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## **Estimation of the cost of equity for mining and cement industries by single-index market model**

### **Introduction**

The cost of capital represents the total cost to the entity or company that will be incurred in order to raise and/or secure funding in order for it to acquire, develop and maintain its future sources of income (Lilford et al. 2002). The cost of capital is the minimum rate of return (in %) on the company's investments that can satisfy both shareholders (the cost of equity) and debtholders (the cost of debt). The cost of capital is thus the company's total cost of financing (Vernimmen... 2009). The cost of equity or the expected rate of return on the firm's common stock usually needs the application of finance models. The common model using in mineral valuation and mining project evaluation is CAPM. In this paper an attempt is made to select an alternative model. Therefore, the single-index market model is selected.

This model is one of the most frequently-used tools of modern business finance. Conceptually, this model is a linear relationship which associates the return from investment in a security with a market factor and a random error term. By assumption, the error term is independent across securities and of the market factor. These assumptions are invoked so that the model is consistent with the notion of diversification. The extent of the common factor's influence on the returns of a given security is known as systematic (or undiversifiable) risk. It may, under certain conditions, be measured by the coefficient associated with the market factor in single-index market factor. It is this parameter, known variously as the

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coefficient beta<sup>1</sup>, from which much of the importance of the single-index market model derives. Since its development, various groups, including stock brokers, investment managers, academics and others, have expended significant amounts of time and resources towards the estimation of the single-index market model and its beta parameter in particular. These estimates have been applied to portfolio selection, attempts at empirical verification of pricing models and numerous other empirical tests of financial and economic hypotheses. In these applications, estimation has almost universally been carried out by means of the ordinary least square technique applied to time series data using a stock market index as a substitute for the unobservable market factor (Riding 1983). Finally the selected model is applied on mining and cement companies listed in Tehran Stock Exchange.

## **1. Estimation of the cost of equity**

Several models are developed for estimating the cost of equity. However, description and comparison of all of them is not possible in this paper. To select the appropriate method(s) to estimate the cost of equity, applied models are studied in practice and then the popular methods are discussed.

### 1.1. Estimation of the cost of equity in practice

In two recent decades a number of surveys into the capital budgeting practice have been conducted in different countries. These surveys covered a range of issues; such as which capital budgeting techniques were used, how firms ranked the importance of these techniques, and how discount rates were determined. Some of these surveys (e.g. Jog et al. 1995; Kester... 1999; Gitman... 2000; Graham et al. 2001; Brounen... 2004; McLaney... 2004; Truong... 2008) have studied the methods that used by the firms in Australia, US, Canada and a number of European countries to determine the discount rate. Truong... (2008) presented a brief comparison of these findings. Taheri ... (2009a) summarised the results and concluded that the CAPM is the most common method among the practitioners.

### 1.2. Estimation of the cost of equity in mining companies

The methods for estimating the cost of equity in mining companies is not studied in practice. However, the study on the texts on mineral valuation or mineral project evaluation demonstrates that the CAPM is general model for estimating the cost of equity (Gentry et al. 1984; Ballard 1994; Barnett et al. 1994; Jackson 1994; O'Connor et al. 1994; Smith 2002; Lilford 2006).

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<sup>1</sup> Risk depends on exposure to macroeconomic events and can be measured as the sensitivity of a stock's returns to fluctuations in returns on the market portfolio. This sensitivity is called the stock's beta.

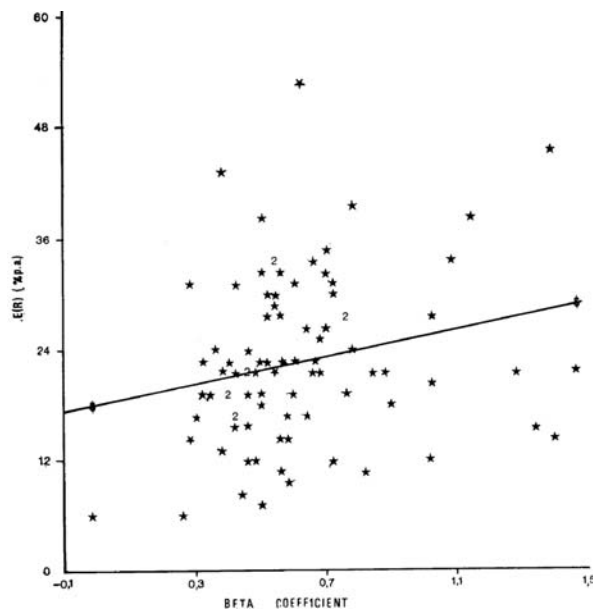


Fig. 1. Cross-sectional regression of E(R) on beta for 87 individual shares

Rys. 1. Regresja E(R) beta dla 87 indywidualnych udziałów (Gilbertson 1980)

Gilbertson (1980) applied the CAPM for important mining and mining-finance shares of Johannesburg Stock Exchange(JSE). The least-square best-fit line by the cross-sectional regression of the expected returns for a large number of individual shares is given by the Equation (1).

$$E(R) = 18.5\% + \beta (6.8\%) \quad (1)$$

As might be expected from the wide scatter of the points in Fig. 1, this relationship is not statistically sufficient and the results are not favorable. However, an alternative method is not proposed.

