An Optimization Model for the Determination of Qualitative Capacity Requirements as Part of a Four Level Approach for the Planning of Production Networks

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Planning and design of companywide production networks are the challenges for many manufacturers in a globalized world. The overall planning task to be tackled is the optimal dimensioning of the network consisting of production sites and their economic interrelations. This complex task is divided into four sub-tasks according to the time frame of the decisions, scope of consideration and the decision hierarchy of the company. Mathematical optimization models are used to represent and solve the individual sub-problems. The model of the task on the top level is presented in detail. Coordination processes are defined to link the individual levels to a hierarchical overall process. This process is implemented into a prototype planning system and evaluated using case studies.

1. INTRODUCTION

This paper presents an optimization model for the determination of qualitative capacity demands as part of an approach for the planning of companywide production networks. This approach deals with the dimensioning and the configuration of the individual production sites with a given network structure. Dimensioning in general is the definition of all restrictions according to capacity and throughput and requires the consideration of time (cf. [1, 6]). In the context of the planning of production networks this is the determination of the performance of the individual sites and actual flows inside of the network. The performance is determined under consideration of qualitative and quantitative capacity aspects. These aspects are specified in detail by the determination of required machinery and workforce.

2. PROBLEM STATEMENT

A production network consists of the production sites of a company that are connected by logistic interrelations. It is characterized by the distribution of production processes among several production sites leading to complex logistic interdependencies and an increased need of coordination (cf. [4, 15, 18]). Furthermore it cannot be regarded as fixed and unchangeable, because adjustments are required due to changes in surrounding conditions. Since initiated adjustments take effect in later periods only, it is not sufficient to react on changes in surrounding conditions when they occur. Therefore it is necessary to initiate adequate adjustments in advance to assure optimal production in a well configured network. Finding appropriate adjustment measures is to be carried out by planning. Planning can be defined as the anticipation of future events by systematic preparation of decisions and decision making (cf. [5, 12]). The concept that documents the results of this process is called plan (cf. [1]).

Systematic preparation of decisions requires a formal definition of the planning task. It can be defined by its prerequisites, objectives and designated results. One of the prerequisites of the overall planning task is the current state of the production network. It is the starting point for the planning process and is given by the network structure consisting of production sites and their possible relationships. Further prerequisites are the future demands for the products and the surrounding conditions. The factual objective is the determination of an adequate dimensioning for the production network. Formal objectives are the requirements for the solution quality of the determined plan and the time needed to find the solution. The result is a plan that fulfills the factual and formal objectives and consists of determined qualitative and quantitative capacity requirements as well as required machinery and workforce. The planning method defines the decision making process. It uses a model of the production network that is supposed to be a formal representation of all relevant aspects of the production network according to the planning task. It includes a formal definition of the rules and parameters used by the planning method to find and evaluate alternatives.

Regarding the structure of the overall planning task four planning levels can be identified:

- Level 1: Determination of qualitative capacity requirements: The first subtask is the determination of qualitative capacity requirements, which determines the long-term performance of the production sites and the entire production network. This is done by assigning production processes to sites, which defines the products that can be produced at each production site.
- Level 2: Determination of quantitative capacity requirements: At this level the task is to find a good or optimal distribution of the production to the locations based on the quantitative demand. Additionally, quantitative determination of flows in the network takes place.
- Level 3: Conversion of quantitative capacity requirements into machinery: On level three quantitative capacity requirements are converted into machinery. With the objective of cost efficient fulfillment of demands decisions are made on the optimal development of machine assets and on the outsourcing of individual products.
- Level 4: Conversion of quantitative capacity requirements and machinery into workforce: On the last level of the hierarchy quantitative capacity requirements and machinery are converted into workforce. Therefore decisions on the employment and on the qualification of personnel have to be made.

The task on top level of the hierarchy is the determination of qualitative capacity requirements, which determines the long-term performance of the production sites and the entire production network. This is done by assigning production processes to sites, which defines the products that can be produced at each production site. They build the framework for the decisions to be made on the second level, the determination of quantitative capacity requirements. At this level the task is to find a good or optimal distribution of the production to the locations based on the quantitative demand. Additionally, quantitative determination of flows in the network takes place. Based on these levels three and four are planned. On level three they are converted into machinery. With the objective of cost efficient fulfillment of demands decisions are made on the optimal development of machine assets and on the outsourcing of individual products. Machine capacities planned at this level and quantitative capacity requirements from level 2 are converted into workforce on level four. Therefore decisions on the employment and on the qualification of personnel have to be made. In addition to this top-down interaction the influence of subordinate levels on superordinate levels can be enabled by the anticipation of lower levels during the planning of higher levels. On the other hand it is possible to implement feedback loops.

3. STATE OF THE ART

3.1. SOLUTION METHODS FOR DIMENSION PLANNING

Qualitative capacity aspects are usually considered as a part of an extensive method. A model presented by Bundschuh (cf. [3]) includes qualitative aspects by the allocation of equipment to production sites. The allocated equipment defines the ability of a production site to produce a certain set of products. Kriesel (cf. [8, 16]) presents a method for strategic location and production planning. His planning system allows decisions on productions sites, the allocation of production processes and products to production sites and the dimension of resources.

Quantitative capacity aspects can be found in different models for the optimization of production networks. A model presented by Henrich (cf. [13], [10]) allows the allocation of end products and quantities to production sites under consideration of capacity limits. Ferber (cf. [9], [10]) extends this model to allow the planning of capacities using capacity stages. Bihlmaier et al. (cf. [2, 14]) develop a two stage stochastic optimization model for strategic and tactical production planning (cf. [2, 11]). The first stage focuses on strategic decisions like the allocation of products and quantities to production sites while tactical decisions on workforce are anticipated by linear approximation.

