

# Reverse Logistics Inventory Model for Reusable Product Parts

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Until recently logistics systems supported only processes carried out in classical material flow from producer to final user. Recently it has been a remarkable growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. Reuse of product parts can bring direct advantages to the company because it reduces costs associated with acquiring new components. Main goal of this paper is to create the reverse logistics inventory model that uses the reliability theory to describe reusability of product parts with assumption that recovered components are used in a production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in production and inventory management. It gives the base to optimize some of the process parameters: the threshold work time of returns, the warranty period for products containing reused elements or new components order size.

**Keywords:** reverse logistics, reusable product parts, inventory model.

## 1. INTRODUCTION

Until recently logistics systems supported only processes carried out in classical material flow from producer to final user. Recently it has been a remarkable growth of interest in optimizing logistics processes that supports recapturing value from used goods. The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal is called reverse logistics. Reverse logistics has become one of the logicians' key areas of interest. It enjoys ever-increasing interest of many industrial branches. Nowadays a growing number of companies realize the meaning of that field of logistics. Reuse of products or product parts can bring direct advantages to the company because it reduces costs associated with acquiring new components by using recycled materials or

recovered components instead of expensive raw materials.

Literature survey that has been done around the theme of the reverse logistics area, allowed to set out this article aims and objectives.

In the reverse supply chain the issue of: timings, quantities and conditions of returned and reusable components make production planning difficult [20]. The majority of models assume that demand for new products and returns quantity are independent Poisson random variables [21] and very few use the reliability theory to estimate the number of reusable products. Murayama et al. [18]-[20] propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" [11]. Only few models use the reliability theory to

diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. [18]-[20]).

In literature the field of reverse logistics is usually subdivided into three areas: inventory control, production, recovery and distribution planning. The model presented in this paper is a inventory model and hence the literature survey refers to this area. Basic inventory model in reverse logistics is built on following assumptions [3]:

- the manufacturer meets the demand for the final products;
- the manufacturer receives and stores the products returned from the final user;
- demand for the new products can be fulfilled by the production or by the recovery of returned products;
- recovered products are as good as new ones;
- the goal of the model is to minimize the total costs.

Inventory models in reverse logistics can be divided into deterministic and stochastic models.

Deterministic models presented in the literature are mainly the modifications of the EOQ model (e.g. [1,2,15,22,23,25,26,28,32]). There are also some models based on dynamic programming which are the extensions of the classical Wagner Whitin model (e.g. [12,13,24,27]).

In the class of stochastic models there are two model groups:

- models, in which demand for the new product is a consequence of the return;
- models, in which the demand for new products does not depend on the number of returns, but the number of returns may depend on the previous demand.

In the first group there are models of repair systems. These are closed systems in which the number of elements remains constant. The main goal of such a system is to keep sufficient number of spare parts to provide the required level of availability of the technical system (e.g. [16,29,30]). The second group of stochastic inventory models can be divided into continuous (e.g. [4-6,14,17,33-35]) and periodic review models (e.g. [7,8,10,31]). A more detailed

description of inventory models in reverse logistics can be found in [21].

Literature review allows to summarize the current state of knowledge and to define the main shortages of existing logistics models that deal with the reverse logistics problem. The majority of models assume that demand for new products and returns quantity are independent Poisson random variables [21]. Few authors examine the relationship between the demand, and the number of returns but there are no inventory models in reverse logistics that use reliability theory to assess the number of returns and very few use the reliability theory to estimate the number of reusable products. Murayama et al. [18]-[20] propose the method to predict the number of quantities of returned products and reusable components at each time period by using series system reliability models. The condition aspect of returns in a reverse logistic system is usually omitted by using assumption that all the returns are reusable and usually "as good as new" [11]. Only few models use the reliability theory to diversify returned elements' reusability, but they don't give any guidelines for the way to optimize the threshold value of components' residual life (e.g. [18]-[20]). Most of created models assume single component product.

Main goal of this paper is to create the reverse logistics inventory model that uses the reliability theory to describe reusability of product parts with assumption that recovered components are used in a production process but they aren't as good as new ones. The model allows to estimate the potential profits of the reusing policy in production and inventory management. It gives the base to optimize some of the process parameters: the threshold work time of returns, the warranty period for products containing reused elements or new components order size. Presented model is an extension of the one described in [9].

