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Investigation of parametric models of differential equations systems stability

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Abstract. We study the practical stability of systems of ordinary differential equations depending on parameters for the numerical calculation of the stability regions of the set of initial conditions and parameters considered in the given structural form. This approach can significantly extend the range of the investigated problems related to the problem of sensitivity.

Key words: practical stability, tolerances on the parameters, the perturbation parameter of the system.

INTRODUCTION

The problem of designing real dynamic objects includes a number of pressing problems in the theory of sensitivity and stability: the calculation of tolerances for parameters, guaranteed by the sensitivity of stability [1–3]. Within this framework, the stability analyses of parametric systems are encouraged to be implemented with the help of Lyapunov's second method for a finite time interval [4]. At the same time, the parameters can take arbitrary constants or variables that characterize the features of the system, including the initial conditions. In order to obtain numerical algorithms for an analysis of the initial conditions of stability and phase, restrictions are set in concrete form.

The aim is to develop algorithms for calculating the sets of initial conditions and parameters in a given structure of the methods of practical stability for parametric systems.

Statement of problems of practical stability for systems depending on parameters. Assume that the motion of the object is described by the system of ordinary differential equations of the form:

$$\frac{dx}{dt} = f\left(x,t,\alpha\right) \ ,t \in \left[t_0,T\right], \qquad (1)$$

where: $x(t,\alpha)$, α − the vectors of states and parameters of dimension n and m respectively; $f(x,t,\alpha)$ − n-dimensional vector function that satisfies the conditions of existence and uniqueness of solutions for any $\alpha \in G_\alpha$.

Definition 1. Unperturbed solution $x(t,0) = 0$ of system (1) named $\left\{G_0^x, G_0^\alpha, \Phi_t, t_0, T\right\}$ - stable if the trajectory of system (1) do not exceed permissible values for a set $\Phi_t, t \in \left[t_0,T\right]$ of initial conditions $x(t_0,\alpha)$ from the field G_0^x and arbitrary $\alpha \in G_0 \subset G_\alpha$.

At the same time $f(0,t,0) \equiv 0$, $0 \in \Phi_t, t \in [t_0,T]$.

According to the common productions of applications it is sometimes considered advisable to introduce consistent dynamical restrictions on phase coordinates and parameters $\Phi_{t,\alpha}$, $t \in [t_0,T]$ and evaluate the region $G_0^{x,\alpha}$ of initial conditions and system parameters (1).

Definition 2. Unperturbed motion $x(t,0) = 0$ of system (1) will be called $\left\{G_0^{x,\alpha}, \Phi_{t,\alpha}, t_0, T\right\}$ - stable, if $x(t)$, $\alpha \in \Phi_{t,\alpha}$, $t \in [t_0,T]$ only just $x(t_0)$, $\alpha \in G_0^{x,\alpha}$.

Let us take into account disturbing factors in the study of practical stability of the system of the form:

$$\frac{dx}{dt} = f\left(x,t,\alpha\right) + R\left(x,t,\alpha\right),\ t \in \left[t_0,T\right], \qquad (2)$$

where: permanent disturbance $R(x,t,\alpha)$ are selected from some areas Ω_R.

Definition 3. System (2) is called internally $\left\{G_0^x, G_0^\alpha, \Phi_t, t_0, T, \Omega_R\right\}$ - stable, if $x\left(t,\alpha\right) \in \Phi_t, t \in \left[t_0,T\right]$ for arbitrary perturbations of initial conditions and parameters that satisfy relations

$$R\left(x,t,\alpha\right) \in \Omega_R,\ x(t,\alpha) \in G_0^x, \alpha \in G_0^\alpha \subset G_\alpha.$$

Definition 4. System (2) is a foreign $\left\{D_0^x, G_0^\alpha, \Phi_t, t_0, T\right\}$ - stable, if there is at least one point of time $t_1 \in [t_0,T]$ for which $x(t_1,\alpha) \in \Phi_{t_1}$ for any $R(x,t,\alpha) \in \Omega_R$, $x\left(t_0,\alpha\right) \in D_0^x, D_0^x \supset \Phi_{t_0}, \alpha \in G_0^\alpha \subset G_\alpha$.