Planning of machinery is the main aspect of the first model of a hierarchical planning method developed by Timm (cf. [23]). It allows deciding on machine assets and on outsourcing of products. Bundschuh (cf. [3]) also presents a detailed version of the model for the planning of equipment that allows deciding on the extension and reduction of equipment.

The second model of the method developed by Timm (cf. [23]) allows deciding on workforce based on the results of the first model (cf. [23]). Decisions on workforce are also included in the model presented by Bundschuh (cf. [3]). It allows deciding on primary personnel directly involved in the production process and on secondary and overhead personnel.

3.2.HIERARCHICAL PLANNING

Hierarchical planning in general is characterized by the decomposition of a complex planning task into a set of less complex tasks with clearly defined hierarchical relations. According to Steven hierarchy, decomposition and aggregation can be identified as the elements of hierarchical planning (cf. [22]). Hierarchy is the division of an extensive task into vertically arranged planning levels. Among these levels there are well-defined relations of super- and sub-ordination. The upper level is allowed to pass instructions to the lower level. Decomposition is the division of a complex planning task, which cannot be solved as a whole, into less complex interdependent sub-tasks. Aggregation of input and output data into groups is used to reduce the amount of data, the size of the model and the uncertainties of the data. It can also be used to influence the type of decisions. Decomposition and hierarchy have to be performed under consideration of the decision hierarchy of the company to allow better acceptance of planning results (cf. [7, 17, 21]).

Schneeweiss [20] introduces a general characterization of hierarchical planning structures. A hierarchical planning system consists of “different kinds of subsystems having particular interrelations and outputs” [20]. One main characteristic of this system is the anticipation of the base-level by the top-level. According to this anticipation an optimal instruction is communicated to the base-level. Taking this instruction into account the base-level derives an optimal reaction that is passed to the top-level creating a feedback loop. This cycle is repeated until it results in a final decision.

4. CONCEPT

4.1. MODELS FOR THE INDIVIDUAL LEVELS

Mathematical optimization models (cf. [19]) have been defined for the individual planning levels. The presentation of all models is out of scope of this paper, so only the model for the determination of qualitative capacity requirements and the objective functions of the other models are presented here. Table 1 presents a list of sets used in the models. The production sites of the production network to be planned are represented by the set of sites S. The Set of products E represents all end-products, components and parts that are produced. The set of technologies T represents the production processes in the production network with products being the input and the output of a technology. Technologies with similar production processes are grouped to a technology group from the set of technology groups G.

Table 1: List of sets

Symbol	Definition
$P = \{p_0, \dots, p_n\}$	Periods (period p_0 defines the initial state)
S	Production sites
L	Countries
E	Products
T	Technologies
G	Technology groups
$K = \{k_0, \dots, k_n\}$	Capacity stages
W	Machines
M	Employees

The time structure and the granularity of the models are defined by the set of periods P with period p_0 being the initial state. Since the individual levels require different time structures and granularities the set P is valid in the context of the particular model only. The sets K, W and M represent the capacities to be planned at the different levels. Table 2 presents a list of all cost parameters of all models. Some parameters are relevant for several models; others are used for a single model only. Table 3 presents a list of the decision variables of all models

Table 2: Cost parameters

Symbol	Definition
c_{tsp}^{T}	costs for technology t at site s in period p
c_{tsp}^{T+}	costs for adding technology t to site s in period p
c_{tsp}^{T-}	costs for removing technology t from site s in period p
c_{gsp}^{TG}	costs for technology group g at site s in period p
c_{gsp}^{TG+}	costs for adding technology group g to site s in period p
c_{gsp}^{TG-}	costs for removing technology group g from site s in period p
$c_{ess'p}^{LGfix}$	fixed costs for transportation of product e from site s to site s' in period p
c_{es}^{Pfix}	fixed costs for production of product e at site s
c_{es}^{Pvar}	variable costs for production of product e at site s
c_{ktsp}^{KS}	costs of capacity stage k of technology t at site s in period p
c_{kgsp}^{KSG}	costs of capacity stage k of technology group g at site s in period p
$c_{kk'ts}^{KA}$	costs for change from capacity stage k to stage k' of technology t at site s
$c_{kk'gs}^{KA}$	costs for change from capacity stage k to stage k' of technology group g at site s
$c_{ess'p}^{LG}$	costs for transportation of one unit of product e from site s to site s' in period p
c_{es}^{V}	penalty cost per unit of product e at site s for demands that exceeds available capacity
c_{w}^{F}	fixed costs per period for the availability of machine w
c_{e}^{K}	costs for purchasing one unit of product e
c_{w}^{N}	costs for acquirement of machine w
c_{w}^{P}	costs for production for one time unit using machine w
c_{w}^{R}	setup costs for one time unit for machine w
c_{m}^{M}	wage of employee m per period
c^{ME}	costs for hiring one employee
c^{MR}	costs for releasing one employee
c_{q}^{Q}	costs for the training of an employee to acquire qualification q
c_{m}^{U}	costs for overtime for one time unit of employee m

