

# Transportation Telematics Systems Operation Efficiency Modeling

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## Abstract

In the paper is presented a method of assessing of the exploitation efficiency of transport telematics systems. In order to obtain as an overall assessment of transport telematics systems, as the method for evaluating was accepted the multi-state analysis of the exploitation process. Then was elaborated the model of exploitation process of the telematics system. The problem of fundamental importance in the study of efficiency is to determine the partial measures of effectiveness. Using the characteristics of the exploitation process, a model of exploitation efficiency of telematics system was elaborated and the measures of its evaluation are presented.

## 1. Introduction

Efficiency of the telematics system can be assessed only if a positive answer exists to the question – does the system can perform the tasks imposed on it? A positive response demonstrates the usefulness of the system to perform the tasks including the possible implementation of telematics services by the technical equipment, qualified staff, proper organization of the process of operation and maintenance, etc. [9]. In reference to technical equipment it must taken into account its failure, the complexity of the service, repair and renewal opportunities. An important factor influencing the effectiveness is the reliability of technical equipment, but it should taken into account also the possibility of a reduced quality of work as a result of inferior equipment reliability.

During the exploitation of technical equipment can be distinguished periods:  
– Ability for work and waiting for the implementation of telematic services;

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- Ability for work and implementation of telematic services;
- Planned maintenance, inspections, repairs;
- Failure repairs.

The primary methodology issue in assessing the efficiency of exploitation of telematics system is adequate to the reality building of a mathematical model involving the largest possible number of combinations of different factors, but allowing in each case include only those factors that correspond to the task of the research team. As a method for assessing the efficiency of service, in accordance with the considerations among others in [2], [10], was adopted the analysis of the stochastic process of exploitation of telematics systems. This leads to the construction of the model of exploitation process and designation of selected characteristics of this process, which in themselves may constitute a partial measures of assessing or may be part of more general dependences of the assessment of exploitation efficiency. It is necessary to draw attention to the fact that the construction of the model even very complex telematics system or subsystem can not be a problem like gaining up to date real input data, ie these values which in the model are subjected to appropriate transformations. The problem of primary importance in the study of efficiency is to determine the overall efficiency and/or sub-functions of efficiency. These functions are dependent on temporary or asymptotic probabilities of staying of the system exploitation process in the states of exploitation. Privileged class of theoretical models to assess the effectivity of telematics systems are semi-Markov processes [1], [3].

## 2. Model of the Exploitation Efficiency of Telematics Systems

Considering the specificity of the telematics system for the implementation of telematic services as a system composed of three main subsystems – Fig. 1 – (broadcasting subsystem, transmission subsystem, the subsystem that receives the telematics information) as well as taking into consideration the purpose of its action and possibilities of using of evaluation results in exploitation practice, as a method for evaluating the efficiency of telematics system, is proposed the analyze multi-state stochastic process analyze of its exploitation is proposed. There is therefore a need to build a theoretical model of the exploitation process of telematics system.

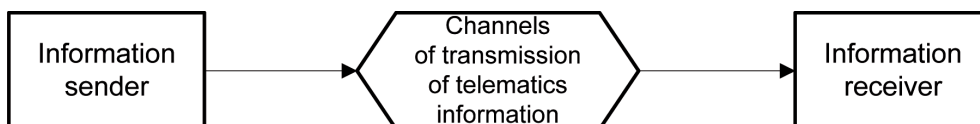


Fig. 1. Block diagram of telematic system

Modelling of the exploitation process of telematics system was performed with the following assumptions:

- The considered telematics system is a renewable system with a long duration of action. The average time of correct operation between successive failures of the system is much shorter than the time of exploitation. It is assumed that the system is after the adaptation and prior of aging period;
- The functional surplus expressed by the possibility of realization of telematic services in other ways is not taken into consideration. Preparing of the alternative ways of connecting takes time, so there would be delay in the implementation of services in relation to the planned time of its beginning;
- Failure of one of the marked-out element of system during its operation of the system result in passing all the system to the sub-set of the states of inability and therefore the intermediate states are not marked-out;
- The probability of occurrence of two or more failures at the same time is close to zero.

Having a set of functional states, as well as a serial functional architecture of telematics system for the implementation of the task of handling the demands of telematic services, the following states of exploitation of telematics system are marked-out:

- $s_1$  – the state of waiting of the able system for realization of the task;
- $s_2$  – state of realization of the task;
- $s_3$  – the state of waiting for repair;
- $s_4$  – the state of repair.