This paper objectives are achieved by creating the simulation model that describe analyzed processes. Created model makes possible to describe analyzed processes with various probability distributions. The next step is a sensitivity analysis of created model and verification process. The article ends with conclusions and directions for further research.

## 2. MODEL DESCRIPTION

In order to evaluate the influence of various reusing and supplying processes parameters, the simulation model was constructed. Because of the complexity of the problem and limited possibilities of its analytical analysis the Monte Carlo method was adopted to model the production system. The simulation was created in GNU Octave, a high-level interpreted language for numerical calculations. Created model is based on the following assumptions:

- A company produces the object composed of two elements (A and B). The product fails when one of components fails – series reliability structure.
- A failure of each component occurs independently on other components' failures.
- If the product fails during the warranty period, it is returned to the manufacturer and he has to pay some penalty cost (e.g. the cost of a new product).
- The products are returned as soon as their lives are ended and reusable B components are stored in a stock until new production batch running, when they may be reused.
- Also new B components are stored. We assume periodic review of the new B components inventory.
- Under this policy, manufacturer orders a variable quantity of new B components every fixed period of time in order to maintain an inventory position at a predefined base stock level. This policy is also known as the order-up-to level.
- We assume fixed and equal to one, lead time for external order.
- The component B of the product may be reused in a new production, if it was not the cause of a product failure and its total work time up to this moment is not greater than some acceptable - threshold time  $T$  (Fig.1).
- Neither failed elements B can be reused in a new production (not repairable) nor any A element. All A components are new in a new production.
- New products are manufactured and sold periodically in established moments.
- Demand for the products is a random variable.

The process of reusing of the component B, dependently on its threshold age  $T$ , is shown in

Figure 1. The usage of recovered components decreases production costs but also increases the risk that additional costs occur because of larger amount of returns during the warranty period.

The number of returns that can return between two moments of a production beginning and may be reused depends on the number of the products that were sold earlier, the length of the period between two consecutive production batch, the length of the warranty period and threshold work time  $T$ . Component B may be returned only during the product warranty period and reused only if it is not older then  $T$ .

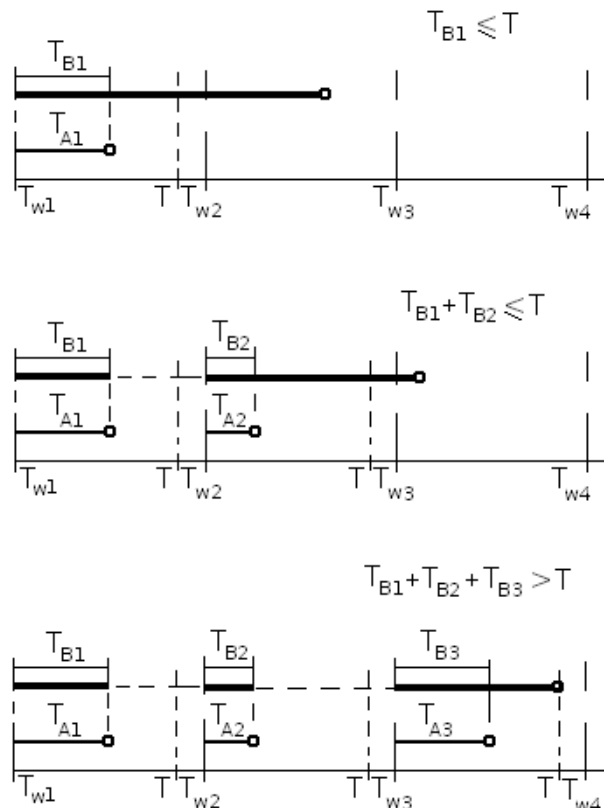


Fig. 1 Process of element reusing in a new production.

Simulation algorithm is realized for the assumption that all necessary co-components (A elements) of reusable elements are always available when needed. It is the simplified assumption that allow to focus on the supply and production process results coming from managing of element B inventory.

Input parameters to the simulation may be divided into groups:

- cost parameters (ordering cost, inventory holding cost, inventory lack cost, purchase cost of a new element B, cost of preparing of the returned B element to reusing, penalty cost resulting from a product failure during warranty period),
- inventory model parameters (maximum level of inventories of new elements, period between orders),
- reliability parameters (probability distribution of A and B elements time to failure, warranty period, threshold age of reusable element),
- demand parameters (probability distribution of demand size in unit period).

The results that were gather from the simulation allow to analyze cost results, inventories levels of new and reusable elements, and customer service level.

