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Mathematical modelling of nonstationary electromechanical processes in Coaxial-Linear Engine

A. ZHILTSOV, I. KONDRATENKO, D. SOROKIN

Prof. V.M. Sinkov, Department of Power Supply, Educational and Research Institute of Energy and Automation, National University of Life and Environmental Sciences of Ukraine, 03041, Kyiv, Geroyiv Oboroni st. 12, Ukraine, tel.+38(044)527-87-29 e-mail: sdima.asp@gmail.com

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Abstract. The mathematical model of the electromagnetic and mechanical processes is developed in a coaxial-linear engine with massive magnetic conductors.

Key words: linear engine, massive magnetic conductor, electromagnetic and mechanical change.

INTRODUCTION

Application of electromechanical impulse systems in the different machine and device processes involves the use of alternating motion. For its realization the coaxial-inductive engines can be used. Due to constructive characteristics they displaced engines based on progressive rotation with digital systems of executive mechanisms.

During design of electro-technical devices it is necessary to look over quite a large quantity of variants for choosing necessary construction, which as a rule takes a lot of time, material and power resources. Therefore, there is a need to work out the mathematical models which will adequately display the processes within the examined devices and also create calculating algorithms and program modules based on them. This approach will make the projecting visibly quicker and cheaper [6, 7].

Nowadays, the most widely applied methods for the calculation of electromagnetic fields are: method of final differences (MFD), method of final elements (MFE), methods based on integral equations or on the theory of chains, and combined methods as well, because they involve the advantages of different methods and are free from defects [1, 8]. Choosing the numerical method the characteristics of research object (in our case the characteristics of electromagnetic system of linear engine) must be considered.

In comparison with CLE, the use of MFD and MFE for modeling connected electromagnetic and mechanical processes cannot be defined as rational because they possess a row of disadvantages (artificial limitation of calculated range, sampling of circumjacent space etc.). Therefore, during the analysis of electromagnetic processes in CLE the most suitable is the method of integral equations for the magnetic field sources: eddy current and linked current of magnetization on the border between steel and air [2, 3, 5, 8].

In the work [9] the mathematical model of electromechanical process in CLE was suggested, where the magnetic circuit was supposed to be constructed of separate bars. This allowed for representing of electromagnetic force in square form with respect to instantaneous counts of currents in puttees and it made modeling significantly easer. But fabricating of magnetic circuit for the similar devices presents a difficult technological problem. A variant for its solving is the fabrication of massive magnetic circuit that requires consideration of eddy current influence on the electromechanical process in the electro drive.

The goal of the work is to work out the mathematical model of the

nonstationary electromechanical process in CLE with the massive magnetic circuit and to propose the algorithm for its numerical realization. The analysis of the influence of material characteristics of CLE constructive elements was performed. Also, the law was analyzed of the influence of voltage changing on the stator relatively to the force-characteristic of engine.

STATEMENT OF THE PROBLEM

In Figure 1.1 *a*, a simplified scheme of CLE is shown which consists of co-axially arranged circular reels and toroidal steel bodies. Each puttee consists of identical reel which connected with next one in such a way that their field is active. We will designate the quantity of reels which stator and inductor puttee consists of as N_{W_1}, N_{W_2}, respectively. We will designate the quantity of winds in each reel of stator and inductor as w_1, w_2. Puttee of the inductor feeds from the source of constant voltage $u_1 = const$, and puttee of the stator feeds from the frequency transformer with voltage $u_2 = u_2(t)$.

One should take that geometrical parameters and also electrical and magnetic characteristics of materials are known: γ_1, γ_2 - conductivity of inductor and stator materials; μ – total magnetic penetration of inductor and stator materials, $\mu = const$; k – coefficient of springs stiffness; m – mass of inductor.

The principle of CLE working is the following: When alternative voltage $u_2(t)$ is given on the stator puttee, the pulsatory magnetic field arises which during interaction processes with current fields of inductor puttee, eddy currents in massive conductors and magnetized currents on the division of magnetic circuits leads to the oscillation of inductor with amplitude Δx_{max}.

