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Chaotic Vibrations of an Object Supported on a Layer of Fibres

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Abstract
The dynamic properties of a mathematical model of a layer of fibres are studied in this paper. It is shown that the layer significantly restricts the amplitude of vibration of a supported mass, which is subject to a compressive oscillating force. For certain frequencies of sufficiently large oscillatory force a chaotic motion may take place.

Key words: *nonlinear vibration, chaos, layer of fibres, fibres compression.*

Introduction

Studies of the behaviour of textiles under compression can be found in works [1 - 3]. A mathematical model of a layer of fibres submerged in a fluid was formulated in paper [4]. The properties of the layer were assumed to be determined by the bending elasticity of fibres and by the resistance to the fluid flow that is squeezed out of the layer. The author showed that filling the layer with a fluid considerably decreases its reaction to an impact force. In this paper, vibrations of a system containing a layer of fibres used as a vibration insulator are studied. The object usually used for this purpose – a helical spring behaves linearly in some range, but when the coils come into mutual contact the discontinuity of this behaviour takes place and an impact force occurs. The purpose of this paper is to introduce the material characteristic which does not have that feature and smoothly restricts the motion.

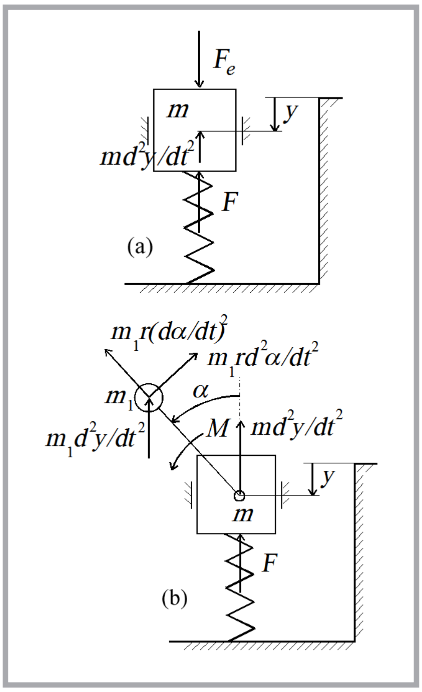

***Figure 1.** Vibrating mass m lying on a layer of fibres (a) subject to an excitation force F_e, (b) subject to excitation by the inertia forces of the rotating mass m_1.*

Equations of motion

The system considered is shown in ***Figure 1***. The mass m lies on the layer of fibres and is subject to a time varying force. Under the action of a compressive force, the fibres gradually lock, when they come into mutual contact [1], giving the reaction force F_r (1). In formula (1) k, c, L, H are material constants, introduced and explained in paper [4]. The position of mass m is measured from the layer of an uncompressed height downward and is denoted by y. In ***Figure 1.b*** mass m_1 is connected to mass m with a link of length r and rotates with respect to mass m, the angle of rotation is denoted by α. The vibrations are excited either by the explicitly given time periodical force F_e (***Figure 1.a***) of known angular frequency ω (2) or the inertia forces (***Figure 1.b***) of a rotating mass m_1. Mass m_1 is driven by the torque M (3) of a motor of known idle angular velocity Ω. In formula (3) T is the motor time constant and C denotes the stiffness of the motor characteristic. The actual angular velocity dα/dt of the motor is a result of the mutual interaction of system elements.

$$F_r = \frac{ky}{\left(1-\frac{y}{L}\right)^3} + \frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}{\left(1-\frac{y}{H}\right)^3}, \qquad y<H,\ \ y<L, \qquad F=F_r \ \ for\ F_r \ge 0, \qquad F=0 \ \ for\ F_r<0. \tag{1}$$

$$F_e = mg + F_1(t),\ \ F_1 = F_0(1-\cos\omega t). \tag{2}$$

$$T\frac{\mathrm{d}M}{\mathrm{d}t} = C\left(\Omega - \frac{\mathrm{d}\alpha}{\mathrm{d}t}\right) - M \tag{3}$$

By requiring that the resultant of all forces (***Figure 1.a***) acting on mass m be equal to zero, one gets the equation of motion (4).

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + F - F_e = 0 \tag{4}$$

Formula (1) into equation (4) gives ordinary nonlinear differential equations (5).

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \frac{ky}{m\left(1-\frac{y}{L}\right)^3} + \frac{c\,\mathrm{sgn}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}{m\left(1-\frac{y}{H}\right)^3} = g + \frac{F_1(t)}{m},\ \ F_r \ge 0, \qquad \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = g + \frac{F_1(t)}{m},\ \ F_r < 0. \tag{5}$$

By requiring that the resultant of all forces (***Figure 2.b***) acting on the system of masses m and m_1 be equal to zero, we get equation (6).

$$(m+m_1)\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - (m+m_1)g + m_1r\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2\cos\alpha + m_1r\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}\sin\alpha + F = 0 \tag{6}$$

The sum of moments of all forces with respect to the centre of rotation gives equation (7).

$$m_1r^2\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2} + m_1\frac{\mathrm{d}^2y}{\mathrm{d}t^2}r\sin\alpha - m_1gr\sin\alpha - M = 0 \tag{7}$$