### 1.3. The CAPM model

The Capital Asset Pricing Model (CAPM) is perhaps the most widely used method of assessing the cost of equity capital. The basis of this method is that the return on an individual company stock may be related to the stock market as a whole by Equation (2).

$$\bar{R} = R_F + \beta \cdot (\bar{R}_M - R_F) \quad (2)$$

where:

- $\bar{R}$  – Expected Yield Rate on an Investment,
- $R_F$  – Risk-Free Rate of Interest,

- $\beta$  – Beta, a Measure of the Systematic Risk for the Investment,  
 $\bar{R}_M$  – Yield Rate on the Market Portfolio,  
 $\bar{R}_M - R_F$  – Market risk premium.

The risk is divided into two components: a systematic portion, called *systematic risk*, and the remainder, which we call *specific* or *unsystematic risk*. The following definitions describe the difference (Ross... 2002):

- A *systematic risk* is any risk that affects a large number of assets, each to a greater or lesser degree.
- An *unsystematic risk* is a risk that specifically affects a single asset or a small group of assets.

The only component of risk that investors care about is systematic risk which is based on the assumption that all unsystematic risks can be eliminated by holding a perfectly diversified portfolio of risky assets (Pratt 2002). The CAPM model assumes that unsystematic risk can be eliminated in a diversified portfolio and is therefore neglected.

The CAPM model explains the cost of equity as the risk-free rate plus a risk premium. The CAPM requires the estimation of three numbers: the risk-free interest rate, the expected equity premium and the expected  $\beta$ . But in practice, a company's beta is estimated from its share returns in the fairly recent past, on the assumption that its past beta provides a good forecast of its future beta. That is, past observations of the excess returns on the share and on the market are treated as a sample of the distribution of possible excess returns in the future. However, the majority of companies use betas estimated by commercial services such as Bloomberg, Value Line or Ibboston Associates (Bruner... 1998; Rutterford 2000).

### 1.3.1. Choice of equity premium

The choice of premium on the stock market is one of the most uncertain aspects of estimating the cost of capital. The two approaches to estimation are to use a long-run historic average, or to estimate the forward-looking premium (Armitage 2005). Use of a historic average is based on the principle that the premium observed over many years in the past provides a good estimate for the premium to be expected over many years in the future. But many, perhaps most academics and practitioners have come to doubt that the premium in the future will be as large as it has been in the past, at least if the historic premiums measured using an estimation period from the twentieth century (Dimson... 2002). So, they place more faith in reasonable expectations about the future than in outcomes observed in recent decades.

### 1.3.2. Limitations and shortfalls

On the whole, empirical tests on the CAPM do not confirm this model as a valid theory. Roll (1977) claims that the CAPM can never be tested, because as the market portfolio

contains every asset in the international economic system, a great fraction of it is non-marketable and therefore unobservable in its returns. In addition, many in the profession have concluded that no one has ever come close to constructing a valid test of the capital asset pricing model and no one ever will. They feel that the CAPM is simply not a testable theory (Elton... 2007).

The practical studies showed that the CAPM is the most popular method in estimating the cost of equity. However, according to shortfalls and problems relating to it, a relatively similar and simpler model i.e. the single-index market model is proposed. This model is used to estimate  $\beta$  for the CAPM because of its simplicity.

#### 1.4. The single-index market model

Since the market model is a single-index model, first the single-index models are described. Essentially, the single-index model assumes security returns are correlated for only one reason, each security is assumed to respond, in some cases more and in other cases less, to the pull of a single factor, which is usually taken to be the market portfolio. As the market portfolio makes a significant movement upward (as measured by any of the widely available stock market indexes), nearly all stocks go up with it (Haugen 1997). This suggests that one reason security returns might be correlated is because of a common response to market changes, and a useful measure of this correlation might be obtained by relating the return on a stock to the return on a stock market index. To state the assumption of the single-index model more precisely, consider Fig. 2, where the returns on an arbitrarily selected stock are related to the returns on the market portfolio. The broken line running through the scatter is the line of best fit (minimizing the sum of the squared vertical deviations of each observation from the line), or an estimate of the stock's characteristic line. The intercept of the characteristic line is given by  $a_i$  and the slope by the beta factor. The rate of return for the stock may be written as Equation (3) (Elton... 2007).

$$R_i = a_i + \beta_i R_m \quad (3)$$

where:

- $a_i$  – is the component of security  $i$ 's return that is independent of the market's performance-a random variable,
- $R_m$  – is the rate of return on the market index-a random variable,
- $\beta_i$  – is a constant that measures the expected change in  $R_i$  given a change in  $R_m$ .

This equation simply breaks the return on a stock into that part due to the market and that part independent of the market.  $\beta_i$  in the expression measures how sensitive a stock's return is to the return on the market. A  $\beta_i$  of 2 means that a stock's return is expected to increase (decrease) by 2%, When the market increases (decreases) by 1%. Similarly, a  $\beta_i$  of 0.5