The modeling of electromechanical process in CLE is a difficult task which is based on united solving of Maxwell's equations in the unlimited range which contains geometrically complicated ferromagnetic massive conductors and equations of inductor motions. Therefore, the problem is divided into several stages: 1) electromagnetic task is to calculate the characteristics of electromagnetic field in CLE when speed of inductor motion is adjusted; 2) mechanical task is to calculate the characteristics of mechanical process when value of electromagnetic force which affects the inductor is adjusted.

MODELING OF ELECTROMAGNETIC PROCESS IN A COAXIALLY-LINEAR ENGINE

In the works [2, 4] based on the integral equations the method of calculating of magnetic field in devices which consist of co-axially arranged circular reels and toroidal steel bodies is examined. When an independent magnetic penetration of material of ferromagnetic body from tensity of magnetic field is admitted, the density of simple layer of currents on their division correspond to the next integral equation [8]:

$$\sigma(Q,t)+\frac{\chi}{\pi}\oint_{l}\sigma(M,t)P(Q,M)dl_M + \frac{\mu}{\mu_0}\frac{\chi}{\pi}\oint_{D}\delta(M,t)P(Q,M)ds_M = -\frac{\chi}{\pi}\oint_{D_W}\delta_W(M,t)P(Q,M)ds_M, Q\in l = l_1\cup l_2, \quad (1)$$

where: $\sigma(Q,t)$ - is a instantaneous density of simple layer of currents in the point Q on the division of magnetic circuits $D = D_1 \cup D_2$; $\sigma(M,t)$ - is analogically in the point M; $\chi = (\mu - \mu_0)/(\mu+\mu_0)$; μ - is total magnetic penetration of material of magnetic circuits; $\mu_0 = 4\pi\cdot 10^{-7}$ H/m – magnetic constant;

$$P(Q,M) = \vec{e}_\alpha \cdot \left[\vec{n}_Q \times \vec{b}(Q,M)\right] = n_z(Q)b_r(Q,M) - n_r(Q)b_z(Q,M).$$

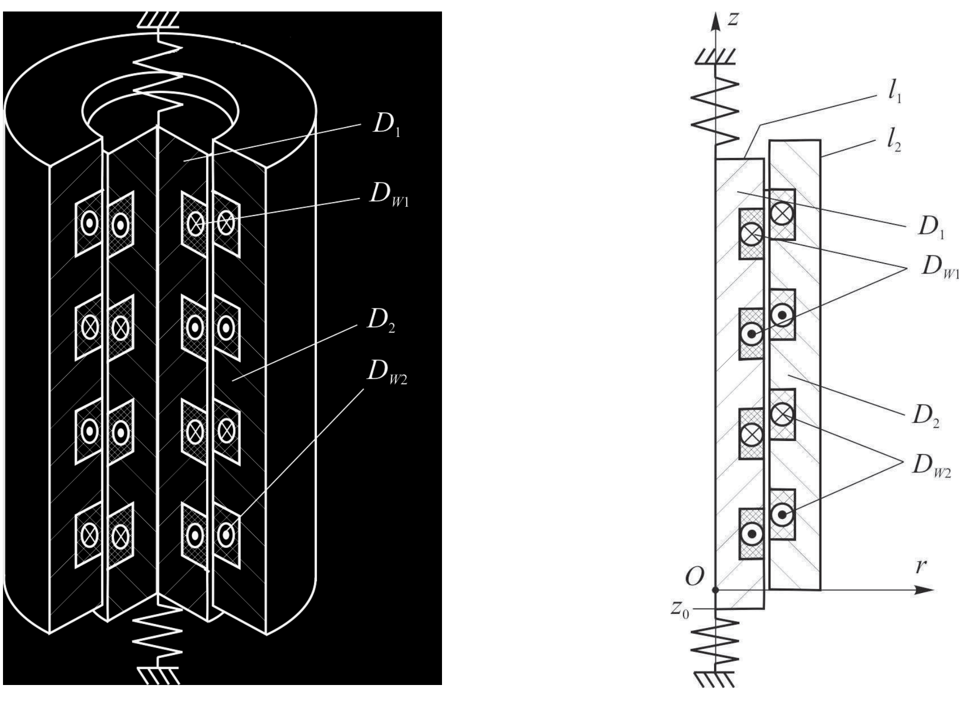

Fig. 1. Simplified electromagnetic scheme of the coaxial-linear engine (a), cut by meridian flat (b): D_1 - magnetic circuit of the inductor, D_{W_1} - puttee of the inductor, D_2 - magnetic circuit of the stator, D_{W_2} - puttee of the stator