The states  $s_i$  form the states space defined by the set  $S$

$$S = \{s_1, s_2, s_3, s_4\} \quad (1)$$

The exploitation process of telematics system may therefore be defined as a function of [1], [3]

$$\mathbf{X}(t) = \{X(s_i, t) \text{ dla } s_i \in S \wedge t \in R_+\} \quad (2)$$

For  $t \in R_+$ ,  $\mathbf{X}(t)$  is a stochastic process  $\{X(t), t \in R_+\}$  with values defined in the set  $S = \{s_1, s_2, s_3, s_4\}$ . This process is a process that is DC-class, it means discrete in the states and continuous in time.

It is assumed that the duration of the exploitation states  $s_i \in S$  is a random variable with any distribution determined in the space of positive real numbers  $R_+$ . Furthermore, it is assumed that the transition of the telematics system to the specified exploitation state does not depend on the history of the states, but only on the current state, in which the system stays. The adoption of these assumptions allows to treat the process of exploitation of the telematics system for  $t \in R_+$  – as a semi-Markov process with a finite number of states  $S$  [2].

Semi-Markov process can be determined by a homogeneous Markov chain with transition matrix  $\mathbf{P} = [P_{ij}]$  and conditional distributions of independent random variables,  $T_{ij}$  [1], [4]:

$$\mathbf{P} = [p_{ij}; i, j \in S], P_{ij} = \lim_{t \rightarrow \infty} Q_{ij}(t)$$

and initial distribution

$$P\{X(\tau_0) = k\} = P_k \quad \text{dla } k \in S$$

$$F_{ij}(t) = P\{\tau_n - \tau_{n-1} < t / X(\tau_{n-1}) = i, X(\tau_n) = j\} \quad \text{dla } i, j \in S$$

where:  $i, j$  – indexes of possible states of the exploitation process  $i, j \in S$ ,

$P_{ij}$  – the probability of the transition of process from state  $i$  to the state  $j$ ;

$F_{ij}(t)$  – distribution function of random variable  $T_{ij}$ ;

$T_{ij}$  – Random variable defining the duration of the state  $i$  under condition that in the next step the process moves to state  $j$ ;

$S$  – set of states of the exploitation process.

In exploitation process telematics system can be described by a set of states  $S$  given by formula (1). State graph of the telematics system exploitation process is shown in Fig. 2. The system able to perform the process realization using transmitted information is referred to telematics information in state  $s_1$ , after the time  $T_{12}$  this state can come in the state  $s_2$ , which is equivalent with the implementation of a telematic transmission services. The probability of such transition is  $P_{12}$ . But there are the cases that the state on stanby  $s_1$  lasting  $T_{13}$  and is ended the failure of element or sub-system resulting large amount of work, before the system can come into the state  $s_3$  corresponding waiting for start to repair the system. Related probability of this transition is  $P_{13}$ , the other transition probabilities are correspondingly  $P_{41}$   $P_{21}$   $P_{34}$  ....

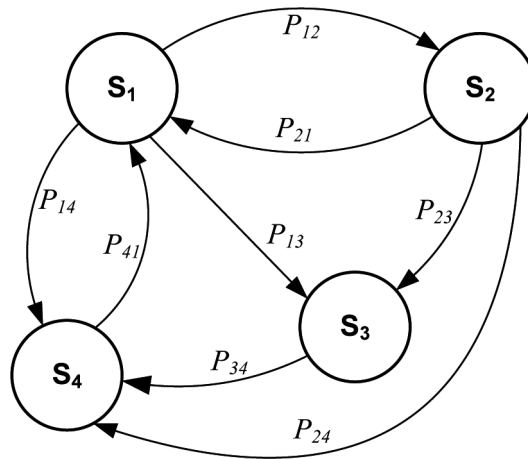


Fig. 2. Exploitation process graph of states for the telematics system, with transition probabilities  $P_{ij}$  between  $i, j$  exploitation processes belonging to  $S$  set of states

If the pending state  $s_1$  ends by failure, which is simple to be removed and requires no special maintenance taking no time and allows to the system after time  $T_{14}$  comes into the state  $s_4$  identifying the effective repair of failure. But the occurrence of failure in the state  $s_1$ , requires to failure removal using the additional maintenance such as waiting for spare parts, waiting for specialist equipment maintenance, etc., and corresponds the system transition to state  $s_3$ , which is the waiting state for repair. The transition from state  $s_1$  to  $s_3$  occur after time  $T_{13}$  and corresponds to the transition probability equal to  $P_{13}$ . From the state  $s_3$  after the time  $T_{34}$  the system comes into the state  $s_4$  of effective repair. The probability of such transition is  $P_{34} = 1$ .