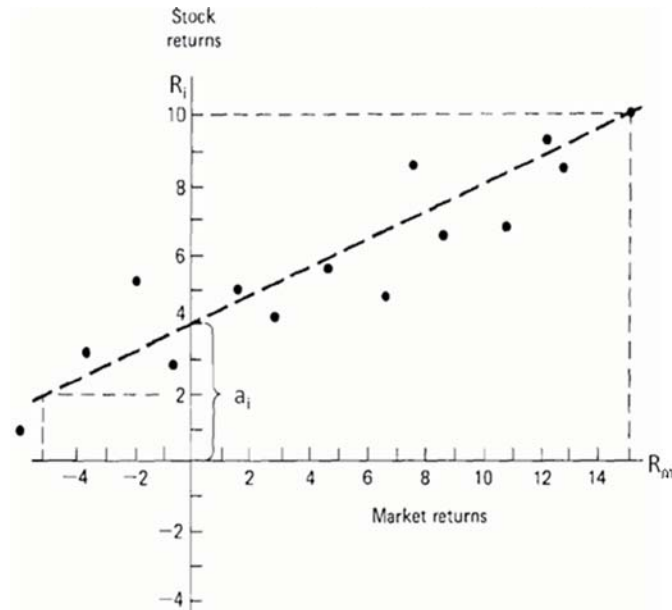


Fig. 2. Relationship between the returns on an individual investment and the returns on the market portfolio (Haugen 1997)

Rys. 2. Zależność pomiędzy indywidualną stopą zwrotu a stopą zwrotu z portfela rynkowego (Haugen 1997)

indicates that a stock's return is expected to increase (decrease) by 1/2 of 1% when the market increases (decreases) by 1%<sup>2</sup>.

The term  $a_i$  represents that component of return insensitive to (independent of) the return on the market. It is useful to break the term  $a_i$  into two components. Let  $\alpha_i$  denote the expected value of  $a_i$  and let  $e_i$  represent the random (uncertain) element of  $a_i$ . Then (Elton... 2007).

$$a_i = \alpha_i + e_i \quad (4)$$

where  $e_i$  has an expected value of zero. The equation for the return on a stock can now be written as Equation (5).

$$R_i = \alpha_i + \beta_i R_m + e_i \quad (5)$$

The single-factor model implicitly assumes that two types of events produce the period-to-period variability in a stock's rate of return. The first type of event is referred to as a *macro event*. Examples might include an unexpected change in the rate of inflation,

<sup>2</sup> The single-index model is illustrated with a stock market index. It is not necessary that the index used be stock market index. The selection of the appropriate index is an empirical rather than a theoretical question.

a change in the Federal Reserve discount rate, or a change in the prime rate of interest. In any case, macro events are broad or sweeping in their impact. They affect nearly all firms to one degree or another, and they may have an effect on the general level of stock prices. They produce a change in the rate of return to the market portfolio, and through the pull of the market, they induce changes in the rates of return on individual securities. Thus, in Fig. 2, if the return to the market portfolio in a given period were equal to – 5 percent, we would expect the return to the stock to be 2 percent. If the market's return were 15 percent instead we would expect the stock's return to be 10 percent. The difference in the stock's expected return can be attributed to the difference in the pull of the market from one period to the other.

The second type of event which produces variability in a security's return in the single-factor model is micro in nature. *Micro events* have an impact on individual firms but no generalized impact on other firms. Examples include the discovery of a new product or the sudden obsolescence of an old one. They might also include a local labor strike, a fire, or the resignation or death of a key person in the firm. These events affect the individual firm alone. They are assumed to have no effect on other firms, and they have no effect on the value of the market portfolio or its rate of return. Micro events do affect the rate of return on the individual security, however. They cause the stock to produce a rate of return which might be higher or lower than normal, given the rate of return produced by the market portfolio in the period. Micro events, therefore, are presumed to cause the appearance of residuals or deviations from the characteristic line.

Other types of events have been assumed away by the model. One might be referred to as an *industry event*, an event which has a generalized impact on many of the firms in a given industry but is not broad or important enough to have a significant impact on the general economy or the value of the market portfolio. Events of this nature also may, conceivably, cause the appearance of a residual, but the single – index model assumes residuals are always caused by micro events. The foregoing scenario is consistent with the assumption that the residuals or shock terms for different companies are uncorrelated with one another. The residuals will be uncorrelated if they are caused by micro events that affect the individual firm alone but not other firms (Haugen 1997).

Once again, note that both  $e_i$  and  $R_m$  are random variables. They each have a probability distribution and a mean and standard deviation. It is convenient to have  $e_i$  uncorrelated with  $R_m$ . Formally, this means that

$$\text{cov}(e_i, R_m) = E [(e_i - 0)(R_m - \bar{R}_m)] = 0 \quad (6)$$

If  $e_i$  is uncorrelated with  $R_m$ , it implies that how well Equation (5) describes the return on any security is independent of what the return on the market happens to be. Estimates of  $\alpha_i$ ,  $\beta_i$ , and  $\sigma^2_{e_i}$  are often obtained from time series-regression analysis. Regression analysis is one technique that guarantees that  $e_i$  and  $R_m$  will be uncorrelated, at least over the period to which the equation has been fit.

The key assumption of the single-index model is that  $e_i$  is independent of  $e_j$  for all values of  $i$  and  $j$  or, more formally,  $\text{cov}(e_i, e_j) = 0$ . This implies ‘that the only reason stocks vary together, systematically, is because of a common co-movement with the market. There are no effects beyond the market (e.g., industry effects) that account for co-movement between securities. It is a simplifying assumption that represents an approximation to reality. The single-index model may be summarized as below (Elton... 2007).