### 3. THE SENSITIVITY ANALYSIS

The aim of the research was to determine which parameters are critical for supply and production process results and to find some general rules that allow easily find parameters satisfactory for needs. For this reason parameters that was concerned as the most meaningful for the cost effects of the reusing policy were tested. The results obtained during the investigation are shown in Figures 2-11.

The basic determinants of every inventory model effectiveness are the model parameters. Analyzed system applies the model where new order is placed in constant periods and ordered quantity depends on the current inventory level and its maximum allowable level. For this reason this two parameters: the period between orders ( $C$ ) and the maximum inventory level ( $S$ ) was assumed as critical parameters in the modeled process and analyzed from various points of view.

Figures 2-3 presents the production system results coming from potential inventory shortages. A customer service level (independently on the way of its calculation) is very sensitive on the maximal inventory level ( $S$ ) and on frequency of deliveries ( $C$ ). It confirms their crucial role in production effects even for analyzed case, when a reusing policy is applied. Growing value of  $S$  gives similar effect as decreasing value of  $C$  – the shortage probability decreases.

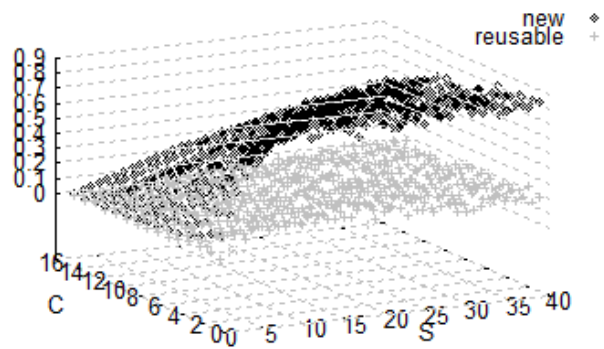


Fig. 2 The percent of a production quantity covered by new and reusable elements from the stock for various lengths of period between orders ( $C$ ) and the maximum inventory levels ( $S$ ).

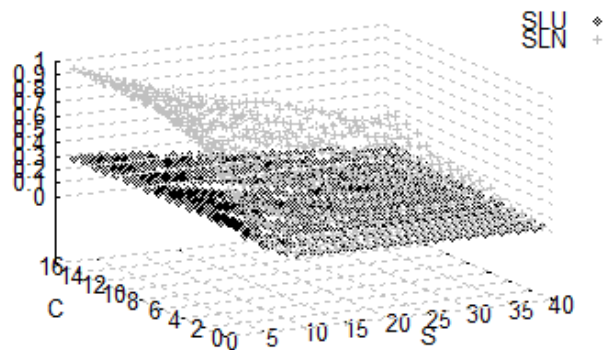


Fig. 3 The percent of all simulated periods when an shortage occurs ( $SLU$ ) and the percent of total demand unsatisfied because of the component B shortages ( $SLN$ ).

When parameters  $S$  and  $C$  are well determined, even up to 20% of total demand (for assumed system) may be covered by reusable elements (Fig. 2). Unexpectedly, the number of used returns in the production also depends on  $S$  and  $C$  values – higher  $S$  and shorter period  $C$  cause higher level of reusing. Higher number of sold products (higher customer service level) gives higher number of reusable returns and savings coming from this fact. The relation of new and “old” elements quantity in the production – when  $S$  and  $C$  are correctly determined – depends on the reliability characteristics of the system.

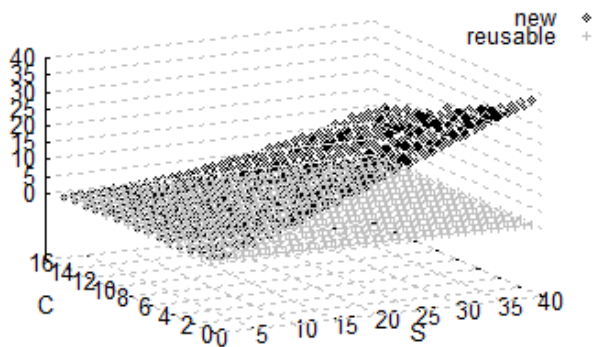


Fig. 4 The average inventory level of new and reusable elements.

Figure 4 presents the average inventory level of new and reusable elements. The inventory level of reusable elements is constant for all tested  $S$  and  $C$  values. The assumption that returns are cheaper and are used in the production as first causes that there is almost no inventories of “old” components.