During the task realization the system may be also damaged and then it comes into a subset of state maintenance states. But when the system failure doesn't require the special arrangements to remove failure, the system from the state  $s_2$  moves to repair state  $s_4$ . The time to be by the system in state  $s_2$ , after which it comes in the state  $s_4$ , occurs after time  $T_{24}$  with transition probability  $P_{24}$ . In case of failure, which removal requires preparation of appropriate technical and organizational means, system moves after the time  $T_{23}$  from state  $s_2$  to the waiting state to be repair  $s_3$  described by probability  $P_{23}$ .

In the case of proper system functioning, after time  $T_{21}$  to complete ongoing task the system moves to the waiting state to realize the task (state  $s_1$ ). The probability of this transition is  $P_{21}$ . There is assumed that from the state  $s_4$  the system can move with the probability equal 1 only to the state  $s_1$ . The time been of the system in the state  $s_4$  is  $T_{41}$  and determines the time duration of system repair. Also, there is assumed that after the system repair the transition is only to the waiting state to realize the task. This is due to the operation manner of the telematics system and due to the fact that the repair ends up restoring the output system capacity such as it was before the failure.

In considered the exploitation model process of telematics system in relation to constant repair intensity, maintenance, scheduled and periodic inspections, the maintenance states are not included into consideration. Besides of it there is also assumed that the random variables denoting the time to be by the system in the state  $i \in S$ , under condition that a transition to the state  $j \in S$  exists, and can define the probability density functions  $f_{ij}(t)$ , the finite expected values  $E[T_{ij}]$ , finite second moments  $E[T_{ij}^2]$  for the times  $T_{12}$ ,  $T_{13}$ ,  $T_{14}$ ,  $T_{23}$ ,  $T_{24}$ ,  $T_{34}$ ,  $T_{21}$ ,  $T_{41}$

Thus, there can be defined

$$f_{12}(t), f_{13}(t), f_{14}(t), f_{23}(t), f_{24}(t), f_{34}(t), f_{21}(t), f_{41}(t),$$

$$E[T_{12}], E[T_{13}], E[T_{14}], E[T_{23}], E[T_{24}], E[T_{34}], E[T_{21}], E[T_{41}],$$

$$E[T_{12}^2], E[T_{13}^2], E[T_{14}^2], E[T_{23}^2], E[T_{24}^2], E[T_{34}^2], E[T_{21}^2], E[T_{41}^2],$$

and transition matrix of the process shown by the graph in Figure 2 is as follows:

$$P = \begin{bmatrix} 0 & P_{12} & P_{13} & P_{14} \\ P_{21} & 0 & P_{23} & P_{24} \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

For such a specified exploitation process of the telematics system the set of indicators characterizing this process can be determined. For these indicators can be included [8]:

- Distribution of time durations of the exploitation process states;
- Stationary probability of process to be in the  $i$ -th state for the embedded Markov chain;
- The intensity of the process to be in the exploitation states;
- Mean distance time between the exploitation states;
- Average time to be by the process in the given subsets of states;
- Stationary probability to be by the process in selected subset of the states.

These indicators don't exhaust all possibilities, which can characterize the exploitation process of telematics system. No less, for practical assessment of operational readiness, exploitation efficiency and other exploitation features of the evaluated system they seem to be sufficient.

### 3. Assessing of the Exploitation Efficiency of the Transport Telematics System

Exploitation efficiency of telematics system, marked as  $Z$  can be formally written as

$$Z = \{\zeta_1, \zeta_2, \dots, \zeta_n\} \quad (4)$$

where:  $Z$ - set of functional and numerical characteristics (indicators of exploitation efficiency) defined on the exploitation process  $X(t)$  of the telematics system.

The set of such specified characteristics enables both evaluation of the system operation and describing the process exploitation. Telematics system actions result in assuring the support of the decision and executive functions, which aimed obtaining the safe and efficient transport tasks for a given type of transport. The safety and efficiency of transport performance is significantly influenced by organization and management of the transport process, the control organization of process and proper operation of technical equipment including the telematics system. In presented work in terms ensuring the support of proper implementation of the transportation tasks with advanced telematics systems technical infrastructure, the security provides the basis for the precising of criteria, which assess the exploitation efficiency of the telematics system. In terms to ensure the safety and efficiency of the transport

process as a principle assessing criterion of the telematics system the reliability of its subsystems and components is assumed. This doesn't exclude, however, the other criteria useful for the evaluation of the exploitation effectivity of telematics system.

To such criteria can be included operational readiness and readiness at the moment (instantaneous readiness) of the evaluated the system. Under the concept of operational readiness is understood to be the property of telematics system, which determines the possibility of staying the system in the states allowing proper implementation of telematics services. Readiness at the moment determines the property of the system which express the possibility to carry out the tasks in a specified time. It seems that the proposed criteria, even they do not exhaust all possible, allow for a fairly comprehensive analysis of telematics system.

