5^{'} + w_5 \leq 12$, $v_5^{'} + w_6 \leq 9$,

$v_5' + w_7 \le 0$, $v_5' + w_8 \le 9$, $v_5' + w_9 \le 11$, $v_6' + w_1 \le 22$, $v_6' + w_2 \le 21$, $v_6' + w_3 \le 19$,
$v_6' + w_4 \le 17$, $v_6' + w_5 \le 15$, $v_6' + w_6 \le 12$, $v_6' + w_7 \le 9$, $v_6' + w_8 \le 10$, $v_6' + w_9 \le 9$,
$u_1, u_{2,}.., u_4, v_1, v_2,.., v_6, v_1', v_2',.., v_6', w_1, w_2,.., w_9$ unrestricted.

4.3. Dual form of the problem with the second objective

By using the variables u_i, v_k, v_k', and w_j, the dual form of the primal problem with the second objective function takes the following form:

$$Max.Q_2 = 4350u_1 + 5340u_2 + 5320u_3 + 4017u_4 + 2900v_1 + 2624v_2 + 3560v_3 + 4213v_4 + 3729v_5 + 1466w_4 + 1759w_5 + 2848w_6 + 1389w_7 + 2136w_8 + 1424w_9$$

Subject To:
$u_1 + v_1 \le 0$, $u_1 + v_2 \le 1$, $u_1 + v_3 \le 3$, $u_1 + v_4 \le 4$, $u_1 + v_5 \le 5$, $u_1 + v_6 \le 6$, $u_2 + v_1 \le 3$,
$u_2 + v_2 \le 2$, $u_2 + v_3 \le 0$, $u_2 + v_4 \le 2$, $u_2 + v_5 \le 3$, $u_2 + v_6 \le 4$, $u_3 + v_1 \le 4$, $u_3 + v_2 \le 3$,
$u_3 + v_3 \le 2$, $u_3 + v_4 \le 0$, $u_3 + v_5 \le 3$, $u_3 + v_6 \le 4$, $u_4 + v_1 \le 5$, $u_4 + v_2 \le 5$, $u_4 + v_3 \le 3$,
$u_4 + v_4 \le 2$, $u_4 + v_5 \le 2$, $u_4 + v_6 \le 2$, $v_1' + w_1 \le 2$, $v_1' + w_2 \le 0$, $v_1' + w_3 \le 1$, $v_1' + w_4 \le 2$,
$v_1' + w_5 \le 3$, $v_1' + w_6 \le 4$, $v_1' + w_7 \le 5$, $v_1' + w_8 \le 6$, $v_1' + w_9 \le 5$, $v_2' + w_1 \le 1$, $v_2' + w_2 \le 1$,
$v_2' + w_3 \le 0$, $v_2' + w_4 \le 1$, $v_2' + w_5 \le 2$, $v_2' + w_6 \le 3$, $v_2' + w_7 \le 2$, $v_2' + w_8 \le 3$, $v_2' + w_9 \le 5$,
$v_3' + w_1 \le 4$, $v_3' + w_2 \le 3$, $v_3' + w_3 \le 2$, $v_3' + w_4 \le 2$, $v_3' + w_5 \le 0$, $v_3' + w_6 \le 2$, $v_3' + w_7 \le 3$,
$v_3' + w_8 \le 4$, $v_3' + w_9 \le 3$, $v_4' + w_1 \le 3$, $v_4' + w_2 \le 4$, $v_4' + w_3 \le 3$, $v_4' + w_4 \le 4$, $v_4' + w_5 \le 2$,
$v_4' + w_6 \le 0$, $v_4' + w_7 \le 3$, $v_4' + w_8 \le 4$, $v_4' + w_9 \le 2$, $v_5' + w_1 \le 5$, $v_5' + w_2 \le 5$, $v_5' + w_3 \le 2$,
$v_5' + w_4 \le 3$, $v_5' + w_5 \le 3$, $v_5' + w_6 \le 3$, $v_5' + w_7 \le 0$, $v_5' + w_8 \le 2$, $v_5' + w_9 \le 2$, $v_6' + w_1 \le 5$,
$v_6' + w_2 \le 6$, $v_6' + w_3 \le 3$, $v_6' + w_4 \le 4$, $v_6' + w_5 \le 4$, $v_6' + w_6 \le 4$, $v_6' + w_7 \le 2$, $v_6' + w_8 \le 0$,
$v_6' + w_9 \le 2$, $u_1, u_{2,}.., u_4, v_1, v_2,.., v_6, v_1', v_2',.., v_6', w_1, w_2,.., w_9$ unrestricted.