Figures 5-7 present cost results of the supply and production process when the reusing policy is applied. All costs were calculated per unit of demand that occurred in simulation time. This calculation allows to make the results independent on real values of the demand and make them more general.

The total cost of the system (Fig. 7) includes costs of: ordering and inventory holding for new elements, costs of new components purchase and preparing to reusing of returns (proportional to the number of used elements presented in Figure 2), cost of penalties coming from inventory shortages and product failures during warranty period (Fig. 6).

The cost results show that, for modeled system, the cost of ordering (Fig. 5) has the greatest meaning for economic effects of system functioning – the total cost is the highest (c.a. 3,5). On the other side, the same solutions (Fig. 7, “shortage minimum”) were found as solutions with the highest customer service level. This fact allow to precise some conclusions:

- the inventory models with the highest customer service level are usually the most expensive,
- the best way to minimize the risk of inventory shortages is to minimize the period between orders, what allow for flexible inventory level control.

The rest of cost components has comparable effect on the total cost of system functioning (cost c.a. 0÷1,5).

The Figure 6 presents the costs of shortages and warranty services and show the same effect as Figure 2. When the cost of inventory lack is high (low part of demand is satisfied), the number of sold product is low. It also causes the lower cost of product failures during warranty period. For modeled system both costs have quite opposite direction and their sum seems nearly constant for all tested  $S$  and  $C$ .

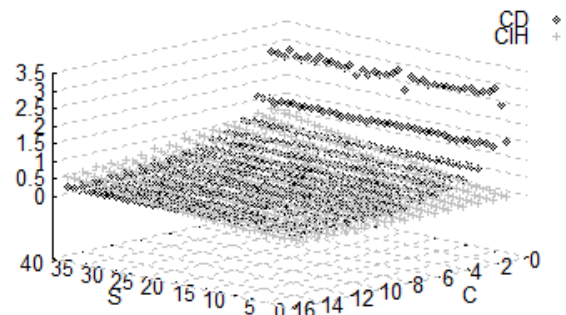


Fig. 5 The total cost of: ordering ( $CD$ ) and inventory holding ( $CIH$ ).

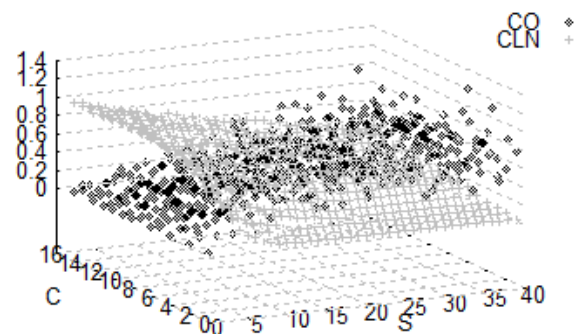


Fig. 6 The total cost of “penalties” resulting from: a product failure during warranty period ( $CO$ ) and the inventory lack ( $CLN$ ) when needed.

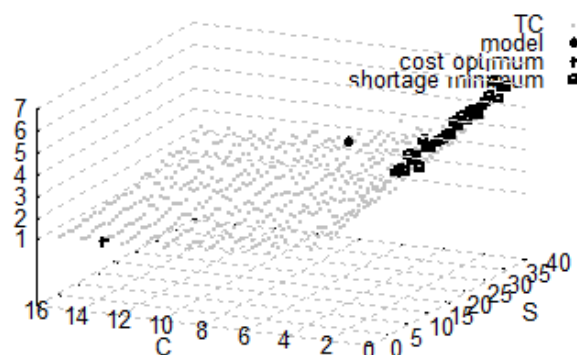


Fig. 7 The total cost ( $TC$ ) and its specific cases: for  $S$  and  $C$  determined according to theoretical rules described in the inventory literature, for the case that gives the lowest cost from all tested possibilities, for the solution that gives the highest customer service level (minimum quantity of shortages).

The summary of the cost analysis is presented in Figure 7. The total cost of supply and production ( $TC$ ) is presented for all tested ranges of  $S$  and  $C$ . The plane of costs shows that there is no explicit minimum. The maximum is determined by the mentioned earlier ordering cost (Fig. 7,  $C = 1$ ). The point found in simulation as the minimum cost gives the solutions with very high percent of unsatisfied demand (Fig. 3) and from the point of view of a real production system cannot be accepted. The better solution is to accept values of  $S$  and  $C$  determined according to classical formulas accessible in the literature from inventory area (Fig. 7, “model”). It gives worse cost results but much better customer service level.