Efficiency ratio  $\zeta$ , which is the element of the set  $Z$  (expression 4), depends on many parameters, in which the dominant role play the properties and technical parameters of the system and the parameters characterizing the impact of the environment. Therefore there is assumed that the efficiency ratio is a function of parameters:

$$\zeta = \zeta(\chi_1, \chi_2, \dots, \chi_k; \gamma_1, \gamma_2, \dots, \gamma_m) \quad (5)$$

where:  $\chi_1, \chi_2, \dots, \chi_k$  – the parameters characterizing the technical properties of telematics;  $\gamma_1, \gamma_2, \dots, \gamma_m$  – parameters characterizing the impact of environment on the functioning of the telematics system.

Dependences of the type (5), however, are not very convenient as the indicators of efficiency of telematics system. This is because a large number of factors affecting the system, which are very difficult to be described. Therefore for the practical assess the efficiency of telematics system some indicators will be introduced in the form of generalized numerical characteristics. To assess the efficiency of telematics system the characteristics describing the process of exploitation by the partial indicators will be introduced. Especially, they will be done by the time distribution to be in the exploitation states, stationary probability to be in the  $i$ -th exploitation state, the intensity of the states of exploitation process, the time intervals between the exploitation states, the average time of process to be in the determined subset of states.

Evaluation of the efficiency of telematics system requires using such indicators which will be presented below. Taking into account the functioning specificity of the transport telematics system as well as evaluation correctness, the following criteria for evaluating will be proposed, they will be the indicators of telematics system exploitation efficiency:

- Indicator of operational readiness;
- Probability of use of the system to implement telematic service;
- The potential ability to perform the telematic service.

### ***Indicator of operational readiness***

Taking into account the telematics system element reliability and its readiness for realization the services the basic operating characteristics of efficiency, the indicator of operational readiness, given by the probability that at time  $t$  the system is in a state of efficiency will be introduced. The value of the telematics system operational readiness rate depends on the probability changes of the exploitation states and the average time durations of these states. Indicator of operational readiness can also be represented as the sum of the boundary probabilities of staying of the system in a subset of the ability states  $S_Z$ . This gives the

$$\zeta_g = \frac{\sum_{i \in S_Z} \pi_i E[T_i]}{\sum_{k \in S} \pi_k E[T_k]} = \frac{E[T_1] + P_{12}E[T_2]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{12}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{12}P_{23}P_{34}E[T_4]} \quad (6)$$

where:  $\pi_i$  – stationary probability of the  $i$ -th state of the embedded Markov chain

$P_{ij}$  – probability of the transition process from the  $i$ -th state to the  $j$ -th state  $i, j \in S$ .

### ***Probability using of the system for realization of telematics services***

For the given services to evaluate the use of telematics as the indicator of exploitation efficiency the probability of the technical use can be assumed. This indicator expresses by synthetic manner the range of system utilization, and by this it can be used to assess the quality of system work as well as process organization supporting the realization of transport tasks. According to assumed model of exploitation process, telematics system may be described by ability and inability states making correspondingly the subsets  $S_Z = \{s_1, s_2\}$  and  $S_N = \{s_3, s_4\}$ .

Let  $W_G$  makes a vector of the boundary probabilities of process to be in distinguishing states

$$W_G = \langle P_Z, P_N \rangle$$

where:  $P_Z$  – process boundary probability to be the system in the subset of the ability states  $S_Z$ ;

$P_N$  – process boundary probability to be the system in the subset of the inability states  $S_N$ .

The boundary probability (for  $(t \rightarrow \infty)$  determining the coordinates of the vector  $W_G$  is as follows:

$$P_Z = \zeta_g = \frac{E[T_1] + P_{12}E[T_2]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{12}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{12}P_{23}P_{34}E[T_4]} \quad (7)$$



$$P_N = \frac{\sum_{i \in S_N} \pi_i E[T_i]}{\sum_{k \in S} \pi_k E[T_k]} = \frac{P_{13}E[T_3] + P_{12}P_{23}E[T_3] + (P_{14} + P_{12}P_{24} + P_{12}P_{23} + P_{13})E[T_4]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{12}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{12}P_{23}P_{34}E[T_4]} \quad (8)$$

where:  $E[T_i]$  – time expected value of process to be in  $i$ -th state;

$\pi_i$  – stationary probability of the  $i$ -th state of the embedded Markov chain;

$P_{ij}$  – probability of process transition from the  $i$ -th state to the state  $j$ -th with  $i, j \in S$

Similarly, the vector of using telematics system in the time of exploitation can be determined. Let  $W_W$  be the vector of the probability distribution determining the time exploitation to use the telematic system to perform the given services

$$W_W = \langle w_Z, w_N \rangle \quad (9)$$

where:  $w_Z$  – exploitation time probability to stay by telematics system in the subset of ability states;

$w_N$  – probability of necessity to realize the service when the telematics system is in the subset of the inability states.