4.4. Finding the optimal solution for the primal problem with the first objective function

After obtaining the initial solutions for both the second and first stages, the generalized table for the two stages with the first objective function takes the construction illustrated in Tab. 3. From this table, the optimum solution for the problem with the first objective is reached after certain number of iterations by applying the stepping stone method same as in classical transportation problems. The same procedure is repeated for the problem with the second objective to obtain its optimum solution. The calculated solutions represent two different points from the set of non-dominated extreme points in the objective space of the problem ($Z_1^{(1)}, Z_2^{(1)}$), ($Z_1^{(2)}, Z_2^{(2)}$).

4.5. The coefficients of the new objective function

Coefficients of the new objective function ($\alpha_1^{(r,s)}, \alpha_2^{(r,s)}$) are calculated and the new form of the problem is reached and its dual form and a new generalized Table are constructed. The optimum solution of the new form is calculated with values of ($\bar{Z}_1, \bar{Z}_2$) which represent a new point in the objective space of the problem. By the same procedure and after a finite number of iterations, the set of all non-dominated extreme points in the objective space and their related efficient solutions will be reached and concluded in Tab. 4.

Tab. 3

The generalized Table of the problem

	v_k	v_1	v_2	v_3	v_4	v_5	v_6		
u_i		M_1	M_2	M_3	M_4	M_5	M_6	M_{fict}	a_i
u_1	D_1	2900	1450						4350
u_2	D_2		1174	3560	606				5340
u_3	D_3				3607	1713			5320
u_4	D_4					2016	353	1648	4017
w_j	O_{fict}						3658		b_j
w_1	O_1	910							910
w_2	O_2	1990	863						2853
w_3	O_3		1761	833					2594
w_4	O_4			1466					1466
w_5	O_5			1261	498				1759
w_6	O_6				2848				2848
w_7	O_7				867	522			1389
w_8	O_8					2136			2136
w_9	O_9					1071	353		1424
	c_k	c_1	c_2	c_3	c_4	c_5	c_6		
	v'_k	v'_1	v'_2	v'_3	v'_4	v'_5	v'_6		

5. CONCLUSIONS

In this paper the mathematical formulation of our case study is introduced with its algorithm of solution which gives the set of non-dominated solutions and its related set of efficient solutions. The presented algorithm can be used for solving any b-criteria two-stage transportation problem from the class of our case study in which the relation between capacities of suppliers (a_i), capacities of warehouses (e_k) and requirements of destinations (b_j) is ($\sum_j b_j < \sum_i a_i < \sum_k e_k$) and with little modifications for phase II, the presented algorithm can be used to solve any bi-criteria two-stage transportation problem with any relation between (a_i, e_k, and b_j).

The solution algorithm gives the set of non-dominated solution and its related efficient solutions. These solutions are illustrated in Tab. 4. The DM can select the preferred one of these solutions according to the company policy for production and distribution.

References

1. Waiel F. Abd El-Wahed, Sang M. Lee.: *Interactive fuzzy goal programming for multi-objective transportation problems.* International J. of Management Science, Omega, 34, 2006, p. 158 – 166.
2. Ringust J.L., Rinks D.B.: *Interactive solutions for the linear multi-objective transportation problems*. European J. of Operational Research, 32, 1987, p. 96 – 106.