Basic equation	$R_i = \alpha_i + \beta_i R_m + e_i$	for all stocks $i = 1, \dots, N$
By construction	1. Mean of $e_i = E(e_i) = 0$	for all stocks $i = 1, \dots, N$
By assumption	1. Factor unrelated to unique return: $E[(e_i(R_m - \bar{R}_m))] = 0$ 2. Securities only related through common response to Market: $E(e_i e_j) = 0$	for all stocks $i = 1, \dots, N$ for all pairs of stocks $i = 1, \dots, N$ and $j = 1, \dots, N$ but $i \neq j$
By definition	1. Variance of $e_i = E(e_i) = \sigma_{e_i}^2$ 2. Variance of $R_m = E(R_m - \bar{R}_m)^2 = \sigma_m^2$	for all stocks $i = 1, \dots, N$

In the subsequent section we derive the expected return, standard deviation, and covariance when the single-index model is used to represent the joint movement of securities. The results are:

1. The mean return,  $\bar{R}_i = \alpha_i + \beta_i \bar{R}_m$
2. The variance of a security's return,  $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2$
3. The covariance of returns between securities  $i$  and  $j$ ,  $\sigma_{ij} = \beta_i \beta_j \sigma_m^2$

Note that the expected return has two components: a unique part  $\alpha_i$  and a market-related part  $\beta_i \bar{R}_m$ . Likewise, a security's variance has the same two parts, unique risk  $\sigma_{e_i}^2$  and market-related risk  $\beta_i^2 \sigma_m^2$ . In contrast, the covariance depends only on market risk. This is what we meant earlier when we said that the single-index model implied that the only reason securities move together is a common response to market movements.

Although the single-index model was developed to aid in portfolio management, a less restrictive form of it-known as the market model-has found increased usage in finance. The term ‘‘market model’’ may be used to describe an equation which has found wide application in the literature of modern finance the original and possibly the simplest form of the market model is the single market model of Sharpe (1963) and Fama (1968). The market model is identical to the single-index model except that the assumption that  $\text{cov}(e_i, e_j) = 0$  is not made.

The model starts with the simple linear relationship of returns and the market (Equation (5)) and produces an expected value for any stock of by Equation (7).

$$\bar{R}_i = \alpha_i + \beta_i \bar{R}_m \quad (7)$$



### 1.4.1. Estimating beta

The use of the single-index model calls for estimates of the beta of a stock. Estimates of future beta could be arrived at by estimating beta from past data and using this historical beta as an estimate of the future beta. There is evidence that historical betas provide useful information about future betas. Furthermore, some interesting forecasting techniques have been developed to increase the information that can be extracted from historical data. Because of this, even the firm that wishes to use analysts' subjective estimates of future betas should start with (supply analysts with) the best estimates of beta available from historical data.

Equation (5) is used for estimating the beta. This equation is expected to hold at each moment in time, although the values of  $\alpha_i$ ,  $\beta_i$ , and  $\sigma_{ei}^2$  might differ over time. When looking at historical data, one cannot directly observe  $\alpha_i$ ,  $\beta_i$ , and  $\sigma_{ei}^2$ . Rather, one observes the past returns on the security and the market. If  $\alpha_i$ ,  $\beta_i$ , and  $\sigma_{ei}^2$  are assumed to be constant through time, then the same equation is expected to hold at each point in time. In this case, a straightforward procedure exists for estimating  $\alpha_i$ ,  $\beta_i$ , and  $\sigma_{ei}^2$ . Notice that Equation (5) is an equation of a straight line. If  $\sigma_{ei}^2$  were equal to zero, then we could estimate  $\alpha_i$  and  $\beta_i$  with just two observations. However, the presence of the random variable  $e_i$  means that the actual return will form a scatter around the straight line. Fig. 3 illustrates this pattern. The vertical axis is the return on security  $i$  and the horizontal axis is the return on the market. Each point on the diagram is the return on stock  $i$  over a particular time interval, for example, one month ( $t$ ) plotted against the return on the market for the same time interval. The actual observed returns lie on and around the true relationship (shown as a solid line). The greater  $\sigma_{ei}^2$ , the greater the scatter around the line, and since we do not actually observe the line, the more uncertain we are about where it is. There are a number of ways of estimating where the line might be, given the observed scatter of points. Usually, we estimate the location of the line using regression analysis.

This procedure could be thought of as first plotting  $R_{it}$  versus  $R_{mt}$  to obtain a scatter of points such as that shown in Fig. 3. Each point represents the return on a particular stock and the return on the market in one month. Additional points are obtained by plotting the two returns in successive months. The next step is to fit that straight line to the data that minimized the sum of the squared deviation from the line in the vertical ( $R_{it}$ ) direction. The slope of this straight line would be our best estimate of beta over the period to which the line was fit, and intercept would be our best estimate of Alpha ( $\alpha_i$ ).

More formally, to estimate the beta for a firm for the period from  $t = 1$  to  $t = n$  via regression analysis use Equation (8)

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\sum_{i=1}^n [(R_{it} - \bar{R}_{it})(R_{mt} - \bar{R}_{mt})]}{\sum_{i=1}^n (R_{mt} - \bar{R}_{mt})^2} \quad (8)$$

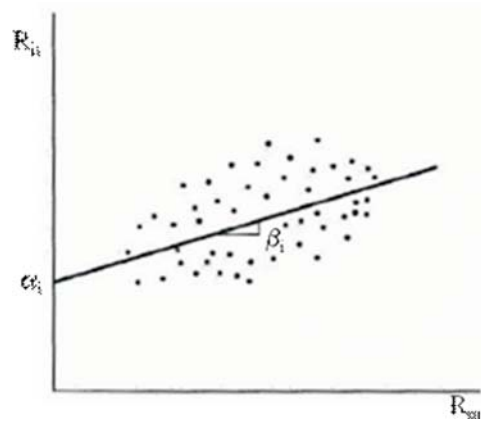


Fig. 3. Scatter diagram of single-index model (Elton... 2007)