Substituting equation (7) into equation (6) gives equation (8).

$$\left(m+m_1-m_1\sin^2\alpha\right)\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - (m+m_1)g + m_1r\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2\cos\alpha + m_1g\sin^2\alpha + \frac{M}{r}\sin\alpha + F = 0 \tag{8}$$

$$\left(m+m_1-m_1\sin^2\alpha\right)\left(\frac{d^2y}{dt^2}\right)+m_1r\left(\frac{d\alpha}{dt}\right)^2\cos\alpha+m_1g\sin^2\alpha+\frac{M}{r}\sin\alpha$$

$$+\frac{ky}{\left(1-\frac{y}{L}\right)^3}+\frac{c\,\mathrm{sgn}\left(\frac{dy}{dt}\right)\left(\frac{dy}{dt}\right)^2}{\left(1-\frac{y}{H}\right)^3}=\left(m+m_1\right)g,\ \ for\ F_r\geq 0$$

$$\left(m+m_1-m_1\sin^2\alpha\right)\left(\frac{d^2y}{dt^2}\right)+m_1r\left(\frac{d\alpha}{dt}\right)^2\cos\alpha+m_1g\sin^2\alpha+\frac{M}{r}\sin\alpha=$$

$$=\left(m+m_1\right)g,\ \ for\ \ F_r<0$$

Equation 9.

Substituting formula (1) into equation (8) yields equation (9).

Numerical results and discussion

The differential equations were solved using the Runge-Kutta method. The calculations were performed by changing the excitation frequency from a zero value upwards and then back downwards by letting the motion achieve steady-state vibrations at each value of the frequency. The maximum amplitudes of vibrations, described by equation (5) for cosine excitation (2), are shown in ***Figure 2.a***. The maximum amplitudes of vibrations excited by the centrifugal force of a motor which drove a rotating mass, as described by equations (3, 7, 9), are shown in ***Figure 2.b***. The following values of parameters were taken: stiffness *k,* defined by $\omega_0 = (k/m)^{0.5}$ = 250/30 rad/s, damping coefficient c/m = 500/80 m^{-1}, material constants L=0.02m, H=0.02m, mass m = 80 kg, force $F_0/(mg)$ = (1/10, 4, 100) and for a rotating mass m_1 = 1 and 20 kg, $\omega_0 = (k/(m+m_1))^{0.5}$ = 250π/30 rad/s, rotating rod length r = 0.01 m, motor constants T = 0.1 s, C = 0.1 Nms.

In ***Figure 2.a***, one may see that an increase in the compressive excitation force by thousand times (100/0.1) resulted only in an approximately three times increase in the mass displacement, which is a result of the increasing stiffness of the layer (1) with an increase in the magnitude of compression y. A similar behaviour can be observed in ***Figure 2.b*** for mass m_1 = 1 kg. For mass m_1 = 20 kg the behaviour of the system significantly differs from that of the other parameters.

In order to explain this difference, the mass motion simulation obtained from the set of equations (3, 7, 9) for m_1 = 20 kg is shown in ***Figure 3*** for Ω/ω_0 = 4.5 and in **Figure 4** for Ω/ω_0 = 7.5. It can be seen in ***Figure 3*** that for Ω/ω_0 = 4.5 the motion is chaotic, which is a result of the loss of contact between the mass and the layer for y < 0, when the contact force F = 0. When the mass is above the layer, without being supported by, it moves freely up and falls back onto the layer, not in accordance with the frequency of oscillation, resulting in chaotic motion [5, 6]. During the contact between the mass and layer, the reaction force F becomes larger than the centrifugal force $F_n = m_1\Omega^2 r$. For Ω/ω_0 = 7.5, as shown in ***Figure 4***, the motion is periodic and the reaction force F is smaller than the centrifugal force $F_n = m_1\Omega^2 r$.

Conclusions

1. The increasing stiffness of the layer resulting from fibres locking restricts the maximum vibration amplitude.
2. For a certain range of excitation frequency, chaotic motion can occur.
3. Over the range of frequency for which chaotic motion takes place, the vibrations can become periodic of limited amplitude and limited reaction force.
4. The material characteristic studied in this paper is free from the locking impact, as opposed to a helical spring, and it can find application for the design of vibration insulators that restrict the amplitude of vibration.