Table 3: Decision variables

Symbol	Definition
b_{tsp}^{T}	1 if technology t is assigned to site s in period p, else 0
b_{tsp}^{T+}	1 if technology t is added to site s in period p, else 0
b_{tsp}^{T-}	1 if technology t is removed from site s in period p, else 0
b_{gsp}^{TG}	1 if technology group g is assigned to site s in period p, else 0
b_{gsp}^{TG+}	1 if technology group g is added to site s in period p, else 0
b_{gsp}^{TG-}	1 if technology group g is removed from site s in period p, else 0
$b_{ess'p}^{LG}$	1 if transportation of product e from site s to site s' occurs in period p
b_{eps}^{EST}	1, if product e is produced at site s in period p, else 0
$b_{ess'p}^{LG}$	1, if product e is transported from site s to site s' in period p, else 0
b_{ktsp}^{KS}	1, if capacity stage k of technology t is selected at site s in period p, else 0
b_{kgsp}^{KSG}	1, if capacity stage k of technology group g is selected at site s in period p, else 0
$b_{kk'tsp}^{KA}$	1, if capacity is changed from stage k to stage k' for technology t at site s in period p, else 0
$b_{kk'gsp}^{KA}$	1, if capacity is changed from stage k to stage k' for technology group g at site s in period p, else 0
$x_{ess'p}^{LG}$	amount of product e transported from site s to site s' in period p
x_{esp}^{V}	amount of product e at site s in period p that exceeds available capacity
b_{wp}^{W}	1, if machine w is available, else 0
b_{wp}^{WN}	1, if machine w is acquired in period p, else 0
x_{ep}^{K}	amount of product e purchased in period p
x_{tp}^{T}	amount of technology t executed in period p
b_{twp}^{RT}	1, if machine w is set up for the execution of technology t in period p, else 0
b_{mp}^{M}	1, if employee m is employed in period p, else 0
b_{mp}^{ME}	1, if employee m is hired in period p, else 0
b_{mp}^{MR}	1, if employee m is released in period p, else 0
b_{mqp}^{MQN}	1, if employee m acquires qualification q in period p, else 0
x_{m}^{U}	overtime of employee m in time units

Table 4: Miscellaneous parameters

Symbol	Definition
b_{tsp}^{TZ}	1 if technology t is allowed to be assigned to site s in period p, else 0
b_{ts}^{T0}	1 if technology t is initially assigned to site s, else 0

b^{TGZ}_{gsp}	1 if technology group g is allowed to be assigned to site s in period p, else 0
b^{TG0}_{gs}	1 if technology group g is initially assigned to site s, else 0
$z_{ee'}$	direct consumption of product e for the production of product e'
n^{B}_{ep}	primary demand for product e in period p
n^{SB}_{ep}	secondary demand for product e in period p $n^{SB}_{ep} = \sum_{e' \in E \setminus e} z_{ee'} \cdot \left(n^{B}_{e'(p+v_e)} + n^{SB}_{e'(p+v_e)}\right)$
n^{BL}_{epl}	primary demand for product e in period p being delivered to country l
n^{SBL}_{epl}	secondary demand for product e in period p being delivered to country l $n^{SBL}_{epl} = \sum_{e' \in E \setminus e} z_{ee'} \cdot \left(n^{BL}_{e'(p+v_e)l} + n^{SBL}_{e'(p+v_e)l}\right)$
n^{TE}_{et}	output of product e from one unit of technology t
n^{TB}_{et}	input of product e for one unit of technology t
$n^{T_{max}}_{tsp}$	maximum number of units of technology t at site s in period p
$n^{TGE_{max}}_{gsp}$	maximum output of products by technology group g at site s in period p
$n^{TA_{max}}$	maximum number of changes allowed for the assignment of technologies
$n^{TAP_{max}}$	maximum number of changes allowed for the assignment of technologies per period
$n^{TAST_{max}}$	maximum number of changes allowed for the assignment of technologies per site
$n^{TGA_{max}}$	maximum number of changes allowed for the assignment of technology groups
$n^{TGAP_{max}}$	maximum number of changes allowed for the assignment of technology groups per period
$n^{TGAST_{max}}$	maximum number of changes allowed for the assignment of technology groups per site
v_e	leadtime of product e in periods
y_l	localization rate of country l
$\vartheta: T \to G$	mapping of technologies to technology groups
$\rho: S \to L$	mapping of sites to countries

Model for the determination of qualitative capacity requirements

The objective of the determination of qualitative capacity requirements is to minimize the costs for the assignment of production processes to site, costs for changes in the assignment and logistic costs. Therefore the objective function (1) sums up the costs for technologies assigned, changes in the assignment of technologies, costs for technology groups assigned, changes in the assignment of technology groups and costs for transportation caused by the assignment. Additionally costs for the amount of any product that exceeds available capacities are added.

$$\min \mathrm{z}_1 = \sum_{p \in P} \left(\sum_{s \in S} \left(\sum_{t \in T} \left(b^{T}_{tsp} \cdot c^{T}_{tsp} + b^{T^{+}}_{tsp} \cdot c^{T^{+}}_{tsp} + b^{T^{-}}_{tsp} \cdot c^{T^{-}}_{tsp} \right) + \sum_{g \in G} \left(b^{TG}_{gsp} \cdot c^{TG}_{gsp} + b^{TG^{+}}_{gsp} \cdot c^{TG^{+}}_{gsp} + b^{TG^{-}}_{gsp} \cdot c^{TG^{-}}_{gsp} \right) + \sum_{e \in E} \sum_{s' \in S} b^{LG}_{ess'p} \cdot c^{LG_{fix}}_{ess'p} \right) + \sum_{e \in E} x^{V}_{ep} \cdot c^{V}_{e} \right) \tag{1}$$

The determination of qualitative capacity requirements has to allow the fulfillment of all demands (primary demands n^{B}_{ep} and secondary demands n^{SB}_{ep}) with regard to quantitative capacity limits for technologies ($n^{T_{max}}_{tsp}$) and technology groups ($n^{TGE_{max}}_{gsp}$) respectively. This is enforced by constraints 2 and 3 while constraint 4 realizes the consistency of the assignment of technologies and technology groups.