Rys. 3. Diagram rozrzutu dla jednowskaźnikowego modelu rynku (Elton... 2007)

And to estimate Alpha use Equation (9)

$$\alpha_i = \bar{R}_{it} - \beta_i \bar{R}_{mt} \quad (9)$$

The values of  $\alpha_i$  and  $\beta_i$  produced by regression analysis are estimates of the true  $\alpha_i$  and  $\beta_i$  that exist for a stock. The estimates are subject to error. As such, the estimate of  $\alpha_i$  and  $\beta_i$  may not be equal to the true  $\alpha_i$  and  $\beta_i$ ; that existed in the period. Furthermore, the process is complicated by the fact that  $\alpha_i$  and  $\beta_i$  are not perfectly stationary over time. We would expect changes as the fundamental characteristics of the firm change. For example,  $\beta_i$  as a risk measure should be related to the capital structure of the firm and, thus, should change as the capital structure changes.

Despite error in measuring the true  $\beta_i$  and the possibility of real shifts in  $\beta_i$ ; over time, the most straightforward way to forecast  $\beta_i$  for a future period is to use an estimate of  $\beta_i$  obtained via regression analysis from a past period. Let us take a look at how well this works.

## 2. Selecting the appropriate model for estimating the cost of equity for the mining and cement industry of Iran

The single-index market model is selected for estimating the cost of equity of the mining and cement industry of Tehran Stock Exchange due to the mentioned problems relating to the CAPM (mentioned in 1.3.1 and 1.3.2) as well as the following items:

- The market premium on TSE is not determined in Iran
- The model is simpler and more appropriate for mining engineers.

## 2.1. Estimating the cost of equity by the single-index market model

Beta coefficients may be estimated via the *market model*. The market model is written as follows, where  $\bar{R}_M$  is the return on the market portfolio.

$$\bar{R}_j = \alpha_j + \beta_j \bar{R}_M + \varepsilon_j \quad (10)$$

$\bar{R}_j$  is the realized rate of return on security  $j$  and  $\bar{R}_M$  is the realized rate of return on the market.  $\alpha_j$  and  $\beta_j$  are constants and  $\varepsilon_j$  is a residual random disturbance term having a Gaussian distribution and an expected value of zero. Under certain simplifying assumptions (Fama 1973), it can be shown that the linear regression coefficient,  $\beta_j$ , that minimizes the variance of the residual terms,  $\varepsilon_j$ , in (10) is identical to the risk measure,  $\beta$ , that was utilized in (2). Hence, the market model provides a direct method for estimating  $\beta$ , via regression analysis, the beta coefficient,  $\beta_j$  which is required to make the CAPM useful in a practical sense.

### 2.1.1. Hypothesis Testing

A statistical analysis is based on a “null” hypothesis (labeled  $H_0$ ) that there is “no effect”. In research terms, the null hypothesis will typically be a statement such as the following: There is no difference in group means, no linear association between two variables, no difference in distributions, and so on.

An experiment is designed to determine whether evidence refutes the null hypothesis. If your evidence (research result) indicates that what you observed was extreme enough, then you would conclude that you have “significant” evidence to reject the null hypothesis. However, if you do not gather sufficient evidence to reject  $H_0$ , this does not prove that the null hypothesis is true, only that we did not have enough evidence to “prove the case.”

In general, null and alternative hypothesis are of the following form:

- A “null hypothesis” ( $H_0$ ) is the hypotheses of “no effect” or “no differences” (i.e., the observed differences are only due to chance variation).
- An alternative hypothesis ( $H_1$ ) states that the null hypothesis is false and that the observed differences are real.

When running a regression by a stats program (e.g. Excel), one purpose is to discover whether the coefficients on independent variables are really different from 0 (so the independent variables are having a genuine effect on your dependent variable) or if alternatively any apparent differences from 0 are just due to random chance. The null (default) hypothesis is always that each independent variable is having absolutely no effect (has a coefficient of 0) and you are looking for a reason to reject this theory.

### 2.1.2. Statistically significance and p-value

In statistics, a result is called statistically significant if it is unlikely to have occurred by chance. The phrase test of significance was coined by Ronald Fisher (Fisher1925). As used in statistics, significant does not mean important or meaningful, as it does in everyday speech. The amount of evidence required to accept that an event is unlikely to have arisen by chance is known as the significance level or critical p-value: in traditional Fisherian statistical hypothesis testing, the p-value is the probability of observing data at least as extreme as that observed, given that the null hypothesis is true. If the obtained p-value is small then it can be said either the null hypothesis is false or an unusual event has occurred.

The significance level is usually denoted by the Greek symbol  $\alpha$ . Popular levels of significance are 10% (0.1), 5% (0.05) and 1% (0.01). If a test of significance gives a p-value lower than the  $\alpha$ -level, the null hypothesis is thus rejected. Choosing level of significance is an arbitrary task, but for many applications, a level of 5% is chosen, for no better reason than that it is conventional (Elliott et al. 2007). A p-value representing the probability that random chance could explain the result. Therefore, a 5% or lower p-value is considered to be statistically significant.

The stats program works out the p-value for the interested statistic (e.g. a correlation). A test statistic is just another kind of effect statistic, one that is easier for statisticians and computers to handle. Common test statistics are t, F, and chi-squared. It's not ever necessary to know how these statistics are defined, or what their values are. All is necessary is the p value, or the confidence limits or interval for your effect statistic (Elliott et al. 2007).