The values of these probabilities are

$$w_Z = P_2$$

$$w_N = 0$$

where:  $P_2$  – stationary probability to occur the realization state performing the telematics service.

$\zeta_i$  index of technical utilization can be determined as a product of boundary stationary probability occurrence to stay by system in ability states and boundary probability occurrence of states realizing the service. Thus, this index can be given by expression

$$\zeta_i = w_Z \cdot P_Z = w_Z \cdot \zeta_g = \frac{P_{12}E[T_1] \cdot E[T_2] + (P_{12} \cdot E[T_2])^2}{(E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{12}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{12}P_{23}P_{34}E[T_4])^2} \quad (10)$$

### **Potential ability to realize the service**

Under given exploitation conditions a functioning quality measure of telematics system can be done by overall exploitation efficiency index, which for given time determines the implementation degree of tasks by mean of specific measures.

Telematics system reliable operation are assured by principle conditions of system functioning at determined possibility of the service realization. In case of some telematics systems, e.g. dispatch communication system, the work efficiency referred to a given section of the railway line can affect the timeliness of realization of transport tasks. If during the system work the failures occurs they may reduce both the degree of transport safety, and reduced possibilities to use the system.

Telematics system can correctly perform the given tasks, if its elements and components are reliable and exploitation process organization is correct. Unreliable components using is not very effective, as in many cases they are responsible for damage of system elements. If activities related to failure finding and repair accomplishing are completed before coming the service request, the system can be effectively used for fulfilling tasks, which transport process aide.

Otherwise, the losses of telematic services implementation occur, which in turn may require additional mobile-organizational performances to complete the transportation and given service tasks. Then, the task can be accomplished using other manners to transmit the telematics or other non telematics information to perform the transport tasks.

For evaluation purposes of telematics system operation quality under such conditions, the general indicator of exploitation efficiency can be used. This indicator can be defined as the ability of task realization and makes a general measure of telematics system functioning. Usually, it is expressed by the probability of events which occur in given time and under given conditions and characterize the effective and targeted use of telematics system to accomplish the telematic services supporting the transport process.

In realization of telematics services there can be considered following possibilities to appear the random events at a given time and under given conditions. After waiting period the system is able to realize the telematic service with a probability  $P_Z$  while the system failure occurs with probability probability when system is in able

$$P_N = 1 - P_Z$$

After time  $T_N$  depending on the complexity of the repair action, the system pass over with to waiting state with probability equal to 1.

During the implementation of telematic services, also, the events damaging the system appear.

From the implementation transport task perspective they are the worst case.

Frequently, these events lead to delays realization of transport tasks and in specific cases: lack or wrong messages, degrades the safety of transport process. If, stationary probability of such event is  $P_{NR}$ , the probability of opposite event  $P_R$  is equal to  $P_R = 1 - P_{NR}$  and has meaning the correct operation of all elements of the telematics system during the implementation of telematic services.

Let for the events the time  $T_N$  waiting when the system to be repaired and period time the system failure repair is less than the time  $T_R$  between the successive requests for service implementations  $\{T_N < T_R\}$ .

Then, the repair will be completed before coming the subsequent request for telematic service and probability of this event is  $P_{UT}$ . Assuming as the probability distribution of repair time the Erlang distribution with shape parameter equal 1, as the distribution verified by practice i.e. [6], we have

$$P_{UT} = P\{T_N < T_R\} = F(t_R) = 1 - \sum_{j=0}^{k-1} \frac{\left(\frac{T_R}{T_w}\right)^j}{j!} \exp\left\{-\frac{T_R}{T_N}\right\} \quad (11)$$

where:  $T_R$  – time interval between the implementation of the successive services;

$T_N$  – time to be the system in the state of removal of the failure.

$P_{UT}$  determines the probability to complete the system repair before coming the successive demand for realization of the telematics service. Taking into account all possible transitions among the states while telematics system waits for realization of task and realize implementation of the service, the probability of realization of the task can be determined. It express the system potential ability to perform a task of transport supporting process. At assumption of system events independence during waiting time and system transition to a service in realization states it can be obtained the following stationary relationship defining the system potential ability to service realization.

$$\zeta_P = P_{OZ}P_{PZ}P_R \quad (12)$$

where:  $P_{OZ}$  – probability of the event that the waiting period for implementation of the service, will be finished in the system state of ability;

$P_{PZ}$  – probability of the event that during the transition of system from the waiting state to the state of realization, any failure will not occur;

$P_R$  – probability of correct operation of the telematics system during the implementation of the service.