3. Steuer R.: *Multiple criteria optimization: theory, computation, and application*. New York; Wiley, 1986.
4. Aneja Y.P., Nair K.P.K.: *Bi-criteria transportation Problems*. Management Science, vol. 25, 1979, p. 1 – 11.
5. Diaz J.A.: *Solving multi-objective problems*. Ekonomicko - Matematicky Obzor, vol. 14, 1976, p. 267 – 274.
6. Diaz J.A.: *Finding a complete description of all efficient solutions to a multi-objective transportation problem*. Ekonomicko - Matematicky Obzor, vol. 26, 1979, p. 23 – 39.
7. Isermann H.: *The enumeration of all efficient solutions for a linear multi-objective transportation problem*. Naval Research Logistic Quarterly, vol. 26, 1979, p. 123 – 139.
8. Kasana H.S., Kumar K.D.: *An efficient algorithm for multi-objective transportation problems*. Asia Pacific Operational Research, 17, 2000, p. 27 – 40.
9. Kallrath J., Wilson J.M.: *Business optimization*. Macmillan Press Ltd., London, 1997.
10. Osman M.S.A., Ellaimony E.E.M.: *An algorithm for solving a class of multistage transportation problems*. Modeling Simulation and Control C., AMSE Press, 1-2, 1984, p. 43-56.
11. Berzina, Istranikova: *The way of solving two-stage transportation problems*. Mathematical Methods in Economics, 1999, pp. 39 – 44.

Tab. 4

Bi-criteria solution in the objective space and its related efficient solutions for the problem

Iteration	L	E	Recorded Points	Efficient Distributions (non zero values)
2	{(1,3), (3,2)}	Φ	$Z^{(3)}=$ (105496 ,20765)	$x_{11}^1=2900,\ x_{12}^1=1450,\ x_{22}^1=1174,\ x_{23}^1=3560,\ x_{34}^1=4213,\ x_{35}^1=65,\ x_{45}^1=1324,\ x_{46}^1=2693,\ x_{11}^2=47,\ x_{12}^2=2853,\ x_{21}^2=30,\ x_{23}^2=2594,\ x_{31}^2=335,\ x_{34}^2=1466,\ x_{35}^2=1759,\ x_{41}^2=498,\ x_{46}^2=2848,\ x_{49}^2=867,\ x_{57}^2=1389,\ x_{68}^2=2136,\ x_{69}^2=557$
3	{(3,2)}	{(1,3)}	$Z^{(3)}=$ (105496 , 20765)	$x_{11}^1=2900,\ x_{12}^1=1450,\ x_{22}^1=1174,\ x_{23}^1=3560,\ x_{34}^1=4213,\ x_{35}^1=1107,\ x_{45}^1=282,\ x_{46}^1=2693,\ x_{11}^2=47,\ x_{12}^2=2853,\ x_{21}^2=30,\ x_{23}^2=2594,\ x_{31}^2=335,\ x_{34}^2=1466,\ x_{35}^2=1759,\ x_{41}^2=498,\ x_{46}^2=2848,\ x_{49}^2=867,\ x_{57}^2=1389,\ x_{68}^2=2136,\ x_{69}^2=557$
4	Φ	{(1,3), (3,2)}	$Z^{(3)}=$ (105496 , 20765)	$x_{11}^1=2900,\ x_{12}^1=1450,\ x_{22}^1=1174,\ x_{23}^1=3560,\ x_{34}^1=4213,\ x_{35}^1=65,\ x_{45}^1=1324,\ x_{46}^1=2693,\ x_{11}^2=47,\ x_{12}^2=2853,\ x_{21}^2=365,\ x_{23}^2=2259,\ x_{33}^2=335, x_{34}^2=1466,\ x_{35}^2=1759,\ x_{41}^2=498,\ x_{46}^2=2848,\ x_{49}^2=867,\ x_{57}^2=1389,\ x_{68}^2=2136,\ x_{69}^2=557$

Received 13.09.2009; accepted in revised form 20.09.2010