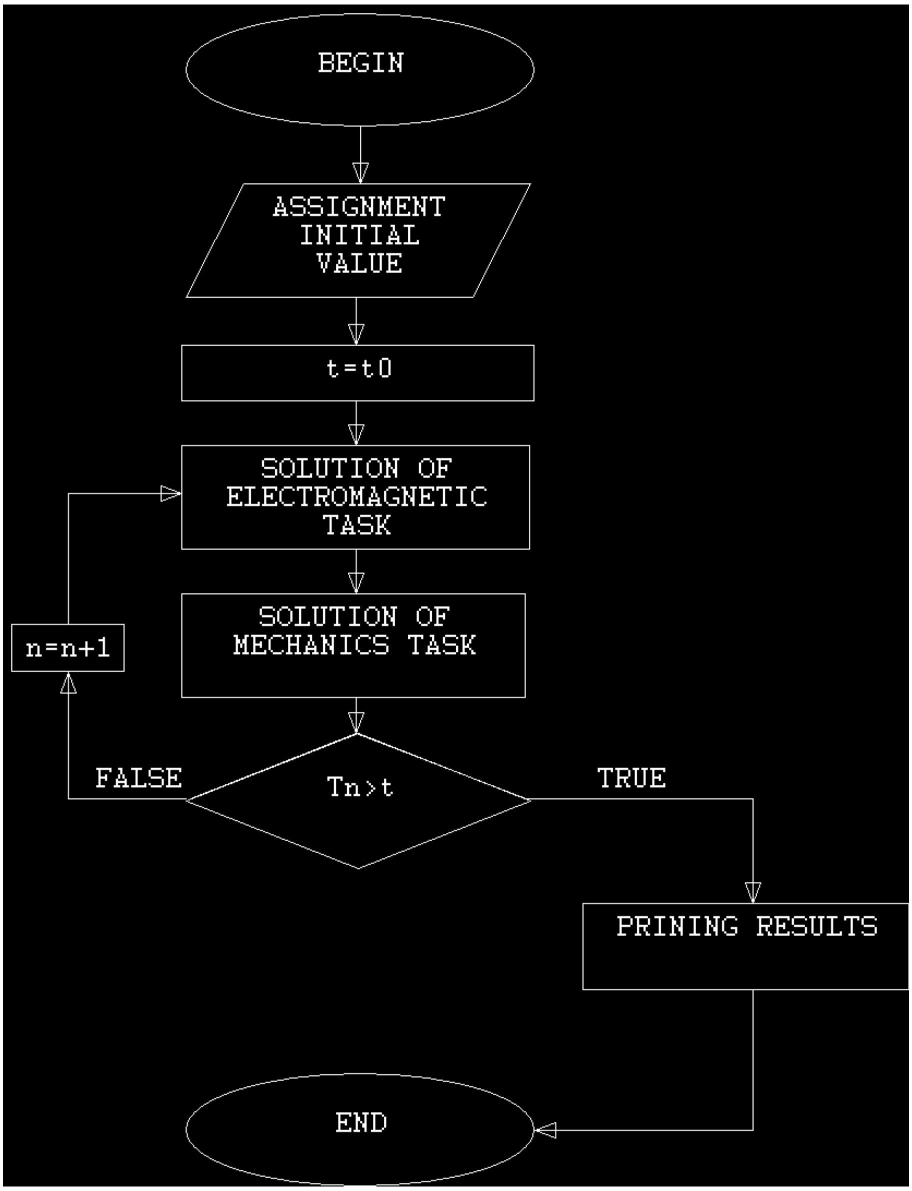

Fig. 2. Integrated bloc-scheme of nonstationary electromechanical process modeling