Using introduced above probability meanings related probabilities can be expressed as follows:

$$\begin{cases} P_{OZ} = P_Z + (1 - P_Z)P_{UT} = P_{12} + (P_{13} + P_{14})P_{UT} \\ P_{PZ} = P_{12} \\ P_R = P_{21} \\ T_R = E[T_1] \\ T_N = E[T_N] \end{cases} \quad (13)$$

where:  $T_R$  – expected time value to stay the telematics system in the able state waiting for service realization;

$T_N$  – expected time value to stay the telematics system in the subset of the states of inability;

$E[T_N]$  – expected time value to stay the process in the subset of the inability states;

$E[T_1]$  – expected time value to stay the process in the waiting state for realization of the service;

$P_{ij}$  – transition probability of the exploitation process of telematics system from  $i$  – th state to the state  $j$  – th, indices  $i, j$  are the successive states of exploitation processes.

Exploitation efficiency ratio indicating the potential ability of telematics system for the service realization under conditions expressed by Eq. (12) and (13) can be written in form:

$$\begin{aligned} \zeta_P &= [P_Z + (1 - P_Z) P_{UT}] P_{PZ} P_R = \\ &= \left\{ P_{12} + (P_{13} + P_{14}) \left\{ 1 - \exp\left(-\frac{E[T_1]}{E[T_N]}\right) \right\} \right\} P_{12} P_{21} = \\ &= \{P_{12} + (P_{13} + P_{14}) [1 - \exp(-Y)]\} P_{12} P_{21} \end{aligned} \quad (14)$$

where:

$$\begin{aligned} Y &= \frac{E[T_1] l_4}{P_3 + P_4} = \\ &= \frac{\{P_{12} E[T_{21}] P_{13} E[T_{13}] + P_{14} E[T_{14}]\} \cdot \{P_{14} + P_{12} P_{24} + P_{21} P_{23} + P_{13}\}}{(P_{13} + P_{12} + P_{23}) E[T_3] E[T_4] + (P_{14} + P_{12} P_{24}) E[T_4]} \end{aligned} \quad (15)$$

$l_4$ , – boundary intensity of the failure removal (repair) is:

$$l_4 = \frac{P_{14} + P_{12} P_{24} + P_{13} + P_{23} P_{34}}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} \quad (16)$$

Using the numerical values of the overall rate of telematics system exploitation efficiency the expected number of tasks (telematic services) can be determined; they are executed by the analyzed system for specified time periods. There is assumed that the expected number of tasks  $E[UT_P(\Delta t)]$  performed in the system haven't losses in the implementation of the requested service. Subsequently, expected number of services provided by the telematics system with no loss in the real system can be described by relationship:

$$E[UT(\Delta t)] = \zeta_P E[UT_P(\Delta t)] \quad (17)$$

where:  $E[UT_P(\Delta t)]$  – expected number of completed services in the system, which works in fixed time interval  $\Delta t$  without losses;

$E[UT(\Delta t)]$  – expected number of services to be the realized in a real telematics system in fixed time interval  $\Delta t$ .

For the transport telematics system shown in block diagram in Fig. 1 the exploitation efficiency was done with the use of indicators introduced above. Data for calculations were obtained for microprocessor and computer railway traffic control systems in studies conducted at the author contribution in the Railway Scientific and Technical Center [6], [10].

Values of some parameters related to the handling of devices have been estimated using the prediction methods presented in [7], [8].

$$P_{12} = 0,9998, E[T_{12}] = 5,05 \text{ min}, P_{13} = 0,00015, E[T_{13}] = 4,88 \text{ min},$$

$$P_{14} = 0,00005, E[T_{14}] = 5,02 \text{ min},$$

$$P_{21} = 0,9997, E[T_{21}] = 2,77 \text{ min}, P_{23} = 0,0002, E[T_{23}] = 1,57 \text{ min}, P_{24} = 0,0001,$$

$$E[T_{24}] = 2,38 \text{ min}, P_{34} = 1, E[T_{34}] = 2,97 \text{ h}, P_{41} = 1, E[T_{41}] = 0,54 \text{ h}$$

According to shown above assumption and evaluations the expected values to be by system in distinguished exploitation states are as follows :

$$E [T_1] = P_{12}E [T_{12}] + P_{13}E [T_{13}] + P_{14}E [T_{14}] = 0,084333 \text{ [h]}$$

$$E [T_2] = P_{21}E [T_{21}] + P_{23}E [T_{23}] + P_{24}E [T_{24}] = 0,085833 \text{ [h]}$$

$$E [T_3] = P_{34}E [T_{34}] = 2,97 \text{ [h]}$$

$$E [T_4] = P_{41}E [T_{41}] = 0,54 \text{ [h]}$$

#### ***Exploitation states stationary probabilities of telematics system***

$P_1$  – probability to be the telematics system in the able state, waiting for realization the task.