The study of possible effect of basic parameters of inventory model ( $S$  and  $C$ ) on the various aspects of system functioning confirms their crucial role in the process. The number of possible variables in the model causes that there is no simple rule, how to determine the best supply policy parameters (no explicit cost minimum). The model analysis shows that the reusing policy has a weak influence on the best inventory model parameters. It is mainly because of the low share of reusable returns in all demand satisfaction.

The following part of the sensitivity analysis of the model focuses on reliability parameters of the reusing policy. The aim of the research is to assess the meaning of reliability parameters, which are key determinants of the reusing policy, for inventory model. The two tested variables are: a warranty period length ( $T_w$ ) and the threshold age of the reusable component that can be used in a the production of new products ( $T$ ).

On the base of previous results, three variants of maximum inventory level ( $S$ ) and period between orders ( $C$ ) were tested (Fig.7, “model, cost\_optimum, shortage\_minimum”), for chosen range of  $T$  and  $T_w$ . Ranges of analyzed vectors of  $T$  and  $T_w$  were set according to guidelines given in literature [9,21 Results of the study are presented in Figures 8-11 and concern the case of  $S$  and  $C$  parameters calculated according to literature formulas ( $S = 38$ ,  $C = 8$ ). The other cases are not presented because of their lower effectiveness – very high shortages in “cost-optimum” case and more expensive results with similar shortages effects in “minimum-shortage case”.

The Figure 8 presents the part of all demand covered by new and reusable elements. Depending on the warranty period length and threshold age of reusable component, even 50% of production may be satisfied by reusable elements. The longer time  $T_w$  and  $T$  means bigger number of returns and:

- the lower cost of new elements purchase,
- the higher level of new elements inventories (Fig. 9) and inventory holding cost, for given inventory model parameters,
- the higher cost of warranty services, resulting from a product failure during warranty period (Fig. 10).

Reliability parameters does not have any influence on the number of inventory shortages and customer service level (Fig. 10). The total cost of system functioning (Fig. 11) strongly depend on reusing policy parameters and is determined by the cost of warranty services. The total effect of reliability parameters on the modeled system depends on cost components relations. Lower unit cost of warranty service makes longer values  $T_w$  and  $T$  profitable. Higher inventory holding cost per unit time should reduce the reusing possibility and increasing unit cost of new element purchase reduce savings from the reusing.

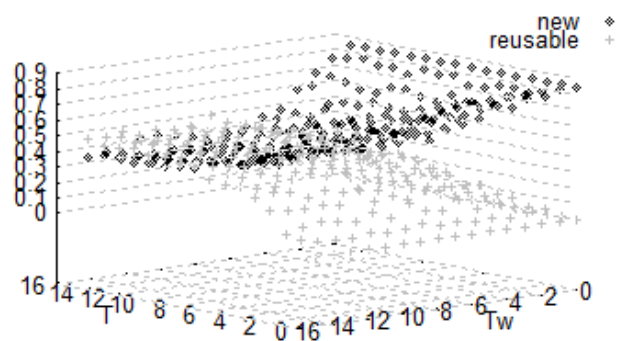


Fig. 8 The percent of a production quantity covered by new and reusable elements from the stock for various lengths of warranty time ( $T_w$ ) and the threshold age of reusable element ( $T$ ).

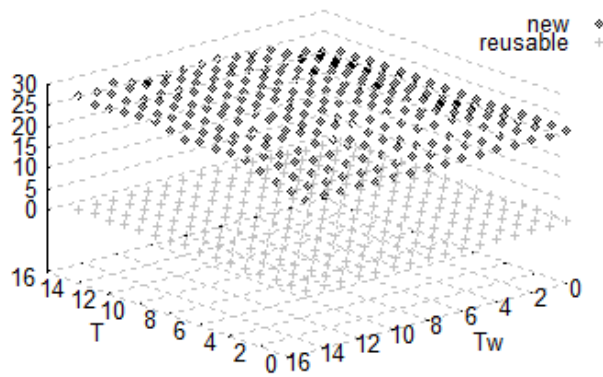


Fig. 9 The average inventory level of new and reusable elements.