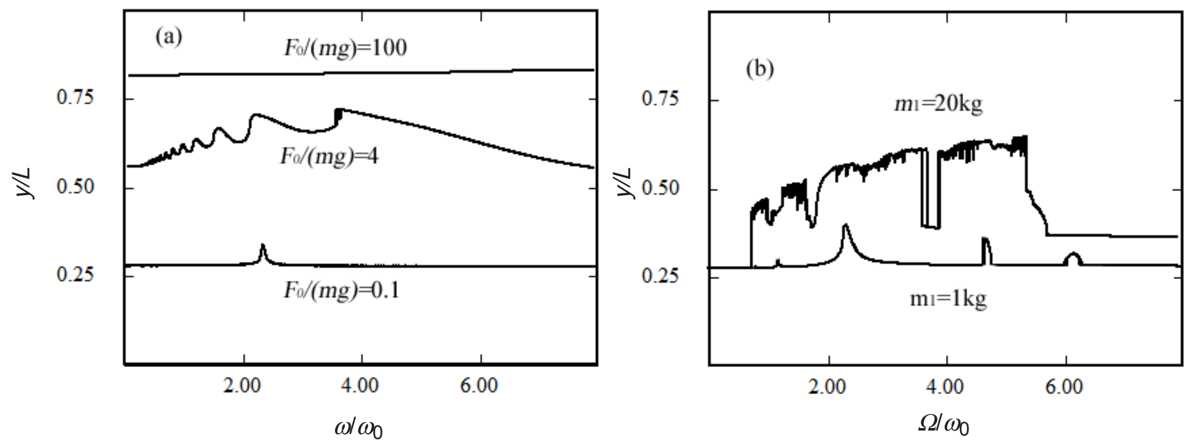

Figure 2. *Maximum steady-state amplitude y of vibrations: (a) versus angular frequency of excitation ω for various magnitudes of the exciting force* F_0*; (b) versus idle motor angular velocity Ω for a rotating mass* m_1 *= 1 and 20 kg.*

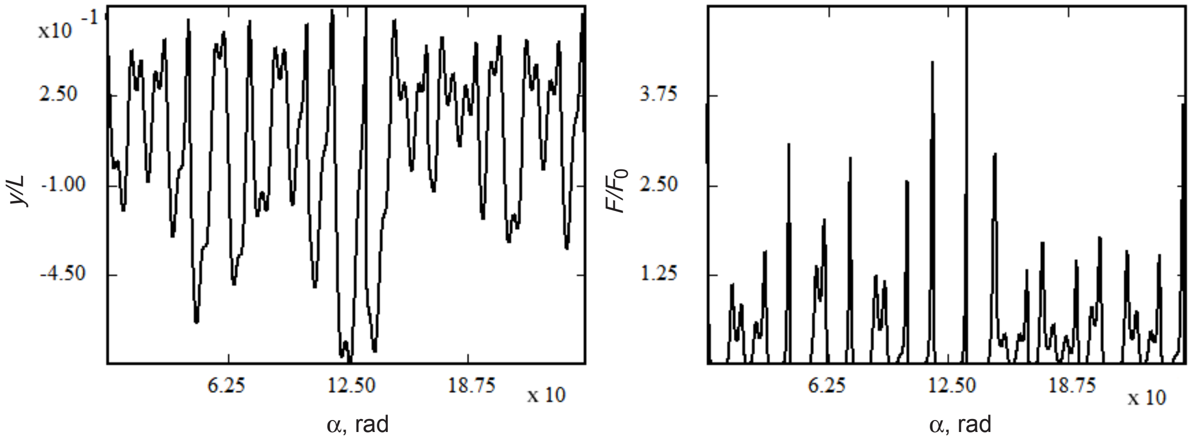

Figure 3. *Chaotic motion – mass displacement y and reaction force versus the angle of revolution α after one million revolutions, for mass* m_1 *= 20 kg and motor idle velocity* $\Omega/\omega_0 = 4.5$.

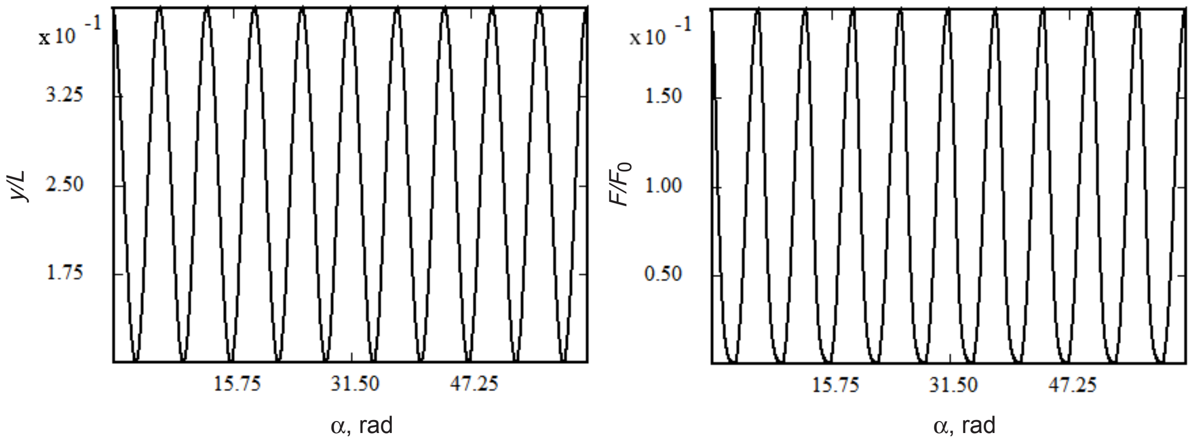

Figure 4. *Periodic motion – mass displacement y and reaction force versus the angle of revolution α after one million revolutions, for mass* m_1 *= 20 kg and motor idle velocity* $\Omega/\omega_0 = 7.5$.

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