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Implementation of Gravity Model to Estimation of Transportation Market Shares

Przemysław Krata*

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Abstract

The theoretical consideration presented in the paper is inspired by market gravity models, as an interesting attitude towards operations research on a market. The transportation market issues are emphasized. The mathematical model of relations, taking place between transportation companies and their customers on the market, which is applied in the course of the research is based on continuous functions characteristics. This attitude enables the use of the field theory notions. The resultant vector-type utility function facilitates obtaining of competitive advantage areas for all transportation companies located on the considered transportation market.

1. Introduction

A transportation market may be considered as a sub-system of any transportation system. The essence of its existence are relationships between customers and transportation service suppliers. Their interactions tend to entering into agreement of both parties – individual customers and transportation companies. Subsequently a carriage of goods takes place. Such a mechanism makes out the need for a subsistence of transportation market as an important element of any transportation system in competitive conditions.

According to the market principle, supply and demand are determined by the price, which is an equilibrium between both [10]. Therefore for many economists the market is a point where goods and services are exchanged and does not have a specific location, since it is simply an abstraction of the relationships between supply and demand [10].

* Gdynia Maritime University Faculty of Navigation, Department of Ship Operation 3, Jana Pawła II Ave, 81-345 Gdynia, e-mail: inhalt@klif.am.gdynia.pl

Since late 20-th of XX century such an abstraction is no longer in force. One of the first researchers investigating spatial aspects of a competition was Hotelling. His early work with the principle of market competition, created the foundations of market area analysis by considering factors such as location and distance decay. Later Hotelling's considerations were developed by Reilly (1931) and factors such as market size were taken into consideration (Reilly's law) permitting to build complex market areas. The continuous development of models in noticed in which the concept of distance is considered concomitantly with the concept of market [6, 10]. One of the milestone was Huff's law (1963), its modification for example by Batty (1978), Yamashiro and Hoshino (1999) and its application for example by Migdał-Nejman and Mudza (2009) [6]. Generally the characteristic feature of a transportation market is the main role of spatial aspects and processes such as carriage of goods, flow of information and making decision regarding location of infrastructure or resources [6].

The typical transportation problem in a technical approach consists in solving the problem of planning the distribution of traffic flow in a transportation system [1]. Usually the outcome of the market competition taking place between transportation companies is assumed a priori as well-known and it constitutes the input data of a classic transportation problem. Although the inherent characteristic of any competitive transportation market is an uncertainty of presumptions regarding any individual customers choices of transportation companies' offer. The question is particularly apparent in any planning tasks since it is difficult to assess the future demand for transport in every selected node of considered transportation system. Thus the matter of competition over transportation market area needs to be modeled.

One of the most useful and steadily developing group of mathematical model is a set of gravity models [6, 7]. The models are a modified version of Isaac Newton's Law of Gravitation and they are applied to predict movement of people, information, and commodities between cities and even continents [11]. The key foundation of gravity models is that elements of considered system may be treated as imaginary masses layouted in the space and interacting with each other [11].

In the paper an application of gravity type model is proposed. It is used for estimation of transportation market shares over the investigated area. The considerations of spatial interactions between transportation companies and their potential customers are restricted to the preliminary stage of interactions preceding shipment of goods, i.e. competition over transportation market area. The investigation is focused on the demand-supply relation and its result expressed as entering into agreement between individual customers and firms. The effect of considered interactions is – from the customer's point of view – commissioning of transport as the final stage of decision-making process and concurrently capturing the element of the market area – from the transportation company point of view.

2. Continuous Function Based Approach to Modeling of Transportation Market

Modern approach to transportation problems is characterized by the wide use of the notion of a system. A system is a functional entirety which consists of elements remaining in mutual relations [1]. The number and characteristics of those elements and the relations between them should guarantee the realization of specific functions by a system [3]. A system – a transportation system as well – is thus an entity isolated from reality, which formulation has the form of relations defined on a set of distinctive elements and relations combining the elements of this entity with environment [1].

The formal description of the system notation as a set of elements and relations between them can take the form of an ordered pair [1]:

$$\boldsymbol{S} = < \boldsymbol{A}, \boldsymbol{R} > \tag{2.1}$$

where:

$\boldsymbol{S}$ – system;

$\boldsymbol{A}$ – set of system elements, $\boldsymbol{A}=\{a_s: s=1, 2, \ldots, n\}$;

$\boldsymbol{R}$ – set of relations defined on system elements, $\boldsymbol{R}=\{\boldsymbol{R}_r: r=1, 2, \ldots, m\}$, $\boldsymbol{R}_r \subset \boldsymbol{A}\times\boldsymbol{A}$;

n – number of elements of the system;

m – number of relations between system elements.