$$n^{B}_{ep} + n^{SB}_{ep} - x^{V}_{ep} \leq \sum_{q=1}^{n^{T}} \sum_{l=1}^{n^{ST}} n^{TE}_{et} \cdot n^{T_{\max}}_{tsp} \cdot b^{T}_{tsp} \qquad \forall e \in E \quad \forall p \in P \setminus p_0 \tag{2}$$

$$\sum_{e \in \left\{x \mid \sum_{t \in \{y \mid \vartheta(y)=g\}} n^{TE}_{xt} \neq 0\right\}} \left(n^{B}_{ep} + n^{SB}_{ep} - x^{V}_{ep}\right) \leq \sum_{l=1}^{ST} n^{TGE_{max}}_{gsp} \cdot b^{TG}_{gsp} \qquad \forall g \in G \quad \forall p \in P \setminus p_0 \tag{3}$$

$$b^{T}_{tsp} \leq b^{TG}_{gsp} \qquad \forall t \in T \quad \forall s \in S \quad \forall p \in P \setminus p_0 \quad g = \vartheta(t) \tag{4}$$

Constraint 5 is used to ensure required quotas for local production.

$$y_l \cdot \sum_{e \in E} \left(n^{BL}_{epl} + n^{SBL}_{epl}\right) \cdot c^{P_{var}}_{e} \leq \sum_{t \in T} \sum_{s \in \{x \mid \rho(x)=l\}} \sum_{e \in E} n^{TE}_{et} \cdot n^{T_{max}}_{etp} \cdot b^{T}_{tsp} \cdot c^{P_{var}}_{e} \qquad \forall l \in L \quad \forall p \in P \setminus p_0 \tag{5}$$

Constraint groups 6-9 are needed to set the variables indicating changes in the assignment technologies and technology groups.

$$\begin{aligned} b^{T^+}_{tsp_i} &\le 1 - b^{T}_{tsp_{i-1}} && \forall t \in T \\ b^{T^+}_{tsp_i} - b^{T}_{tsp_i} &\le b^{T}_{tsp_{i-1}} && \forall s \in S \\ b^{T}_{tsp_i} - b^{T^+}_{tsp_i} &\le b^{T}_{tsp_{i-1}} && \forall p_i \in P \setminus p_0 \end{aligned} \tag{6}$$

$$\begin{aligned} b^{T^-}_{tsp_i} &\le 1 - b^{T}_{tsp_i} && \forall t \in T \\ b^{T^-}_{tsp_i} - b^{T}_{tsp_{i-1}} &\le b^{T}_{tsp_i} && \forall s \in S \\ b^{T}_{tsp_{i-1}} - b^{T^-}_{tsp_i} &\le b^{T}_{tsp_i} && \forall p_i \in P \setminus p_0 \end{aligned} \tag{7}$$

$$\begin{aligned} b^{TG^+}_{gsp_i} &\le 1 - b^{TG}_{gsp_{i-1}} && \forall g \in G \\ b^{TG^+}_{gsp_i} - b^{TG}_{gsp_i} &\le b^{TG}_{gsp_{i-1}} && \forall s \in S \\ b^{TG}_{gsp_i} - b^{TG^+}_{gsp_i} &\le b^{TG}_{gsp_{i-1}} && \forall p_i \in P \setminus p_0 \end{aligned} \tag{8}$$

$$\begin{aligned} b^{TG^-}_{gsp_i} &\le 1 - b^{TG}_{gsp_i} && \forall g \in G \\ b^{TG^-}_{gsp_i} - b^{TG}_{gsp_{i-1}} &\le b^{TG}_{gsp_i} && \forall s \in S \\ b^{TG}_{gsp_{i-1}} - b^{TG^-}_{gsp_i} &\le b^{TG}_{gsp_i} && \forall p_i \in P \setminus p_0 \end{aligned} \tag{9}$$

The following constraints can be used to limit the number of changes allowed for the assignment of technologies and technology groups. It is possible to limit the overall number of changes (constraints 10 and 13), the number of changes per period (constraints 11 and 14) and the number of changes per site (constraints 12 and 15).

$$\sum_{t \in T} \sum_{s \in S} \sum_{p \in P \setminus p_0} \left(b^{T^+}_{tsp} + b^{T^-}_{tsp} \right) \le n^{TA_{max}} \tag{10}$$

$$\sum_{t \in T} \sum_{s \in S} \left(b^{T^+}_{tsp} + b^{T^-}_{tsp} \right) \le n^{TAP_{max}} \qquad \forall p \in P \setminus p_0 \tag{11}$$

$$\sum_{t \in T} \sum_{p \in P \setminus p_0} \left(b^{T^+}_{tsp} + b^{T^-}_{tsp} \right) \le n^{TAST_{max}} \qquad \forall s \in S \tag{12}$$

$$\sum_{g \in G} \sum_{s \in S} \sum_{p \in P \setminus p_0} \left(b^{TG^+}_{gsp} + b^{TG^-}_{gsp} \right) \le n^{TGA_{max}} \tag{13}$$

$$\sum_{g \in G} \sum_{s \in S} \left(b^{TG^+}_{gsp} + b^{TG^-}_{gsp} \right) \le n^{TGAP_{max}} \qquad \forall p \in P \setminus p_0 \tag{14}$$

$$\sum_{g \in G} \sum_{p \in P \setminus p_0} \left(b^{TG^+}_{gsp} + b^{TG^-}_{gsp} \right) \le n^{TGAST_{max}} \qquad \forall s \in S \tag{15}$$