### 2.1.3. Simple Linear Model

The simplest case of linear relationship is  $Y = \alpha + \beta X$ . This is the equation of a straight line: the parameter  $\alpha$  is called the regression constant and represents the intercept of the line with the vertical (Y) axis, and the regression coefficient  $\beta$  represents the slope of the line. Take an arbitrary sample of T observations on X and Y and denote this

$$\{(X_t, Y_t), \quad t = 1, \dots, T\} \quad (11)$$

In the following we implicitly assume the sample is a historical sample taken over some period of time, i.e. a time series. In the simple linear model we include an error term so that the points do not need to lie exactly along a line. Thus we write

$$Y_t = \alpha + \beta X_t + \varepsilon_t, \quad t = 1, \dots, T \quad (12)$$

where  $\varepsilon_t$  is called the error process. A low correlation between X and Y implies that the error process has a relatively high variance; a high correlation between X and Y implies that the error process has a relatively low variance.

If the fitted line is denoted as

$$\hat{Y} = \hat{\alpha} + \hat{\beta}X \quad (13)$$

where  $\hat{Y}$  is estimator of  $Y$ ,  $\hat{\alpha}$  and  $\hat{\beta}$  denote the estimates of the line intercept  $\alpha$  and slope  $\beta$ .

The difference between the actual value of  $Y$  and the fitted value of  $Y$  for the observation at time  $t$  is denoted  $\varepsilon_t$  and this is called the residual at time  $t$ . That is:

$$\varepsilon_t = Y_t - \hat{Y}_t \quad (14)$$

With this definition the data point for  $Y$  at time  $t$  is the fitted model value plus the residual, i.e.

$$\hat{Y} = \hat{\alpha} + \hat{\beta}X + \varepsilon_t, \quad t = 1, \dots, T \quad (15)$$

Several assumptions are involved, these include the following:

1. *Normality*. The population of  $Y$  values for each  $X$  is normally distributed.
2. *Equal variances*. The populations in Assumption 1 all have the same variance.
3. *Independence*. The dependent variables used in the computation of the regression equation are independent. This typically means that each observed  $X$ - $Y$  pair of observations must be from a separate subject or entity.

You will often see the assumptions above stated in terms of the error term  $\varepsilon$ . Simple linear regression is robust to moderate departures from these assumptions, but you should be aware of them and should examine your data to understand the nature of your data and how well these assumptions are met.

#### 2.1.4. Ordinary Least Squares

It is logical to choose a method of estimation that minimizes the residuals in some manner, because then the predicted values of the dependent variable will be as close as possible to the observed values. But choosing the estimates to minimize the sum of the residuals will not work because large positive residuals would cancel large negative residuals. The sum of the absolute residuals could be minimized, as they are in quantile regression.

However, the easiest way to obtain estimators that have simple mathematical properties is to minimize the variance of the residuals, or equivalently to minimize the sum of the squared residuals. This is the ordinary least squares optimization criterion.

The sum of the squared residuals, also called the residual sum of squares and denoted  $RSS$ , may be expressed as

$$RSS = \sum_{t=1}^T \varepsilon_t^2 = \sum_{t=1}^T (Y_t - (\alpha + \beta X_t))^2 \quad (16)$$

Hence the OLS estimators  $\hat{\alpha}$  and  $\hat{\beta}$  are found by solving the optimization problem

$$\min_{\alpha, \beta} \sum_{t=1}^T (Y_t - (\alpha + \beta X_t))^2 \quad (17)$$

This is the OLS criterion.

### 2.1.5. ANOVA and Goodness of Fit

The standard error of the regression, which is derived from the residual sum of squares, measures the goodness of fit of the regression model. A small standard error indicates a good fit, but how small is ‘small’? This depends on the total sum of squares (denoted TSS) which is given by

$$\text{TSS} = \sum_{t=1}^T (Y_t - \bar{Y})^2 \quad (18)$$

TSS measures the amount of variation in the dependent variable Y that we seek to explain by the regression model. It is directly related to the sample variance of Y, indeed

$$\text{TSS} = (T - 1) S_Y^2 \quad (19)$$

There are  $T - 1$  degrees of freedom associated with the total sum of squares. The explained sum of squares (ESS) is the amount of variation in Y that is explained by the regression. It is obtained by subtracting RSS from TSS:

$$\text{ESS} = \text{TSS} - \text{RSS} \quad (20)$$

There are  $T - 2$  degrees of freedom associated with the residual sum of squares, and the number of degrees of freedom associated with the explained sum of squares is the number of explanatory variables in the regression model, which is 1 in this case.

The decomposition of the total variance of the dependent variable into the variance explained by the model and the residual variance is called the *analysis of variance* or ANOVA for short. The results of ANOVA can be summarized succinctly in a single statistic which is called the regression  $R^2$ . This is given by

$$R^2 = \frac{\text{ESS}}{\text{TSS}} \quad (21)$$

So the regression  $R^2$  takes a value between 0 and 1 and a large value indicates a good fit for the model. The regression  $R^2$  is the square of the correlation between the fitted value.

The  $R^2$  of the regression is the fraction of the variation in the dependent variable that is accounted for (or predicted by) the independent variable.

A statistical test can be performed of the significance of the  $R^2$  from a simple linear regression model, using the F statistic (Elliott et al. 2007).