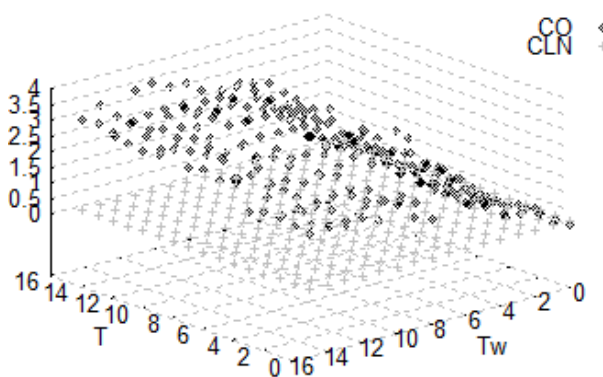


Fig. 10 The total cost of “penalties” resulting from: a product failure during warranty period (*CO*) and the inventory lack (*CLN*) when needed.

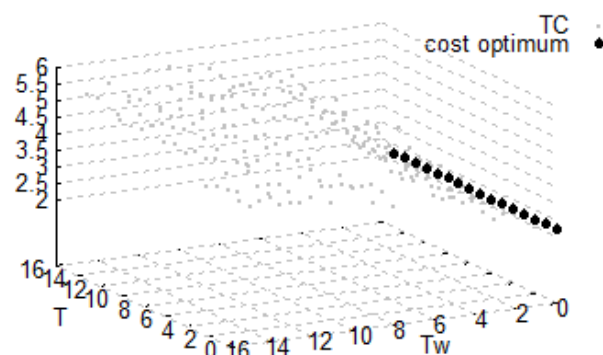


Fig. 11 The total cost (*TC*) and its specific cases: for *S* and *C* determined according to theoretical rules described in the inventory literature, for the case that gives the lowest cost from all tested possibilities, for the solution that gives the highest customer service level (minimum quantity of shortages).

## SUMMARY

Optimal solution of the model strongly depends on the input parameters that may depend on various factors (e.g. the laws that determine the length of the warranty period). Higher customer service level which is achieved by selecting the appropriate parameters of the inventory policy, indicates greater quantity of returns, higher level of total cost of “penalties” resulting from a product failure during warranty period and also the greater savings from the use of cheaper recovery.

There are many possible extensions of the presented model by taking into account:

- parameters associated with the process of collection and transportation of returns. Particularly important may be the consideration of different return sources;
- ability to deliver returns in batches;
- longer than a period, time of recovery;
- random lead time for external orders;
- random times of recovery;
- complex reliability structure of the technical objects.

## REFERENCES

- [1] Dobos. I., Richter. K. 2000. The integer EOQ repair and waste disposal model – Further analysis. Central European Journal of Operations Research 8: 173–194.
- [2] Dobos. I. 2002. The generalization of Schrady’s model: a model with repair, Working Paper No. 7, Department of Business Economics, Budapest University of Economics and Public Administration.
- [3] Fleischmann. M., Bloemhof-Ruwaard. J.M., Dekker. R., van der Laan. E., van Nunen. J.A.E.E., van Wassenhove. L.N. 1997. Quantitative models for reverse logistics: A review. European Journal of Operational Research 103: 1-17.
- [4] Fleischmann. M., Dekker. R., Kuik. R. 2002. Controlling inventories with stochastic item returns: A basic model. European Journal of Operational Research 138: 63-75.
- [5] Fleischmann. M., Kuik. R. 2003. On optimal inventory control with independent stochastic item returns. European Journal of Operational Research 151: 25-37.
- [6] Heyman. D.P. 1997. Optimal disposal policies for a single-item inventory system with return., Naval Research Logistics Quarterly 24: 385-405.
- [7] Inderfurth. K., Simple optimal replenishment and disposal policies for a product recovery system with leadtimes. OR Spektrum 19: 111-122.