Constraint 16 sets the variables indicating a logistic link induced by the assignment of technologies to production sites

$$\left(b^{T}_{ts(p_{i-v_e})} + b^{T}_{t's'p_i} \right) \cdot n^{TE}_{et} \cdot n^{TB}_{et'} \le n^{TE}_{et} \cdot n^{TB}_{et'} \cdot \left(b^{LG}_{ess'p_i} + 1 \right) \qquad \begin{aligned} &\forall e \in E \\ &\forall s, s' \in S \\ &\forall p \in P \\ &\forall t, t' \in T \end{aligned} \tag{16}$$

Constraints 17 and 18 are used to set the initial assignment of technologies and technology groups.

$$b^{T}_{ts0} = b^{T0}_{ts} \qquad \begin{aligned} &\forall t \in T \\ &\forall s \in S \end{aligned} \tag{17}$$

$$b^{TG}_{gs0} = b^{TG0}_{gs} \qquad \begin{aligned} &\forall g \in G \\ &\forall s \in S \end{aligned} \tag{18}$$

Constraint 19 ensures the non-negativity of variable x^{V}_{ep}.

$$x^{V}_{ep} \ge 0 \qquad \begin{aligned} &\forall e \in E \\ &\forall p \in P \setminus p_0 \end{aligned} \tag{19}$$

Model for the determination of quantitative capacity requirements

The objective of this planning level (2) is to minimize costs of production, transportation and capacities. Production costs consist of fixed production costs per product and variable production costs per unit. Transportation costs are calculated per unit of a product that is transported from one site to another. Costs for capacities consist of cost for chosen capacity stages for technologies and technology groups and for changes of capacity stages.

$$\min \mathrm{z}_2 = \sum_{p\in P}\sum_{s\in S}\left[\sum_{e\in E}\left(b_{esp}^{EST}\cdot c_{es}^{P_{fix}} + x_{esp}^{EST}\cdot c_{es}^{P_{var}} + x_{esp}^{V}\cdot c_{es}^{V} + \sum_{m=1}^{n^{ST}} x_{ess'p}^{LG}\cdot c_{ess'p}^{LG}\right) + \sum_{t\in T}\sum_{k\in K}\left(b_{ktsp}^{KS}\cdot c_{ktsp}^{KS} + \sum_{k'\in K} b_{k'ktsp}^{KA}\cdot c_{k'kts}^{KA}\right) + \sum_{g\in G}\sum_{k\in K}\left(b_{kgsp}^{KSG}\cdot c_{kgsp}^{KSG} + \sum_{k'\in K} b_{k'kgsp}^{KAG}\cdot c_{k'kgs}^{KAG}\right)\right] \quad (20)$$

Model for the conversion of quantitative capacity requirements into machinery

On this level the development of machine assets and its utilization is optimized. Also decisions on outsourcing are made. Therefore the objective function (3) minimizes the costs for the setup of the machines, costs for production processes executed on the machines, costs for external procurement, costs for demands exceeding capacities and costs for the acquirement of new machines.

$$\min \mathrm{z}_3 = \sum_{p\in P}\left[\sum_{w\in W}\sum_{t\in T}\left(b_{twp}^{RT}\cdot t_{tw}^{RT}\cdot c_{w}^{R} + x_{tp}^{T}\cdot t_{tw}^{TW}\cdot c_{w}^{p}\right) + \sum_{e\in E}\left(x_{ep}^{K}\cdot c_{e}^{K} + x_{ep}^{V}\cdot c_{e}^{V}\right) + \sum_{w\in W}\left(b_{wp}^{W}\cdot c_{w}^{F} + b_{wp}^{WN}\cdot c_{w}^{N}\right)\right] \quad (21)$$

Model for the conversion of quantitative capacity requirements and machinery into workforce

The model of level four has to determine an optimal workforce to run the machines and to fulfill the quantitative capacity demands. Therefore the objective function (4) minimizes the costs for the setup of machines, the costs for production processes executed on the machines as well as the costs or the employment of personnel, the costs for overtime, the costs for hiring and releasing employees and the costs for training.

$$\min \mathrm{z}_4 = \sum_{p\in P}\left[\sum_{w\in W}\sum_{t\in T}\left(b_{twp}^{RT}\cdot t_{tw}^{RT}\cdot c_{w}^{R} + x_{tp}^{T}\cdot t_{tw}^{TW}\cdot c_{w}^{p}\right) + \sum_{e\in E}\left(x_{ep}^{K}\cdot c_{e}^{K} + x_{ep}^{V}\cdot c_{ep}^{V}\right) + \sum_{m\in M}\left(b_{mp}^{M}\cdot c_{m}^{M} + x_{mp}^{U}\cdot c_{m}^{U} + b_{mp}^{ME}\cdot c^{ME} + b_{mp}^{MR}\cdot c^{MR} + \sum_{q\in Q} b_{mqp}^{MQN}\cdot c_{q}^{Q}\right)\right] \quad (22)$$

4.2.HIERARCHICAL OVERALL PROCESS

Coordination processes

In order to create a hierarchical overall process, coordination processes for anticipation, instruction and reaction are defined.

Anticipations: The determination of qualitative capacity demands anticipates the decisions of the second level by taking into account maximum capacities for production processes and production sites and local content quota for countries. Capacity stages used for the determination of quantitative capacity demands are an anticipation of available machine capacities and workforce, which are planned in detail on the subordinate levels. Machine capacities on level three include an anticipation of the availability of personnel.

Instructions: Planning results of the individual levels define the framework for subordinate levels and form the instruction of the upper level to the lower levels. The optimal allocation of production processes to production sites is the instruction passed from level one to level two. The allocation of products and quantities, which is one result of the model on level two, is the instruction passed from level two to subordinate levels. On level three decisions on machine assets and outsourcing are made and passed as instructions to level four.