Here $\vec{n}_Q = n_r(Q)\vec{e}_r + n_z(Q)\vec{e}_z$ - is directed towards the division of magnetic circuits in the point Q; $\vec{e}_r$ $\vec{e}_\alpha$ $\vec{e}_z$ unit vectors of cylindrical system of axes;

$$b_r(Q,M) = \frac{z_Q - z_M}{r_Q\sqrt{(r_Q + r_M)^2 + (z_Q - z_M)^2}} \left[-K(k) + \frac{r_Q^2 + r_M^2 + (z_Q - z_M)^2}{(r_M - r_Q)^2 + (z_Q - z_M)^2} \right], \quad (2)$$

$$b_z(Q,M) = \frac{1}{r_Q\sqrt{(r_Q + r_M)^2 + (z_Q - z_M)^2}} \left[K(k) + \frac{r_M^2 - r_Q^2 + (z_Q - z_M)^2}{(r_M - r_Q)^2 + (z_Q - z_M)^2} \right], \quad (3)$$

$$\vec{b}(Q,M) = \vec{e}_r b_r(Q,M) + \vec{e}_z b_z(Q,M). \quad (4)$$

If inductor moves progressively along axes Oz with speed $\vec{V}(t) = \vec{e}_z V_z(t)$, integral-differential equations for calculation of eddy currents density in cuts of massive conductors will be next [8]:

$$\frac{\partial}{\partial t}\oint_l \sigma(M,t)T(Q,M)\,dl_M + \frac{\delta_1(Q,t)}{\gamma_1\lambda} + \frac{\mu}{\mu_0}\frac{\partial}{\partial t}\oint_D \delta(M,t)T(Q,M)\,ds_M = -\frac{\partial}{\partial t}\oint_{D_w} \delta_w(M,t)T(Q,M)\,ds_M + \Phi(Q,t),\ Q \in D_1 \quad (5)$$

$$\frac{\partial}{\partial t}\oint_l \sigma(M,t)T(Q,M)\,dl_M +$$
$$+\frac{\mu}{\mu_0}\frac{\partial}{\partial t}\oint_D \delta(M,t)T(Q,M)\,ds_M =$$
$$=-\frac{\partial}{\partial t}\oint_{D_w} \delta_w(M,t)T(Q,M)\,ds_M,\ Q\in D_2, \qquad (6)$$

where: $\lambda=\mu_0/(2\pi)$; $\Phi(Q,t)=\frac{1}{\lambda}\left(\vec{e}_\alpha\cdot\left[\vec{V}(t)\times\vec{B}(Q,t)\right]\right)=$ $=\frac{1}{\lambda}V_z(t)B_r(Q,t)$.

The induction of the magnetic field can be found in the following way:

$$\vec{B}(Q,t)=\frac{\mu}{2\pi}\int_D \delta(M,t)\vec{b}(Q,M)ds_M +$$
$$+\frac{\mu_0}{2\pi}\int_D \delta_w(M,t)\vec{b}(Q,M)ds_M +$$
$$+\frac{\mu_0}{2\pi}\int_D \sigma(M,t)\vec{b}(Q,M)ds_M,$$

where: $\vec{b}(Q,M)$ is defined using equation.

In equations (1), (5), (6) current densities in cuts of stator and inductor puttees remain unknown because it is accepted that voltage on their tips is known. Therefore, the system of equations must be completed with equations that determine the connection between puttee voltages and total current distribution in electromagnetic system. Based on the second Kirhgof's law for the chains of puttee of stator and inductor:

$$u_1=i_1(t)R_1+\frac{d\psi_1(t)}{dt},\quad u_2(t)=i_2(t)R_2+\frac{d\psi_2(t)}{dt}, \qquad (7)$$

where: u_1, $u_2(t)$ - voltage on the stator and inductor puttees; $i_1(t)$, $i_2(t)$ - currents on the puttees of the inductor and stator; $\psi_1(t)$, $\psi_2(t)$ - total linkage with inductor and stator puttees respectively, which can be found by the next way [9]:

$$\psi_1(t)=\sum_{q=1}^{N_{W_1}}\sum_{i=1}^{w_1}2\pi r_{qi}A(Q_{qi},t),$$
$$\psi_2(t)=\sum_{q=1}^{N_{W_2}}\sum_{i=1}^{w_2}2\pi r_{qi}A(Q_{qi},t), \qquad (8)$$

where: r_{qi} - is the centre radius of the i-loop and of q-reel of inductor and stator puttees for the first and second equations respectively; $A(Q_{qi},t)$ - instantaneous value of the vector potential in the centre of the i-loop and of q-reel. Integrating the equation (7) by the time we got the Volter's equations:

$$u_1(t-t_0)=R_1\int_{t_0}^{t} i_1(t)dt+\psi_1(t)-\psi_1(t_0), \qquad (9)$$
$$\int_{t_0}^{t} u_2(t)dt=R_2\int_{t_0}^{t} i_2(t)dt+\psi_2(t)-\psi_2(t_0).$$