#### 2.1.6. Reporting the Estimated Regression Model

The rate of return on the Market Portfolio ( $\bar{R}_M$ ) is estimated by variations to the TSE Dividend and Price Index (TEDPIX) (Taheri... 2009b). In fact, it is the average of yearly variations of TEDPIX. The realized yearly returns of the ordinary shares of mining and cement companies – capital gains plus dividends expressed as a percentage of the opening price – are regressed against the corresponding realized returns on the TSE Dividend and Price Index. Regression analysis is carried out using Excel spreadsheet on forty listed companies. The outputs are illustrated in Table 1 to Table 4. The first part of the output is the regression statistics (Table 1). These are standard statistics which are given by most programs.

TABLE 1

Some of the Excel output for listed mining and cement companies

TABELA 1

Wybrane wyniki z arkuszy Excela dla wyselekcjonowanych przedsiębiorstw górniczych i cementowych

Regression Statistics	Mining Companies	Cement Companies
Multiple R	0.35	0.47
R Square	0.12	0.22
Adjusted R Square	0.12	0.22
Standard Error	0.37	0.36
Observations	249	486

The ANOVA table (Table 2) comes next. The  $R^2$  is calculated using Equation (21). The  $R^2$  measures the proportion of the variability of a dependent variable that is explained by an independent variable or variables. Adjusted  $R^2$  used if there's more than one independent (x) variable. According to This statistic suggests that 12% of the risk (variance) in mining companies comes from market sources (interest rate risk, inflation risk etc.), and that the balance of 88% of the risk comes from firm-specific components. The latter risk should be diversifiable, and therefore unrewarded. Mining companies  $R^2$  is slightly lower than the median  $R^2$  of companies listed on the New York Stock Exchange, which was approximately 21% in 2003 (Damodaran 2004). For cement companies this statistic is slightly higher.

The  $R^2$  is generally of secondary importance, unless the main concern is using the regression equation to make accurate predictions. The p-value (or F significance) gives the

TABLE 2

ANOVA for listed mining and cement companies

TABELA 2

Analiza wariancji (ANOVA) dla wybranych przedsiębiorstw górniczych i cementowych

	Mining companies		Cement companies	
	Degrees of freedom	Sum of squares	Degrees of freedom	Sum of squares
Regression(ESS)	1	1.14	1	6.69
Residual(RSS)	247	8.20	484	23.21
Total(TSS)	248	9.33	485	29.90
F	34.2		139.38	
Significance F	0.0051		9.3E-12	

TABLE 3

Coefficient estimates for mining companies

TABELA 3

Szacunkowe współczynniki dla przedsiębiorstw górniczych

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.205	0.05	4.12	0.0001
$\beta$	0.178	0.06	2.90	0.0051

TABLE 4

Coefficient estimates for cement companies

TABELA 4

Szacunkowe współczynniki dla przedsiębiorstw cementowych

	Coefficients	Standard Error	t Stat	P-value
Intercept	0.251	0.03	9.54	8.95E-12
$\beta$	0.237	0.03	7.28	9.33E-12

degree of correlation between each individual variable and the dependent variable, which is the important thing. In this case the value of the F significance (0.0051 and 9.3E-12) indicates that the null hypothesis should be rejected and concludes that so the independent variable has a genuine effect on the dependent variable and there is a statistically significant linear relationship between the two variables.

Excel compares the t statistic on the variable with values in the *Student's t distribution* to determine the p-value, which is the number that really needs to be looking at. The Student's t



distribution describes how the mean of a sample with a certain number of observations ( $n$ ) is expected to behave.

If 95% of the  $t$  distribution is closer to the mean than the  $t$ -value on the coefficient, then the  $p$ -value will be 5%. This is also referred to a significance level of 5%. The  $p$ -value is the probability of seeing a result as extreme as the one in a collection of random data in which the variable had no effect. A  $p$  of 5% or less is the generally accepted point at which to reject the null hypothesis. With a  $p$ -value of 5% (or 0.05) there is only a 5% chance that results you are seeing would have come up in a random distribution, so with a 95% probability of being correct that the variable is having some effect, assuming the model is specified correctly.

According to Table 3 and Table 4, the coefficient of  $R_m$ , i.e.  $\beta$  is tested at significance level  $\alpha = 0.05$ . ( $H_0: \beta = 0$  against  $H_1: \beta \neq 0$ ). Since the  $p$ -values are 0.0051 and 9.33E-12 respectively, so the null hypothesis is thus rejected  $\beta$  is therefore statistically significant at significance level  $\alpha = 0.05$  as  $p < 0.05$

The coefficient of intercept is tested at significance level  $\alpha = 0.05$ . Since the  $p$ -values are 0.0051 and 9.33E-12 respectively, the null hypothesis is thus rejected,  $\beta$  is therefore statistically significant at significance level  $\alpha = 0.05$  as  $p < 0.05$  and therefore, the realized rate of return on security  $j$  or cost of equity for mining and cement companies listed in TSE can be estimated using the following equations.

For mining companies:

$$R_j = 20.5\% + R_m (17.8\%) \quad (22)$$

For cement companies:

$$R_j = 25.1\% + R_m (23.7\%) \quad (23)$$

According to the forecast of TEDPIX for 2011 the  $R_M$  is estimated 25% so the cost of equity for mining and cement companies is estimated 25.0% and 31.0% respectively.