Similar definitions are introduced and relevant for the stability of system (2) in respect of restrictions on the joint state vector and parameters.

The concepts introduced in this way are sufficiently rich in content, in the sense that includes various formulations of the problems of practical stability and sensitivity, defined by all possible combinations of their constituent sets. Thus, based on Definition 1, the following tasks can be seen e.g.: to assess the known limitations $\Phi_t, t \in [t_0, T]$ of the set G_0^x and G_0^α on the sets G_0^x, G_0^α, to find the estimate $\Phi_t, t \in [t_0, T]$ and if, in particular, the set G_0^α consists of one point − to investigate the problem of $\{G_0^x, \Phi_t, t_0, T\}$- stability [2].

General theorems of practical stability for systems with perturbations. Within the above definitions, using the second method of Lyapunov and unambiguous continuously differentiable functions $V(x,t,\alpha)$, by analysis and evaluation of stability for the unknown or bounded disturbances:

$$R(x,t,\alpha) \in \Omega_R^{(1)} = \{R(x,t,\alpha) :$$
$$: \left(\int_{t_0}^{T} \left\| R(x(t,\alpha),t,\alpha) \right\|^p d\tau \right)^{\frac{1}{p}} \le \overline{R}, \alpha \in G_\alpha \Bigg\},$$

$$R(x,t,\alpha) \in \Omega_R^{(2)} = \{R(x,t,\alpha) :$$
$$: \left| R_i(x,t,\alpha) \right| \le \overline{R}_i(t,\alpha), i = 1,2,...,n, \alpha \in G_\alpha \}.$$

In particular, there are the following theorem.

Theorem 1. If the system (1) there is positive-expert procedures for variable x on $[t_0,T]$ function $V(x,t,\alpha)$ and the number $0 < \varepsilon < 1$ for which the conditions:

$$\{x: V(x,t,\alpha) \le 1\} \subset \Phi_t, t \in [t_0, T], \alpha \in G_0^\alpha,$$
$$\left(\frac{dV(x,t,\alpha)}{dt} \right)_{(1)} + \sum_{i=1}^{n} \left| \frac{\partial V(x,t,\alpha)}{\partial x_i} \right| \overline{R}_i(t,\alpha) \le 0. \qquad (3)$$

for arbitrary $x(t) \in \Phi_t / \{x: V(x,t,\alpha) < 1 - \varepsilon\}$, $t \in [t_0, T]$, $\alpha \in G_0^\alpha$,

$$G_0^x \subset \{x: V(x(t_0,\alpha),t_0,\alpha) < 1\}, \alpha \in G_0^\alpha,$$

the system (2) internally $\{G_0^x, G_0^\alpha, \Phi_t, t_0, T, \Omega_R^{(2)}\}$- rack.

Theorem 2. If the system (1) found positive-expert procedures for variable x function $V(x,t,\alpha)$ and number $A > 0$such that:

$$\{x: V(x,t,\alpha) \le 1\} \subset \Phi_t, t \in [t_0, T], \alpha \in G_0^\alpha,$$

$$D_o^x \subset \{x: V(x(t_0,\alpha),t_0,\alpha) < A\}, \alpha \in G_0^\alpha,$$

for any n-dimensional functions $\psi(t,\alpha) \in E_n / \Phi_t, t \in [t_0, T]$, $\psi(t_0,\alpha) \in D_0^x$, $\alpha \in G_0^\alpha$ the inequality:

$$\int_{t_0}^{t} \left[\frac{\partial V(\psi(\tau,\alpha),\tau,\alpha)}{\partial \tau} + \right.$$
$$\left. + grad_x^* V(\psi(\tau,\alpha),\tau,\alpha) f(\psi(\tau,\alpha),\tau,\alpha) \right] d\tau +$$
$$+ \left(\int_{t_0}^{t} \left\| grad_x^* V(\psi(\tau,\alpha),\tau,\alpha) \right\|^q d\tau \right)^{1/q} \cdot$$
$$\cdot \overline{R} < 1 - A,$$

the system (2) external $\{D_0^x, G_0^\alpha, \Phi_t, t_0, T, \Omega_R^{(1)}\}$ - bar.