Reactions: The definition of reactions requires the definition of a ratio indicating the need to react. Since the overall planning process and the individual levels have to ensure the production of all demands, the ratio of demands that cannot be produced using the determined plan is an adequate indicator. Its value based ratio of the objective function is used to allow the comparison of different products. Equations (5), (6), (7) and (8) show the ratios for the four levels. Since reactions are directed towards upper levels, K_1 is not used for reactions. Additionally threshold values K_2^{max}, K_3^{max} and K_4^{max} are defined. A reaction occurs if the ratio of a plan is above the threshold value.

$$K_1 = \frac{\sum_{p \in P} \sum_{e \in E} x_{ep}^V \cdot c_e^V}{z_1} \tag{23}$$

$$K_2 = \frac{\sum_{p \in P} \sum_{e \in E} \sum_{s \in S} x_{esp}^V \cdot c_{es}^V}{z_2} \tag{24}$$

$$K_3 = \frac{\sum_{p \in P} \sum_{e \in E} x_{ep}^V \cdot c_e^V}{z_3} \tag{25}$$

$$K_4 = \frac{\sum_{p \in P} \sum_{e \in E} x_{ep}^V \cdot c_e^V}{z_4} \tag{26}$$

Hierarchical planning procedure

The combination of the coordination processes described before results in a hierarchical planning procedure. On each level the corresponding mathematical optimization model is used to solve the planning task. On the first level production processes are assigned to production sites. This assignment is the instruction passed to the subordinate level, the determination of quantitative capacity requirements. Here quantitative capacity demands are determined by assigning products and quantities to production sites and passed as instruction to subordinate levels. If ratio K_2 exceeds the threshold value K_2^{max}, a feedback process is initiated allowing the adjustment of the parameters of level one for the next planning run. On level 3 quantitative capacity requirements are converted into machinery. Machine capacities planned at this level and quantitative capacity requirements are the instructions passed to level 4 and converted into workforce there. As before feedback processes are initiated if K_3 or K_4 exceeds the threshold values K_3^{max} or K_4^{max} respectively.

5. RESULTS

The approach presented in this paper was implemented into a software prototype to prove the usability for real world problems. It consists of a database for input and output data, IBM ILOG CPLEX 12.1 to solve the optimization models and a Java program to build the model from input data using the CPLEX Java API. The program was parallelized to improve performance.

This prototype was tested for different examples. One example is briefly presented below.

In this example 3794 products from 28 product groups are produced distributed to 7 production sites.

Table 5: Size of the current production network

Location	product classes	products
A	21	2715
B	12	546
C	7	509
D	10	271
E	7	46
F	5	22
G	2	10

A new site is going to start production in the second quarter of the first year planned. It is possible to allocate production processes of 41 products of product group 1 and 43 components of product group 1K to this site. At the beginning of quarter 3 of the second year the second part of the new site is going to start production.

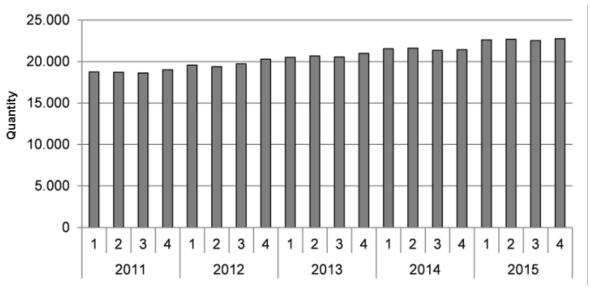
Table 6: Maximum production quantities in units per quarter

Location	product class 1	product class 1K	product class 2	product class 2K
B	9.000	13.000	20.500	27.500
D	2.100	2.000	2.000	2.500
X	1.400*	1.800*	1.500**	2.000**

*from 2nd Quarter 2011
**from 3rd Quarter 2012

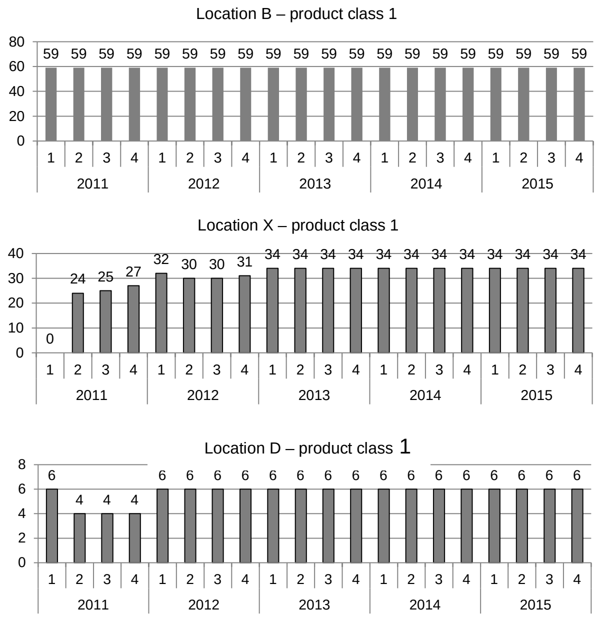
From then on the option to allocate production processes of 51 products of product group 2 and 40 components of product group 2K should be evaluated.

On the first level the allocation of production processes to the new production site and its effects on other parts of the production network were determined.