This probability has the form:

$$P_1 = \frac{E[T_1]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{21}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{21}P_{23}P_{34}E[T_4]} = 0,49197$$

In the assumed model of exploitation process for  $s_2$  being the state of realization task the stationary probability is:

$$P_2 = \frac{P_{12}E[T_2]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{21}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{21}P_{23}P_{34}E[T_4]} = 0,49224$$

$P_3$  – probability of waiting of the telematics system to be repair is expressed as:

$$P_3 = \frac{P_{13}E[T_3] + P_{12}P_{23}E[T_3]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{21}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{21}P_{23}P_{34}E[T_4]} = 0,30208$$

Probability to stay by the telematics system in the repair state  $s_4$  is expressed by the relationship:

$$P_4 = \frac{P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{21} P_{23} E[T_4] + P_{13} E[T_4]}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} = 0,00157$$

***Process intensity to be in exploitation state***

The boundary intensities of the exploitation states for the analyzed telematics system can be calculated from the formula:

$l_1$  – the boundary intensity of telematics system to be in the waiting state to realize the task is:

$$l_1 = \frac{1}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} = 0,09728$$

$l_2$  – border intensity of the realization of service in the telematics system has the from:

$$l_2 = \frac{P_{12}}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} = 0,09649$$

$l_3$  – border intensity of waiting of the failed system to carry out the repair

$$l_3 = \frac{P_{13} + P_{12} P_{23}}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} = 0,00017$$

$l_4$ , – boundary intensity of removal of the failure

$$l_4 = \frac{P_{14} + P_{12} P_{24} + P_{13} P_{34} + P_{21} P_{23} P_{34}}{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]} = 0,17548$$

***Time intervals between the j-hi exploitation states***

The expected values of the intervals between specified exploitation states is expressed by the formula:

$$E[T_{11}] = E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{12} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4] + P_{13} P_{34} E[T_4] + P_{12} P_{23} P_{34} E[T_4] = 0,1713 \text{ [h]}$$

$$E[T_{22}] = \frac{E[T_1] + P_{12} E[T_2] + P_{13} \cdot E[T_3] + P_{12} \cdot P_{23} \cdot E[T_3] + P_{14} \cdot E[T_4] + P_{12} \cdot P_{24} E[T_4]}{P_{12}} + \frac{P_{13} \cdot P_{34} E[T_4] + P_{12} \cdot P_{23} \cdot P_{34} E[T_4]}{P_{12}} = 0,1712 \text{ [h]}$$

$$E[T_{33}] = \frac{E[T_1] + P_{12} E[T_2] + P_{13} E[T_3] + P_{21} P_{23} E[T_3] + P_{14} E[T_4] + P_{12} P_{24} E[T_4]}{P_{13} + P_{12} \cdot P_{23}} + \frac{P_{13} P_{34} E[T_4] + P_{21} P_{23} P_{34} E[T_4]}{P_{13} + P_{12} \cdot P_{23}} = 489,55 \text{ [h]}$$

$$E[T_{44}] = \frac{E[T_1] + P_{12}E[T_2] + P_{13} \cdot E[T_3] + P_{12} \cdot P_{23} \cdot E[T_3] + P_{14} \cdot E[T_4] + P_{12} \cdot P_{24}E[T_4]}{P_{14} + P_{12} \cdot P_{24} + P_{21} \cdot P_{23} + P_{13}} +$$

$$+ \frac{P_{13} \cdot P_{34}E[T_4] + P_{12} \cdot P_{23} \cdot P_{34}E[T_4]}{P_{14} + P_{12} \cdot P_{24} + P_{12} \cdot P_{23} + P_{13}} = 342,69 \text{ [h]}$$

***The average time of process staying in the subset of exploitation states***

For such specified exploitation process the expected value of random variable denoting the time of continuing ability to work the considered telematics system is:

$$E[T_+] = \frac{\sum_{i \in S_Z} P_i}{\sum_{i \in S_+} \frac{P_i}{E[T_i]} \cdot \sum_{j \in S_N} P_{ij}} = \frac{P_1 + P_2}{l_1(P_{13} + P_{14}) + l_2(P_{23} + P_{24})} = 340,33 \text{ [h]}$$

In turn the expected value of staying of the telematics system in the subset of inability states is expressed as follows:

$$E[T_-] = \frac{\sum_{i \in S_N} P_i}{\sum_{i \in S_-} \frac{P_i}{E[T_i]} \cdot \sum_{j \in S_Z} p_{ij}} = \frac{P_3 + P_4}{l_4(P_{41} + P_{24})} = 1,89 \text{ [h]}$$