A salient element of transportation systems investigation is the description of relations of systems and their environment [1]. These relations are expressed by the demand for goods shipment uttered by customers belonging to the system's environment and the response to such demand (demand-supply relation) [1]. In this formulation the transportation market system is investigated and the transportation system is an element of the considered market system, and it remains in relations with other elements, including customers expressing their demand for shipment of goods.

It is often difficult to conduct the research of physically existing systems and sometimes even impossible [1]. Therefore, the modeling and the investigation of models behavior is commonly used instead of real objects and systems examination. In most applications it is not necessary to consider all details of the examined system and the degree of model simplification is determined by the aim of the conducted research [3]. Nowadays modeling of transportation systems and transportation market systems is usually performed in discrete formulation [1, 5]. The discrete attitude towards the modeling of systems and the definition of a system is based on the relations characterizing discrete sets (formula 2.1) and it results in a specific formal notation. The sets of elements, relations, variables and constraints are defined. Subsequently all sets are indexed, while the Cartesian product of sets determines possible arrangements of elements that can interact. The mathematical model of a system is designed for all possible relations between its elements. A variable is called discrete if it can take selected numerable values, which means that it is possible to number those values and present them as a sequence [9].

The most often the values of measurable variables used in the model of a system appear to be continuous [9]. They can take any value from a given interval or a group of intervals. The consideration and the model proposed in the paper are based on continuous functions and a naming convention commonly used while describing many physical phenomena, especially gravity effects [8]. The effect of modeling based on continuous functions is that the relations taking place between customers and suppliers in the considered transportation market system are described in a different way than in discrete models.

The field theory based models of physical interactions operates local defined continuous variables [8]. Such local variable is defined in the entire considered space and its value (or value and direction – for vector variable) may be different at any point of the space. The introduction of a local variable notion in itself implicates the infinite number of points described by that variable and, therefore, the impossibility of indexed addressing. Consequently, the modeling of relations taking place in any systems described by local continuous variables is performed in a different way than the modeling of systems based on discrete variables only. Such an attitude towards modeling of the transportation market system is applied in the paper.

The two-dimensional coordinate system is used, which satisfies the assumption that there is no need for the use of the three-dimensional space coordinate system. In other words the geometry of the transportation market area is reduced to the orthogonal projection of the earth surface. As the analyzed area is much smaller than the earth extent, the geoid curvature is omitted. The rectangular axis is used and every point is defined by the grid coordinates.

In a classic discrete formulation the set $\boldsymbol{A}$ of elements of the transportation market system (formula 2.1) may be represented by the sum of sets:

$$\boldsymbol{A} = \boldsymbol{PT} \cup \boldsymbol{KL} \tag{2.2}$$

where:

$\boldsymbol{PT}$ – set of transportation companies;

$\boldsymbol{KL}$ – set of customers.

The set of transportation companies may be defined as:

$$\boldsymbol{PT} = \left\{pt_k : k = 1, 2, 3, ..., n_{pt}\right\} \tag{2.3}$$

where the vector of particular company's characteristics is given by:

$$pt_k = \left\langle w_k^1, w_k^2, ..., w_k^f \right\rangle \tag{2.4}$$

where:

n_{pt} – number of transportation companies on the considered market;

f – number of features characterizing every transportation company on the market;

w_k^f – value of f-numbered characteristic of k-numbered transportation company.

It is assumed that transportation companies' features describing the source of their offer on the transportation market fair enough are:

- magnitude of every single transportation company offer,
- customer service level,
- unit price of transportation service,
- offered velocity of goods shipment.

For the sake of making no mention of the units of a/m variables the relative description of variables is applied. The relative offer $Q_{R(j',i')}$ of a transportation company located in the point of coordinates (j', i') is described by the formula:

$$Q_{R(j',i')} = \frac{Q_{(j',i')}}{Q_T} \tag{2.5}$$

where:

$Q_{(j',i')}$ – offer of a transportation company located in the point of coordinates (j', i');

Q_T – total offer of all transportation companies over the considered transportation market;

(j', i') – coordinates of offer' source location (a transportation company).

The relative unit price $c_{R(j',i')}$ offered by a transportation company located in the point of coordinates (j', i') is given by the formula:

$$c_{R(j',i')} = \frac{c_{(j',i')}}{c_A} \tag{2.6}$$

where:

$c_{(j',i')}$ – unit price offered by a transportation company located in the point of coordinates (j', i');

c_A – average unit price on the transportation market.