Theorem 3. If the system (1) there is positive-expert procedures on $[t_0,T]$ a function $V(x,t,\alpha)$ that satisfies:

$$\{x, \alpha: V(x,t,\alpha) \le 1\} \subset \Phi_{t,\alpha}, t \in [t_0, T],$$

(3) when $x(t), \alpha \in \Phi_{t,\alpha} / \{x, \alpha: V(x,t,\alpha) < 1 - \varepsilon\}$, $t \in [t_0, T]$,

$$G_0^{x,\alpha} \subset \{x, \alpha: V(x(t_0,\alpha),t_0,\alpha) < 1\},$$

the system (2) is $\{G_0^{x,\alpha}, \Phi_{t,\alpha}, t_0, T, \Omega_R^{(2)}\}$ - internally stable.

Algorithms for calculating areas of practical stability.

Consider the linear system:

$$\frac{dx}{dt} = A(t)x + G(t)\alpha + f(t), t \in [t_0, T], \qquad (4)$$

where permanent disturbance $f(t)$ is assumed to be known or unknown, but bounded by the norm.

Let $G_0^x = \{x: W(x) < 1\}, G_0^y = \{y: W(y) < 1\}$, where $y = \begin{pmatrix} x \\ \alpha \end{pmatrix}$, and $W(x)$, $W(y)$ − positive-definite functions which are closed surface level $W(x) = 1$, $W(y) = 1$.

Proposition 1. For $\{G_0^x, G_0^\alpha, \Phi_t, t_0, T\}$- system stability (4) of known perturbations and should be enough to have made the ratio:

$$\{x: W[X(t_0,t)(x(t,\alpha) - G_1(t)\alpha - a(t))] < 1\}$$
$$\subset \Phi_t, t \in [t_0, T], \alpha \in G_0^\alpha.$$

Here $G_1(t) = \int_{t_0}^{t} X(t,\tau) G(\tau) d\tau$; $a(t) = \int_{t_0}^{t} X(t,\tau) f(\tau) d\tau$; $X(t,t_0)$ − normalized by moment t_0 fundamental matrix of solutions of homogeneous system (4) at $\alpha = 0$:

$$\frac{dX(t,t_0)}{dt} = A(t)X(t,t_0), X(t_0,t_0) = E. \qquad (5)$$

Proposition 2. For system (4) was $\{G_0^{x,\alpha}, \Phi_{t,\alpha}, t_0, T\}$ - stand at the set $f(t)$ and should be enough to:

$$\{y: W[Y(t_0,t)(y(t) - \tilde{a}(t))] < 1\} \subset \Phi_{t,\alpha}, t \in [t_0, T],$$

where: $Y(t,t_0)=\begin{pmatrix} X(t,t_0) & G_1(t) \\ 0 & E \end{pmatrix}$,

$$\tilde{a}(t)=\int_{t_0}^{t} Y(t,\tau)\tilde{f}(\tau)d\tau,\ \tilde{f}(t)=\begin{pmatrix} f(t) \\ 0 \end{pmatrix}. \quad (6)$$

Based on the above general statements of theorems and algorithms developed constructive evaluation of firmness of the initial sets $G_0^x = \{x \mid x^*Bx \le c^2\}, G_0^\alpha = \{\alpha \mid \alpha^* B_\alpha \alpha \le c_\alpha^2\}, G_0^{x,\alpha} = \{x,\alpha \mid x^*Bx+\alpha^* B_\alpha \alpha \le c^2\}$ when specific constraints on phase coordinates and parameters [2, 3]:

$$\Phi_t=\Gamma_t=\left\{x:\left|l_s^*(t)x\right|\le 1, s=1,2,...N\right\},\ t\in[t_0,T],$$

$$\Phi_t=\Psi_t=\{x:\psi(x,t)\le 1\}, t\in[t_0,T],$$

$$\Phi_{t,\alpha}=\Gamma_{t,\alpha}=\left\{x,\alpha:\left|l_s^*(t)x+m_s^*\alpha\right|\le 1, s=1,2,...N\right\},\quad t\in[t_0,T],$$

$$\Phi_{t,\alpha}=\Psi_{t,\alpha}=\{x,\alpha:\psi(x,t,\alpha)\le 1\}, t\in[t_0,T].$$

This region of initial conditions $G_0^x, G_0^{x,\alpha}$ are evaluated numerically.