### Conclusions

The studies on the texts on mineral valuation or mineral project evaluation demonstrate that the CAPM is a general model for estimating the cost of equity. In spite of shortfalls and problems relating to this model, no alternative method is proposed. To estimate the cost of equity of the mining and cement industry of Tehran Stock Exchange, the alternative method, i.e. the single index model is applied because of the limitations and shortfalls of the CAPM as well as the lack of commercial services for determining the market premium. The regression analysis as well as the statistical analysis using  $F$  and  $t$  statistics is carried out by Excel spreadsheet. The results showed that  $p$ -value is lower than 0.05 in all the tests, so the null hypothesis is rejected and conclude that the parameters are jointly statistically significant

at significance level of 0.05. Therefore, the independent variable (the rate of return on the market index) has a genuine effect on the dependent variable (the rate of return for the stock) and there is a statistically significant linear relationship between the two variables at significance level of 5%. Finally, the cost of equity for mining and cement companies is estimated 25.0% and 31.0% respectively.

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**SZACOWANIE KOSZTU KAPITAŁU WŁASNEGO DLA ZAKŁADÓW GÓRNICZYCH I CEMENTOWNI  
W JEDNOWSKAŹNIKOWYM MODELU RYNKU**

**Słowa kluczowe**

Koszt kapitału własnego, koszt kapitału, jednowskaźnikowy model rynku, model wyceny aktywów kapitałowych (CAPM), TSE

**Streszczenie**

Koszt kapitału przedsiębiorstwa jest używany przez taksatorów przy wyznaczaniu przyszłych zdyskontowanych przepływów pieniężnych podmiotów w celu uzyskania przyszłej ich wartości. Koszt kapitału jest zatem określany przez średni ważony koszt różnych źródeł finansowania, którymi są zwykle akcje, papiery dłużne i instrumenty preferencyjne. Szczególnie trudnym i ważnym elementem jest szacowanie kosztów akcji, które zazwyczaj wymaga zastosowania modeli finansowych. Badania materiałów dotyczących wyceny minerałów i oceny ekonomicznej projektów inwestycyjnych związanych z surowcami mineralnymi pokazują, że model wyceny aktywów kapitałowych (CAPM) jest ogólnym modelem używanym dla szacowania kosztu kapitału własnego. Jednakże, w związku z brakami i problemami z nim związanymi, proponuje się zastosowanie stosunkowo podobnego i prostszego modelu, tj. jednowskaźnikowego modelu rynku. Jednowskaźnikowy model rynku stanowi ważne narzędzie współczesnych badań w zakresie finansów. Główną zaletą tego modelu stanowi parametr „beta”, który w najlepszy możliwy sposób mierzy wrażliwości stopy zwrotu z papierów wartościowych na zmiany zachodzące na rynku. W celu oceny kosztu wkładu własnego dla przedsiębiorstw cementowych

i górniczych znajdujących się na Teherańskiej giełdzie papierów wartościowych (TSE) wybrano jednowskaźnikowy model rynku, uwzględniając niedostatki i problemy związane z modelem wyceny aktywów kapitałowych, a także niedobór usług określających rynkową premię ryzyka. Analiza regresji, a także analiza statystyczna, zostały przeprowadzone z użyciem arkusza kalkulacyjnego MS Excel. Istotność statystyczna modelu została zbadana przy użyciu testów statystycznych t i F. Wyniki wykazały, że zmienna niezależna (stopa zwrotu z indeksu rynkowego) ma wpływ na zmienną zależną (stopa zwrotu z giełdy), w związku z czym istnieje istotna statystycznie liniowa zależność między dwoma zmiennymi na poziomie istotności 5%. Koszt wkładu własnego dla potrzeb przedsiębiorstw górniczych i cementowych jest szacowany pomiędzy 25,0% i 31,0%. Przy znajomości kosztu wkładu własnego, obliczenie wysokości stopy dyskonta nie powinno stanowić problemu.

**ESTIMATION OF THE COST OF EQUITY FOR MINING  
AND CEMENT INDUSTRIES BY SINGLE-INDEX MARKET MODEL**

Key words

Cost of equity, Cost of capital, Single-index market model, CAPM, TSE

Abstract

The corporate cost of capital is used by valuers to discount future flows of income from an entity in order to derive a present-day, forward-looking value of that entity. The cost of capital is therefore determined as the weighted cost of the various sources of funding, being typically equity, debt and preference instruments. The tricky and important part is estimating the cost of equity, which usually needs the application of finance models. The study on the texts on mineral valuation or mineral project evaluation demonstrates that the capital asset pricing model (CAPM) is a general model for estimating the cost of equity. However, according to shortfalls and problems relating to it a relatively similar and simpler model i.e. the single-index market model is proposed. The single index market model is an important tool in contemporary research in finance. Much of the importance of the model follows from its "beta" parameter which, ideally, measures the sensitivity of returns on a security to changes in a market model. To estimate the cost of equity of the mining and cement companies listed in Tehran Stock Exchange (TSE) The single-index market model is selected because of the shortfalls and problems of the CAPM as well as the lack of commercial services for determining the market premium. The regression analysis as well as the statistical analysis is carried out using Excel spreadsheet. The statistic significance of the model is tested using t and F test statistics. The results showed that the independent variable (the rate of return on the market index) has a genuine effect on the dependent variable (the rate of return for the stock) and there is a statistically significant linear relationship between the two variables at significance level of 5%. Finally, the cost of equity for mining and cement companies is estimated 25.0% and 31.0% respectively. Knowing the cost of equity, calculating the discount rate will not be very difficult.