The relative transport velocity $p_{R(j',i')}$ offered by a transportation company located in the point of coordinates (j', i') is described as follows:

$$p_{R(j',i')} = \frac{p_{(j',i')}}{p_A} \tag{2.7}$$

where:

$p_{(j',i')}$ – transport velocity offered by a transportation company located in the point of coordinates (j', i');

p_A – average transport velocity on the transportation market.

Instead of the discrete formulation, a locally defined continuous functions are applied in the proposed model. The spatial distributions of values of the functions create the continues fields in the considered space. The field concept is understood as a feature of the considered space and the sufficient descriptor defining the field is the possibility to attribute specific characteristic to every selected point of the space, for instance the relative offer, relative unit price or relative velocity which may be obtained according to the formulas (2.5), (2.6), (2.7). Thus, the field of characteristics of transportation companies location (sources points of an offer) is described by the formula:

$$PT = PT(Q_{R(j',i')}, SL_{R(j',i')}, c_{R(j',i')}, p_{R(j',i')}) \tag{2.8}$$

where:

PT – field of transportation company characteristics;

Q_R, SL_R, c_R, p_R – fields describing a transportation company;

(j', i') – coordinates offer' source location (transportation companies).

Analogically to the formulas (2.3) and (2.4) the set of customers located in the considered area of the transportation market may be classically defined as:

$$\boldsymbol{KL} = \left\{kl_g : g = 1, 2, 3, ..., n_{kl}\right\} \tag{2.9}$$

where the vector of a particular customer's characteristics is:

$$kl_g = \left\langle w_g^1, w_g^2, ..., w_g^h \right\rangle \tag{2.10}$$

where:

n_{kl} – number of customers on the transportation market;

h – number of features characterizing every customer on the considered market;

w_g^h– value of h-numbered characteristic of g-numbered customer.

The function describing the customers features field is given by the formula:

$$KL = KL(q_{(j,i)}) \tag{2.11}$$

where:

KL – field of customers characteristics;

q – field of demand for cargo shipment;

(j, i) – coordinates of customers.

Although the core foundation of competition for a comparable service is price, there are several spatial strategies that impact the price element [10]. One of the most important aspect is the time of shipment of goods hence the price-time models are commonly used for logistic issues modeling. As the transport velocity offered by a transportation company is taken into account in the proposed model it may be classified as a variety of the price-time model as well.

The other most common strategies impacting the price are market coverage and range expansion [10]. They both are directly associated with the concept of distance between customers and service offerers on the market. It is assumed that the greater is the distance between a customer and a transportation company the weaker their interaction is. The comprehensive description of an influence of the distance on the relation customer-supplier may be given by the distance decay curve [10]. The applied one in the model is shown in Fig. 1. It presents the decay of an offer impact of any transportation company in terms of distance. The applied function is r^{-1} where r is the considered distance.

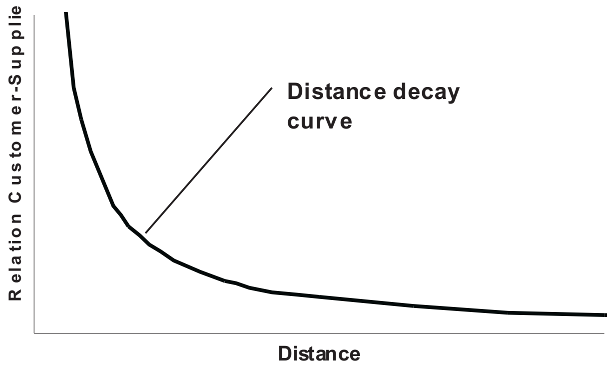

Fig. 1. Distance decay curve applied in the model

It is assumed that the relation between a transportation company and a customer consists in exerting an influence on a customer by a transportation company. The interaction aims at commissioning of goods shipment by a customer to a specified supplier, which is named an “offer influence” [4]. Thus, the offer influence is a one-way interaction of transportation companies to customers expressing their demand for goods’ shipment.