So, to system (4) was internally $\{c,B,c_\alpha,B_\alpha,\Gamma_t,t_0,T\}$ - stable in the presence of known perturbations, necessary and sufficient that inequality was done:

$$c^2 \le \min_{t\in[t_0,T]} \cdot \min_{s=1,2,...,N} \cdot \min_{\alpha\in G_0^\alpha} \frac{\left(1-\left|l_s^*(t)\left(a(t)+G_1(t)\alpha\right)\right|\right)^2}{l_s^*(t)Q^{-1}(t)l_s(t)},$$

$$\left|l_s^*(t)\left(a(t)+G_1(t)\alpha\right)\right|<1,\ s=1,2,...,N,\quad \alpha\in G_0^\alpha,\ t\in[t_0,T]$$

$$(Q^{-1}(t)=X(t,t_0)B^{-1}X^*(t,t_0)).$$

Terms and foreign $\{c,B,c_\alpha,B_\alpha,\Gamma_t,t_0,T\}$ - and $\{c,B,c_\alpha,B_\alpha\Psi_t,t_0,T\}$ - stability under unknown disturbance can be represented respectively as follows:

$$c^2 \le \max_{t\in[t_0,T]} \cdot \min_{s=1,2,...,N} \cdot \frac{\left(1-a_s(t)\right)^2}{l_s^*(t)Q_1(t)l_s(t)+m_s^*B_\alpha^{-1}m_s+2l_s^*(t)G_1(t)B_\alpha^{-1}m_s},$$

$$c^2 \le \max_{t\in[t_0,T]} \cdot \min_{\bar{x}\in\Psi_t'} \cdot \min_{\alpha\in G_0^\alpha} \frac{\left(g^*(\bar{x},t)\left(\bar{x}-G_1(t)\alpha-a_{\bar{x}}(t)\right)\right)^2}{g^*(\bar{x},t)Q^{-1}(t)g(\bar{x},t)} \quad (7)$$

$$(Q_1(t)=X(t,t_0)B^{-1}X^*(t,t_0)+G_1(t)\cdot B_\alpha^{-1}\cdot G_1^*(t)).$$

Similar relations can lead to joint dynamic limits $\Gamma_{t,\alpha}$, $\Psi_{t,\alpha}$, $[t_0, T]$ for various types of disturbances.

You can also explore the system (4) for the case when the initial conditions, continuous disturbance and parameters selected from the region

$$S_c(t)=\left\{x(t_0),\alpha,f(t):\ x^*(t_0)Bx(t_0)+ +\alpha^*B_\alpha\alpha+\int_{t_0}^{t} f^*(\tau)G_1(t)f(\tau)d\tau\le c^2\right\}. \quad (8)$$

Thus, the criterion $\{S_c(t), \Gamma_t, t_0, T\}$ - stability takes the form [2]

$$c^2 \le \min_{t\in[t_0,T]} \cdot \min_{s=1,2,...,N} \cdot \frac{1}{l_s^*(t)R(t)l_s(t)}, \quad (9)$$

where: the symmetric matrix $R(t)$ satisfies by Cauchy problem

$$\frac{dR(t)}{dt}=A(t)R(t)+R(t)A^*(t)+G_1^{-1}(t)+ +G(t)B_\alpha^{-1}G^*(t), R(t_0)=B^{-1}. \quad (10)$$

As part of this trend can be seen a number of problems associated with the construction of regulatory actions that would ensure the sustainability of the system of the specified type [3]. So, using the concept of sustainability in the direction, upper bounds areas are obtained of the initial conditions for the vector of states and parameters for specific restrictions on phase coordinates.

The above formulation allows for a unified position – exploring the various practical stability in the nature of the problem of sensitivity and solving them numerically, to conduct a comprehensive analysis of parametric systems.

CONCLUSIONS

On the basis of the general theorems, numerical algorithms were developed for calculation of stability regions for systems with parametric perturbations. A number of pressing problems were considered in the theory of sensitivity associated with the design of real dynamic objects, including the formulation of A.

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