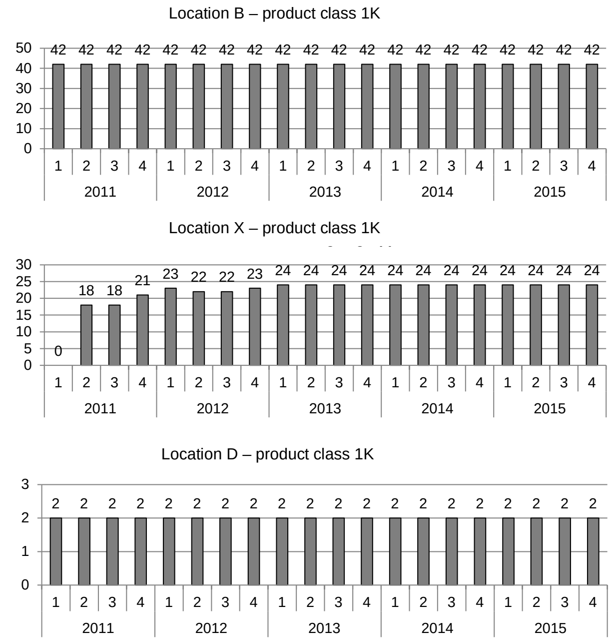


Picture 1: Demand curve for product group 2 to 5 years on a quarterly basis

A planning horizon of five years with a granularity of quarters was chosen for this level. The results showed the startup of production at the new site. Starting with the assignment of 42 processes in second quarter the number of processes assigned to the new site increases to 58 processes for the product groups 1 and 1K.

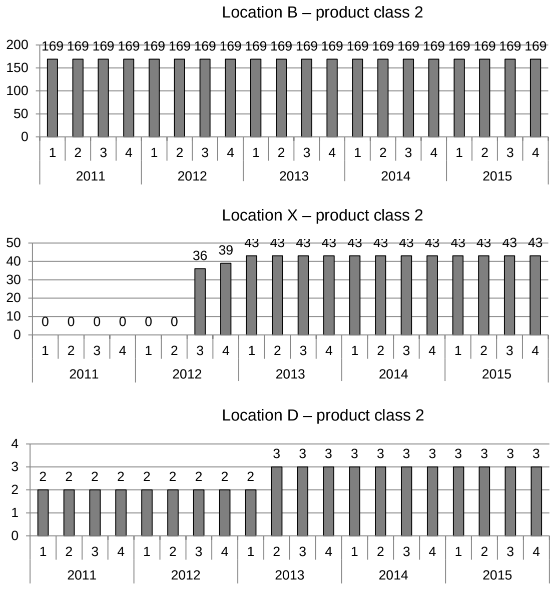


Picture 2: Number per site associated production processes for products from the product group 11

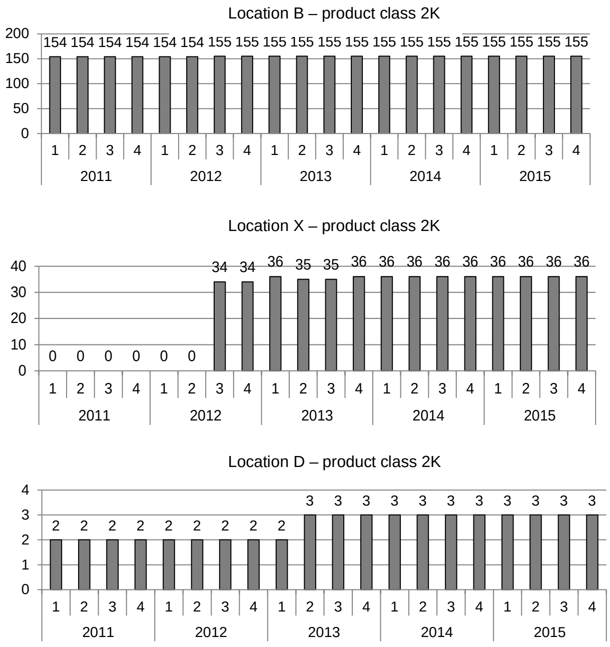


Picture 3: Number per site associated production processes for products from product group 1K

A similar startup is planned for product groups 2 and 2K starting in third quarter of the second year.

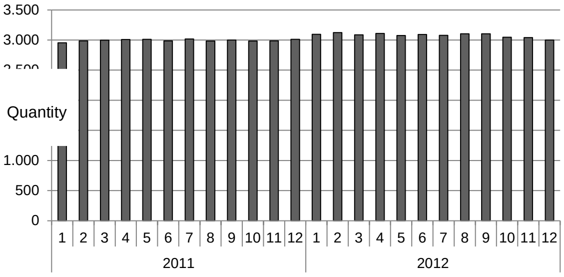


Picture 4: Number per site associated production processes for products from the product group 2

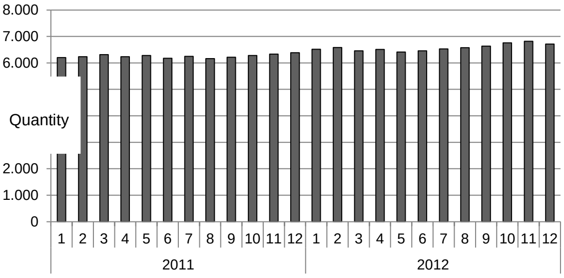


Picture 5: Number per site associated production processes for products from product group 2K

Based on the allocation of production processes quantitative capacity requirements are determined by assigning production quantities to production sites. At this level a planning horizon of 24 months was chosen.