Considering PT and KL fields of the transportation market system - formulas (2.8) and (2.11) – having effect on the value of the offer influence, taking into account the distance decay function r^{-1}, furthermore assuming elasticity of demand χ characterizing the sensibility of the investigated transportation market to the change of the unit price of service and the coefficient λ of price priority when choosing a service offerer (analogous to the coefficient modeling priority regarding time availability of an offer in Huff’s model [6]), the vector function of an offer influence $\overline{F}$ is constructed in a form given by the formula:

$$\overline{F}_{(j,i)} = \overline{F}\left(KL(q_{(j,i)}), PT(Q_{R(j',i')}, SL_{(j',i')}, c_{R(j',i')}, p_{R(j',i')}), r_{(j,i)}\right) = M \cdot \frac{Q_R \cdot q}{r \cdot (1 + \frac{q^2}{Q_R^2})} \cdot \overline{e_r} \tag{2.12}$$

by the coefficient$M$:

$$M = 2 \cdot SL \cdot p_R \cdot c_R^{(\chi+\lambda)} \tag{2.13}$$

where:

M – coefficient describing the source of an offer (transportation company);

Q_R – value of an relative offer of a transportation company;

q – value of the demand for goods' shipment expressed by a customer;
r – distance between a customer and a supplier (source of an offer);
$\overline{e_r}$– radial unit vector;
SL – service level;
c_R – relative unit price of transportation service;
p_R – relative transport velocity;
χ – elasticity of demand;
λ – coefficient of price priority when choosing of an service offerer.

The quality-type verification of the vector function $\overline{F}$ in the form (2.12), (2.13) is published in [4]. The function $\overline{F}$ formulates a notion of the offer influence vector field defined in a flat geometric space comprising transportation market system area. The field of an offer influence constitutes quality measure factor of relations between an individual customer and an individual transportation company on the transportation market in the mathematical model of the considered relation [4].

3. Utility Function as an Vector Field

When modeling transportation issues the multiobjective optimization of decision making is emphasized [1, 2]. It results from a wide variety of constraints (technical, economical, ecological and others) as well as a variety of interest of individual participants of the transportation process [1]. The main difficulty in realization of multiobjective goal function is the fact that very rarely there is the convergence of particular criteria extremum [2].Solving the multiobjective optimization problem of vector-type goal function supplying the extremum values of all partial goal components (which are components of the vector) is called utopian and it is usually impossible to be obtained due to contradictive aims [2].

The postulate of including all important system features and optimization criteria in the transportation market system analysis, which is equivalent to the requirement of multiobjective optimization, is fulfilled by the use of multiattributive method based on the American inspiration and applying an utility function [1]. The method may be also called the method of synthesis to the single criterion omitting incomparability and it consists in the aggregation of all particular criteria (points of view) to one utility function, which is the base for optimization process later on [1]. So the multiobjective character of optimization is reached at the stage of the transportation market system modeling and the model is fundamental to utility function deduction.

The utility function applied in the model is based on the offer influence field described by the vector function (2.12). It is transformed into the offer intensity field. The offer intensity field is a kind of field description focused on a transportation service supplier and its relative offer Q_R. It is independent from a customer and his demand q. The directionality of the considered relation (radial direction of the offer influence vector function) is also taken into account.

The offer intensity $\overline{E}$ is defined similarly to the classical intensity definition and may be presented in the following form [8]:

$$\overline{E} = \lim_{q \to 0} \frac{\overline{F}}{q} \tag{3.1}$$

where:

$\overline{F}$– vector of an offer influence;

q – demand for goods shipment expressed by a customer.

On substituting the formula (2.12) and determining the limit, the offer intensity vector $\overline{E}$ is obtained as:

$$\overline{E} = \frac{M \cdot Q_R}{r} \cdot \overline{e_r} \tag{3.2}$$

by the symbols applied like in the formula (2.12).

The offer intensity function (2.2) is applied as the utility function aggregating all essential goal-type variables, being the partial criteria of the proposed model of interactions taking place on a the transportation market. The utility function refers to the multiattribute utility theory and the distribution of its vectors $\overline{E}$ is the base for further optimization of decision making process.

The vectors of the offer intensity may be superposed, so the offer intensity at any arbitrarily chosen point of the researched space containing any number of sources of an offer can be obtained as a vector sum of offer intensities coming from every single transportation company according to the formula [3]:

$$\overline{E}(j,i) = \sum_{k=1}^{n_{pt}} \overline{E_k} \tag{3.3}$$

where:

$\overline{E}(j,i)$ – offer intensity field in the point (j,i) of transportation market area;

$\overline{E}_k$ – offer intensity vector generated by k-numbered transportation company;

n_{pt} – number of transportation companies located in considered market area.