- [8] Inderfurth. K., van der Laan. E. 2001. Leadtime effects and policy improvement for stochastic inventory control with remanufacturing., *International Journal of Production Economics* 71: 381-390.
- [9] Jodejko-Pietruczuk A., Plewa M. (2011). The model of reusability of series system product. *Summer Safety and Reliability Seminars*, July 03-09, 2011, Gdańsk-Sopot, Poland.
- [10] Kelle. P., Silver. E.A. 1989. Purchasing Policy of New Containers Considering the Random Returns of Previously Issued Container. *IIE Transactions* 21 (4): 349-354.
- [11] Kiesmüller. G.P. 2003. A new approach for controlling a hybrid stochastic manufacturing/remanufacturing system with inventories and different leadtimes. *European Journal of Operational Research* 147: 62-71.
- [12] Kleber. R., Minner. S., Kiesmüller. G. 2002. A continuous time inventory model for a product recovery system with options. *International Journal of Production Economics* 79: 121-141.
- [13] Konstantaras. I., Papachristos. S. 2007. Optimal policy and holding cost stability regions in a periodic review inventory system with manufacturing and remanufacturing options. *European Journal of Operational Research* 178: 433-448.
- [14] Korugan. A., Gupta. S. M. 1998. A Multi-Echelon Inventory System with Returns. *Computers & Industrial Engineering* 53 (1-2): 145-148.
- [15] Mabini. M.C., Pintelon. L.M., Gelders. L.F. 1992. EOQ type formulations for controlling repairable inventories. *International Journal of Production Economics* 28: 21-33.
- [16] Muckstadt. J.A. 1973. A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science* 20 (4): 472-481.
- [17] Muckstadt. J.A., Isaac. M.H. 1981. An analysis of single item inventory systems with returns. *Naval Research Logistics Quarterly* 28: 237-254.
- [18] Murayama T., Shu L. H. (2001). Treatment of Reliability for Reuse and Remanufacture, *Proceedings of the 2nd International Symposium on Environmentally Conscious Design and Inverse Manufacturing (EcoDesign'01)*, Tokyo, Japan.
- [19] Murayama T., Yoda M., Eguchi T., Oba F. (2005). Adaptive Production Planning by Information Sharing for Reverse supply Chain, *Proceedings of the 4th International Symposium on Environmentally Conscious Design and Inverse Manufacturing (EcoDesign'05)*, Tokyo, Japan.
- [20] Murayama T., Yoda M., Eguchi T., Oba F., *Production Planning and Simulation for Reverse Supply Chain*, Japan Society Mechanical Engineering International Journal, Series C, Vol. 49, No. 2.
- [21] Plewa M., Jodejko-Pietruczuk A. (2011). The reverse logistics model of single-component product recovery. *European Safety and Reliability Conference*, Troyes, France.
- [22] Richter. K. 1994. An EOQ repair and waste disposal model. *Proceedings of the Eight International Working Seminar on Production Economics* 3: 83-91. Inngs/Innsbruck.
- [23] Richter. K. 1997. Pure and mixed strategies for the EOQ repair and waste disposal problem. *OR Spectrum* 19: 123-129.
- [24] Richter. K., Sombrutzki. M. 2000. Remanufacturing planning for the reverse Wagner/Whitin models. *European Journal of Operational Research* 121 (2): 304-315.
- [25] Richter. K. 1996a. The EOQ repair and waste disposal model with variable setup numbers. *European Journal of Operational Research* 95: 313-324.
- [26] Richter. K. 1996b. The extended EOQ repair and waste disposal model. *International Journal of Production Economics* 45: 443-447.
- [27] Richter. K., Weber. J. 2001. The reverse Wagner/Whitin model with variable manufacturing and remanufacturing cost. *International Journal of Production Economics* 71: 447-456.
- [28] Schrady. D.A. 1967. A deterministic inventory model for repairable items. *Naval Research Logistics Quarterly* 14 (3): 391-398.
- [29] Sherbrooke. C.C. 1971. An Evaluator for the Number of Operationally Ready Aircraft in a Multilevel Supply System. *Operations Research* 19: 618- 635.
- [30] Sherbrooke. C.C. 1968. METRIC: A Multi-echelon Technique for Recoverable Item Control. *Operations Research* 16: 122-141.
- [31] Simpson V.P. 1978. Optimum solution structure for a repairable inventory problem. *Operations Research* 26 (2): 270-281.
- [32] Teunter. R.H. 2001. Economic ordering quantities for recoverable item inventory system. *Naval Research Logistics* 48: 484-495.
- [33] Van der Laan. E.A., Dekker. R., Salomon. M. 1996. Product remanufacturing and disposal: A numerical comparison of alternative control strategies. *International Journal of Production Economics* 45: 489-498.
- [34] Van der Laan. E.A., Dekker. R., Salomon. M., Ridder. A. 1996. An (S,Q) inventory model with remanufacturing and disposal. *International Journal of Production Economics* 46-47: 339-350.
- [35] Van der Laan. E.A., Salomon. M. 1997. Production planning and inventory control with remanufacturing and disposal. *International Journal of Operational Research* 102: 264-278.

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