Picture 6: Demand curve for product group 1 for 2 years on a monthly basis



Picture 7: Demand curve for product group 2 for 2 years on a monthly basis

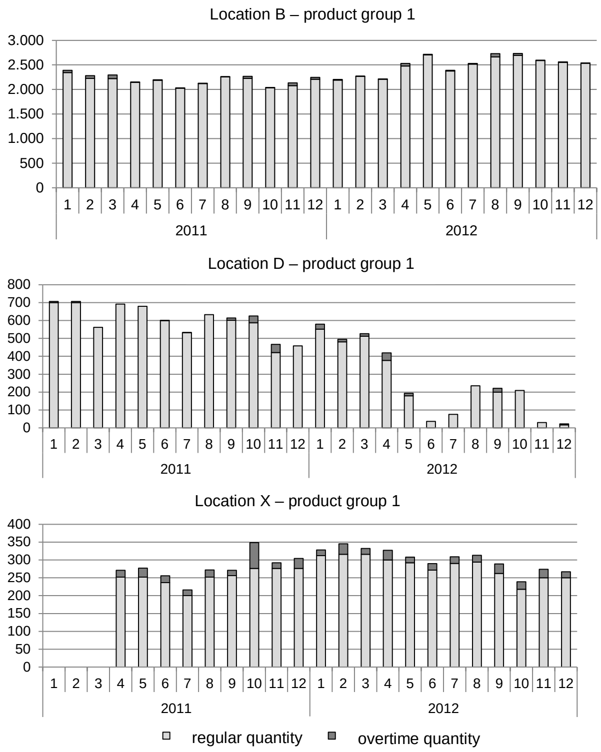
The results showed that due to the assignment of quantities to the new site there is a drop in assigned quantities at two other sites producing the same products as the new one.

Picture 8: Allocation amount for a product group 1

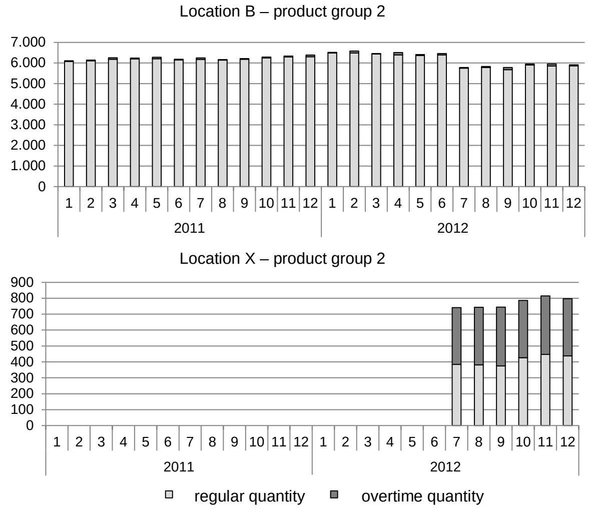
Picture 9: Allocation amount for product group 2

For one of the existing sites planning level three was executed to analyze the effects on the machinery of this site. At this site 32 machines are available to produce the demands for 434 products. Here a planning horizon of 52 weeks was chosen. Due to a high level of demands the results show a high utilization of machinery at this site.

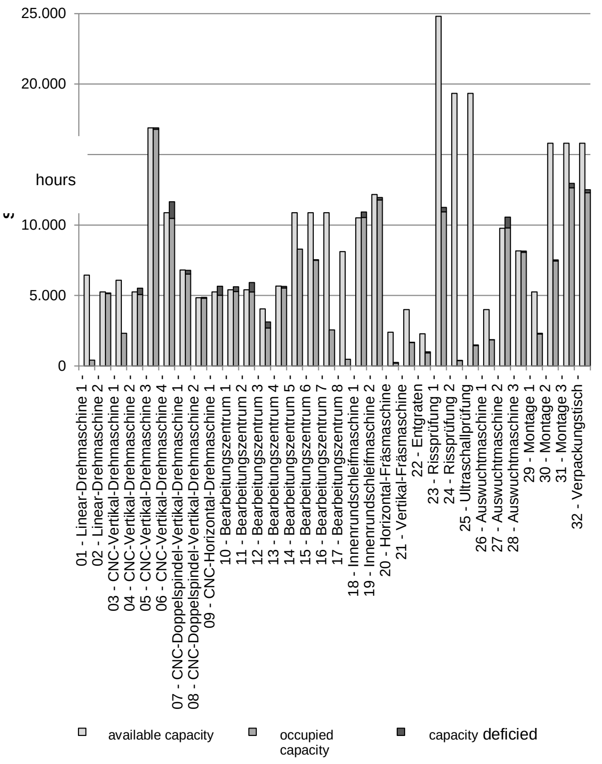
Picture 10: Capacity utilization of the tools - Cumulative representation for 2011



An extension of machinery is not necessary, because of the relieving effect of new site on the capacity situation.



In a last step workforce was planned for the same site. The results showed that the existing 173 employees are not sufficient and that it necessary to hire 12 additional employees. Additionally, 4 of



the existing employees need to acquire a second qualification to allow flexible assignment.

6. CONCLUSION AND FURTHER WORK

In this paper a model for the determination of qualitative capacity requirements is presented as part of a four level approach for the planning of production networks was introduced. The overall planning task was decomposed into four hierarchically interrelated planning levels. On each level a mathematical optimization model is used to solve the planning task. The entire model of the top level and the objective functions of the other models and hierarchical coordination processes are presented. The resulting hierarchical overall planning process and its evaluation based on a case study are described. The evaluation was executed on a prototype implementation.

This planning system is supposed to be part of the system environment of a company. Thus there is the need to define interfaces to existing systems, e.g. to retrieve master data (products, bill of material etc.) needed as input for planning. There is also the need for a graphical user interface allowing a user-friendly control of the system, analyses of results and interaction with the system in the purpose of the hierarchical planning process. Thus a partial implementation of such a graphical user interface has been realized, but further extension is needed.

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