The vector potential of the magnetic field which belongs to the equations, can be found using the relation:

$$A(Q,t)=\frac{\mu_0}{2\pi}\int_{D_W}\delta_W(M,t)T(Q,M)ds_M+$$
$$+\frac{\mu}{2\pi}\int_D \delta(M,t)T(Q,M)ds_M+$$
$$+\frac{\mu_0}{2\pi}\oint_l \sigma(M,t)T(Q,M)dl_M. \qquad (10)$$

If current densities in puttees of inductor and stator are known, the eddy current densities of massive conductors and density of current of magnetization on the magnetic circuit divisions, instantaneous electrodynamical force which affects the inductor can be found:

$$\vec{F}(t)=\int_{D_1}\vec{\delta}_1(Q,t)\times\vec{B}_2(Q,t)ds_Q+$$
$$+\int_{D_{W_1}}\vec{\delta}_{W_1}(Q,t)\times\vec{B}_2(Q,t)ds_Q+$$
$$+\int_{l_1}\vec{\sigma}(Q,t)\times\vec{B}_2(Q,t)dl_Q, \qquad (11)$$

where: $\vec{B}_2(Q,t)$ instantaneous magnetic induction, which is caused by currents in the stator puttee, eddy currents in magnetic circuit of the stator and by the currents of magnetization on its division.

THE MODELLING OF THE MECHANICAL PROCESS IN COAXIAL-LINEAR ENGINE

For the defining of position and speed of inductor in the field of electromagnetic force according to the second Newton's law the Koshy's problem is entered:

$$m\frac{d^2z}{dt^2}=-2kz-mg+F_z(t), \qquad (12)$$

$$v=\frac{dz}{dt}, \qquad (13)$$

$$z(0)=z^{(0)},\quad v(0)=v^{(0)}, \qquad (14)$$

where: m – mass of inductor; $z=z(t)$ - coordinate of inductor position as function of time t; k – coefficient of springs stiffness; $g=9{,}8$ m/s^2 - acceleration of gravity; $F_z(t)$

- instantaneous value of Z-projection of electromagnetic force (11); $v = v(t)$ - instantaneous speed of inductor motion; $z^{(0)}$, $v^{(0)}$ - initial position of the inductor and its initial speed.

The equations (12), (13) were integrated and integral-differential equation were obtained:

$$m\left(v(t)-v\left(t_0\right)\right)=-2k\int_{t_0}^{t} z(t)\,dt-mg\left(t-t_0\right)+\int_{t_0}^{t} F_z(t)dt, \quad (15)$$

$$z(t)-z\left(t_0\right)=\int_{t_0}^{t} vdt. \quad (16)$$

Knowing the initial position of the inductor, its speed on the initial moment, the law of changing of Z-projection of electromagnetic force, which can be found from solution of electromagnetic problem by the way of solving system of equations (15), (16) we can find the inductor position in any moment of time.

For numerical solution of integral-differential equation systems for finding the functions the piecewise-constant approximation can be used. First, they can be approximated by the dimensional variables with using the method of complete averaging. Then, for approximation by the time equations (5), (6) which contain fluxions by time, they can be reduced to the Volter's equations and after that integrals by time can be replaced with cubature formulas with coefficients; equation (15) is approximated analogically. After that the iterative process of solution of linked electromagnetic and mechanical problems can be organized (Fig. 2).

CONCLUSIONS

The mathematical model of linked no stationary electromagnetic and mechanical processes in coaxial-linear drive with massive magnetic circuit was worked out. The solution of electromagnetic problem was determined as a system of integral-differential equations for the density of eddy currents in the cuts of massive conductors and for currents of magnetization on the divisions of magnetic circuits with considering of inductor motion. The last one was completed with equations for chains of inductor and stator using the second Kirhgof's law. The solution of the problem of inductor motion in the field of electromagnetic force was determined as an integral-differential equation using the second Newton's law. Using the worked out mathematical model the program was made based on *FORTRAN* language and using this model an analysis of influence of material characteristics of CLE constructional elements, law of voltage changing on the draught characteristic of the engine was carried out.

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