The superposition rule is an important element of the proposed model of the transportation market system, because it allows for the consideration of transportation markets comprising more than one customer and more than one supplier of the transportation service. The resultant distribution of the offer intensity field is the final form of the utility function which is the key notion in the described model.

4. Procedure of Determination of Market Shares

The formulas (3.2) and (3.3) enable to determine the distribution of the offer intensity vectors $\overline{E}$ in the entire area of the transportation market being modeled.

As the offer intensity is applied as the utility function aggregating many decision-making criteria to the single criterion, the well-known single-variable optimization techniques can be used.

For the purpose of the model presentation the exemplary hypothetical transportation market is depicted. It comprises five competitive transportation companies. The example is not an image of the practically existing transportation market, so the coordinates of the area do not refer to any geographical latitude and longitude of any region. The elasticity of demand χ is assumed constant as being a feature characterizing the market in a general way and independent from the actual location of the examined point of the market. The coefficient λ of price priority when choosing a service offerer is assumed constant as well. The exemplary characteristics and location of five mentioned companies on the imagined transportation market is shown in the Table 1.

Table 1

Distribution and characteristics of transportation companies on the market area (example)

Transportation company	Coordinate		$Q_{R(j',i')}$	*SL*	c_R	p_R
	j'	i'	[%]	[–]	[%]	[%]
1	*2*	*3*	*4*	*5*	*6*	*7*
PT1	35	50	19	0.8	73	99
PT2	50	11	7	0.9	87	83
PT3	17	27	9	0.85	87	92
PT4	52	32	42	0.75	116	106
PT5	26	14	23	0.9	102	99

The distribution of the offer intensity vectors $\overline{E}$ on the modeled exemplary market is determined with regard to the formulas (3.2) and (3.3). The resultant spatial distribution of values of the offer intensity vectors is shown in Fig. 2.

One of the most important features of the offer intensity field are its vectors directions. Therefore the map of the offer intensity vectors is generated and they are presented in the form of symbolic arrows plotted in the grid coordinate system. The length and the direction of the arrows show the value and the direction of the local offer intensity vectors (Fig. 3). For the purpose of the increase in clarity of the figure the lines of equal values of the offer intensity are plotted.

On the basis of the formula (3.2) it can be affirmed that the customer should commission the shipment of goods to the transportation company which is the closest one, the cheapest, offering the highest service level and the highest transport velocity (shortest time of shipment). The simultaneous fulfilling of all such requirements is usually impossible and the partial criteria do not reach their extreme values concurrently. Therefore the proposed solution of the choice problem is the optimization based on the search for the extreme utility function aggregating all partial criteria, which is the offer intensity function. The search for the optimal choice of trans-