***Stationary probability of process staying in selected state subset***

In the considered model the set  $S_Z$  the sub-states  $s_1$  and  $s_2$  are distinguished and the boundary probability of ability state is described expression:

$$P_+ = \frac{\pi_1 E[T_1] + \pi_2 E[T_2]}{\sum_{k \in S} \pi_k \cdot E[T_k]} = P_1 + P_2 = 0,98841$$

while the random inability states of the telematics system belong to the set  $S_N = \{s_3, s_4\}$

and the system boundary probability to stay in the inability subset of the states is:

$$P_- = \frac{\pi_3 E[T_3] + \pi_4 E[T_4]}{\sum_{k \in S} \pi_k \cdot E[T_k]} = P_3 + P_4 = 0,03202$$

***Indicator of operational readiness***

The boundary indicator of the operational readiness of telematics system is dependent on the changes of probabilities for exploitation states and the average time durations of such states. Then, an indicator of operational readiness can be represented as the sum of the boundary probabilities to stay the system in the ability states. This gives:

$$\zeta_g = \frac{\sum_{i \in S_Z} \pi_i E[T_i]}{\sum_{k \in S} \pi_k E[T_k]} = \frac{E[T_1] + P_{12}E[T_2]}{E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{21}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{21}P_{23}P_{34}E[T_4]} = 0,99237$$

***Probability of system using for realization of telematics service***

Taking into account formulas (7) and (10) the indicator of technical use takes the value

$$\zeta_t = w_z \cdot P_z = \frac{P_{12}E[T_1] \cdot E[T_2] + (P_{12} \cdot E[T_2])^2}{(E[T_1] + P_{12}E[T_2] + P_{13}E[T_3] + P_{21}P_{23}E[T_3] + P_{14}E[T_4] + P_{12}P_{24}E[T_4] + P_{13}P_{34}E[T_4] + P_{21}P_{23}P_{34}E[T_4])^2} = 0,49631$$

***Potential ability of the service realization***

The exploitation efficiency ratio indicating the potential ability to implement telematics services according to (12) and (14) can be written as:

$$\begin{aligned} \zeta_P &= [P_Z + (1 - P_Z) P_{UT}] P_{PZ} P_R = \\ &= \left\{ P_{12} + (P_{13} + P_{14}) \left\{ 1 - \exp\left(-\frac{E[T_1]}{E[T_N]}\right) \right\} \right\} P_{12} P_{21} = \\ &= \{P_{12} + (P_{13} + P_{14}) [1 - \exp(-Y)]\} P_{12} P_{21} = 0,99681 \end{aligned}$$

where:

$$\begin{aligned} Y &= \frac{E[T_1] l_4}{P_3 + P_4} = \\ &= \frac{\{P_{12}E[T_{21}] P_{13}E[T_{13}] + P_{14}E[T_{14}]\} \cdot \{P_{14} + P_{12}P_{24} + P_{21}P_{23} + P_{13}\}}{(P_{13} + P_{12} + P_{23}) E[T_3] E[T_4] + (P_{14} + P_{12}P_{24}) E[T_4]} = 11,7164 \end{aligned}$$

Presented results of calculations can be used not only for evaluation of the exploitation properties of telematics system technical equipment, but also for evaluation of the organization process of exploitation. This is given by the ratio of operational readiness, as well as the expected value to stay the system in a subset of maintenance states: waiting to be repaired and repair of damage. Especially, an attention supposed to be paid to the large value of waiting time for system repair. This time can be reduced by better organization of the system exploitation in area of maintenance organization.



## 4. Conclusion

A number parts of transport telematics systems is characterized by complex, expanded and redundant structure. This structure is subjected changes dependent on changes on the input of the system and actual working correctness of system components. Therefore, in the paper an attention was focused on defining the functional state of these systems and the probabilities determining to be by the system in given states. In order to achieve an overall assessment of transport telematics systems operations as evaluation method the multi-state analysis of the exploitation process was assumed. In the studies of system efficiencies the primary importance problem is determining the efficiency of partial measures of efficiency. From this point of view using the characteristics of the exploitation process a new model of exploitation efficiency of telematics system and its verification quality was presented.

The presented method of exploitation process modeling of the telematics systems, especially, developed algorithms determining system characteristics and efficiency indicators, can be useful in exploitation process planning and the process organization. Mainly, in developing mechanisms, which ensure availability and continuity telematics services aided realization of transport tasks. In proposed methodology method, however, the aspects of economic efficiency analysis of telematics systems exploitation were not included. Instead of that, the proposed model by introducing the indicators of correct functioning of telematics systems can be also used to formulate the indicators to assess the selected economic criteria, when modifications related to determination of costs and benefits resulting from correct work of the system exploitation states will be done.

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