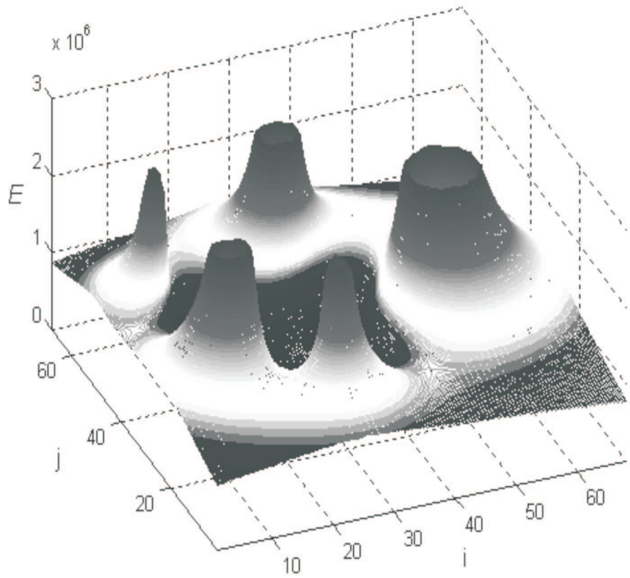

Fig. 2. Spatial distribution of values of utility function vector filed

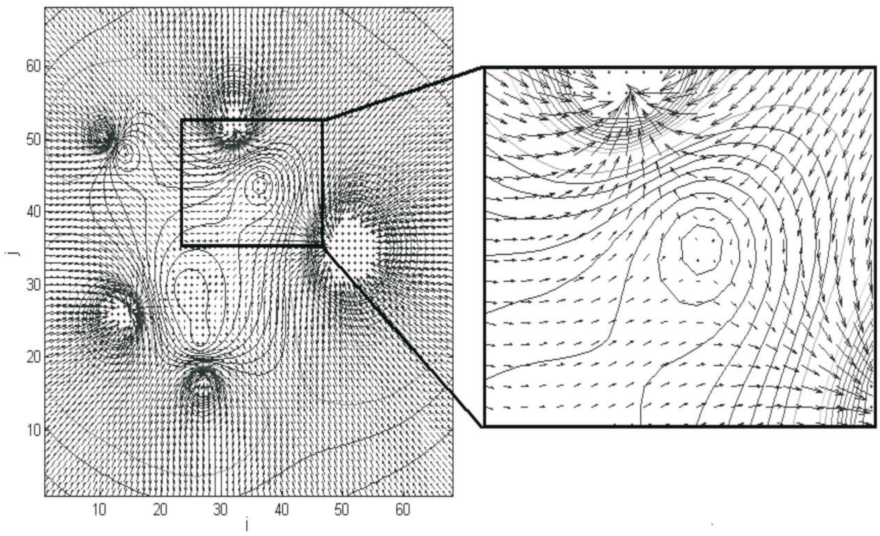

Fig. 3. Map of utility function vectors (projection)

portation service supplier is described in [3]. The algorithm consists in proceeding from the location of a customer step by step according to the local direction of the utility vector i.e. offer intensity vector (Fig. 3). Following the algorithm for all possible locations of a customer on the whole considered transportation market, the optimal choice of transportation company is determined. As the result all points of the market are assigned to one of the transportation companies on the market. From the other point of view every transportation company offering service on the market captures a part of the market area which is shown in Fig. 4 for exemplary data given in the Table 1.

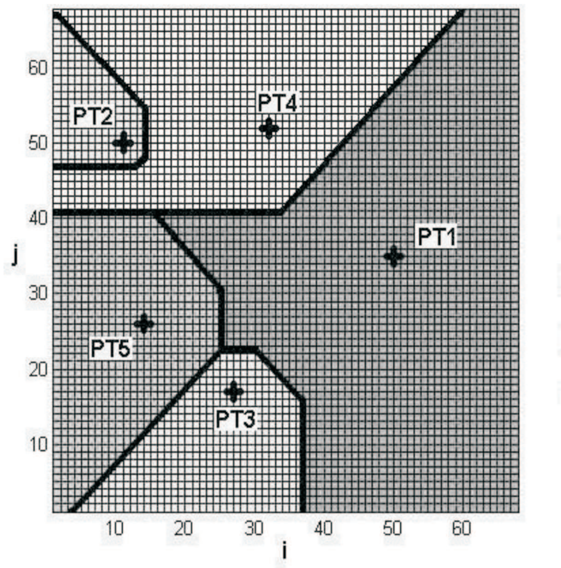

Fig. 4. Tributary market area captured by individual transportation companies

The tributary market area captured by every transportation company reflects its potential possibility of gaining profits on agreements made with customers located within the area. Thus, the notion of market area potential *MAP* is introduced and it is defined according to the formula:

$$MAP_{(PTk)} = \int_{Sk} KL(q_{(j,i)}) \tag{4.1}$$

where:

S_k – tributary market area captured by transportation company PT_k;

PT_k – k–numbered transportation company;

KL – function of customer characteristics;

q – the demand for goods' shipment expressed by a customer located in the point of coordinates (j, i);

(j, i) – coordinates of customers location on the tributary market area captured by the transportation company PT_k.

The modeling of market area potential can be useful information enabling a strategy planning for existing transportation companies and for future investments as well.

5. Summary and Conclusions

The gravity model of relation taking place between transportation companies and their customers on the transportation market enables the multiobjective optimization of the decision regarding the choice of a transportation service supplier for a customer located in the investigated market area. The model is a variety of price-time model comprising the emphasized space geometry element. The proposed method refers to the multiattribute utility theory.

The model and the optimization algorithm can be widely used in the course of transportation system modeling and optimization since it allows to obtain the burden in any specified node of a transportation network, which is the input data to models of traffic flow distribution. In such a conception, the results of the optimization can be taken as the preliminary stage preceding the classical transportation problem formulation. It is especially important while solving the planning problem where it is problematic to arbitrarily assume the realistic distribution of demand for transportation service in a transportation network.

One of the advantage of the model is a convenient presentation of optimization results in the form of a sketch of tributary market areas captured by every transportation company. The interpretation may be similar to the Huff's diagrams of probability or Laksmanan-Hansen's charts of potentials [7]. Moreover, the notion of the market area potential *MAP* can be a rational descriptor enabling comparison of different market strategies for existing transportation companies and for new funded ones.

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