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Analysis of three-phases matrix reactance frequency converter with two pulsations of control signal

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Abstract: This paper presents a method of calculation of steady-state processes in three-phases matrix-reactance frequency converters (MRFC's), in which voltages and currents are transformed by control signals with two pulsations. A solution of nonstationary differential equations with periodic coefficients that describe this system is obtained by using Galerkin's method and an extension of equations of one variable of time to equations of two variables of time. The results of calculations are presented in an example of three-phases MRFC with buck-boost topology and compared with a numerical method embedded in the program Mathematica.

Key words: matrix-reactance frequency converter, non-stationary periodic differential equations, Galerkin's method, extension of differential equations

1. Introduction

The ability to control alternating current (AC) voltages is one of the most desirable features of AC systems. Frequency converters used to control AC voltages can be built on structures of a matrix converters (MC) or matrix-reactance chopper (MRC). These systems allow to control both frequency and magnitude of an output voltage [1-3]. Mathematical models of converters are widely used to analyze the theoretical properties of such systems. MRFC are based on unipolar MRC structures. Each unipolar MRC has two synchronous-connected switch sets (Fig. 1). In this way we obtain the ability to change the frequency and magnitude of output voltages.

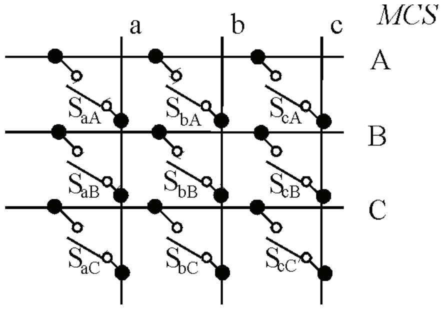

Fig.1. Three-phase matrix frequency converter

The description of the MRFC control strategy is presented in Fig. 2.

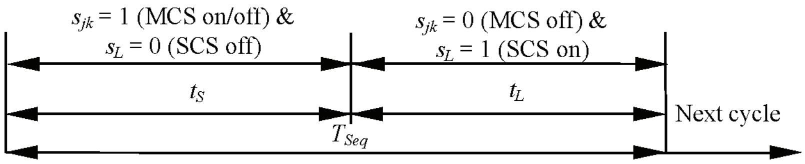

Fig. 2. General form of the control strategy

The switching functions s_{jk} are defined in such a way that $s_{jk} = 1$ when the switch is turned on and $s_{jK} = 0$ when the switch is turned off, where $j = \{a, b, c\}$, $K = \{A, B, C\}$. In each sequence period T_{Seq} there are two time intervals, t_s and t_L. In the interval t_s the synchronous connected switches (SCS) are off, whereas matrix connected switches (MCS) operate in accordance with a control strategy.

The main aim of this paper is to present a method for steady-state process analysis in circuits of three-phases MRFC, in which a transformation of voltages and currents is realised with the use of a low-frequency transfer matrix $\mathbf{M}(t)$, dependent on two different pulsations. The steady-state process in a circuit of MRFC is calculated by using the Galerkin method [5] and by extending a differential equation of one variable of time in the differential equation in two variables of time [6]. The solution is obtained assuming an incommensurable ratio between pulsations of the input and output voltages. The results of calculations are obtained on an example of MRFC with buck-boost topology and compared with a numerical method.

2. The control strategy

In line with Venturini control strategy [4] input and output currents and voltages in MRFC systems are transformed by means of the low-frequency transfer matrix $\mathbf{M}(t)$:

$$\mathbf{M}(t) = \begin{bmatrix} d_{aA} & d_{aB} & d_{aC} \\ d_{bA} & d_{bB} & d_{bC} \\ d_{cA} & d_{cB} & d_{cC} \end{bmatrix}, \tag{1}$$

where:

$$d_{aA} = d_{bB} = d_{cC} = (1 - D_S)[1 + 2q\cos(\omega_m t)]/3,$$

$$d_{aB} = d_{cA} = d_{bC} = (1 - D_S)[1 + 2q\cos(\omega_m t - 2\pi/3)]/3,$$

$$d_{aC} = d_{bA} = d_{cB} = (1 - D_S)[1 + 2q\cos(\omega_m t - 4\pi/3)]/3,$$

$D_S = t_S/T_{Seq}$ is a pulse duty factor, $\omega_m = \omega_L - \omega$, ω, ω_L are pulsations of the supply and load voltages, q is a voltage gain.

Let us calculate steady-state processes in which case of the matrix $\mathbf{M}(t)$ is presented in the form of the sum of matrices:

$$\mathbf{M}(t) = \mathbf{M}_{\omega_m}(t) + \mathbf{M}_{\omega_\sigma}(t), \tag{2}$$

where $\mathbf{M}_{\omega_m}(t)$ is the matrix with the pulsation ω_m, whose coefficients are the same as in (1), and the matrix $\mathbf{M}_{\omega_\sigma}(t)$ with a pulsation ω_σ has the form:

$$\mathbf{M}_{\omega_\sigma} = \begin{bmatrix} d'_{aA} & d'_{aB} & d'_{aC} \\ d'_{bA} & d'_{bB} & d'_{bC} \\ d'_{cA} & d'_{cB} & d'_{cC} \end{bmatrix},$$

$$d'_{aA} = d'_{bB} = d'_{cC} = (1 - D_S)[1 + 2q' \cos(\omega_\sigma t)]/3,$$

$$d'_{aB} = d'_{cA} = d'_{bC} = (1 - D_S)[1 + 2q' \cos(\omega_\sigma t - 2\pi/3)]/3,$$

$$d'_{aC} = d'_{bA} = d'_{cB} = (1 - D_S)[1 + 2q' \cos(\omega_\sigma t - 4\pi/3)]/3,$$

where $\omega_\sigma = i\omega_L - \omega$, $i = 2, 3, .., q'$ is a voltage gain.

Then input and output currents and voltages are transformed by the transfer matrix $\mathbf{M}(t)$ that depends on two different pulsation and coefficients of this matrix take the form:

$$d_{aA} = d_{bB} = d_{cC} = 2(1 - D_S)\ [1 + q \cos(\omega_m t) + q' \cos(\omega_\sigma t)]/3,$$

$$d_{aB} = d_{cA} = d_{bC} = 2(1 - D_S),\ [1 + q \cos(\omega_m t - 2\pi/3) + q' \cos(\omega_\sigma t - 2\pi/3)]/3,$$

$$d_{aC} = d_{bA} = d_{cB} = 2(1 - D_S),\ [1 + q \cos(\omega_m t - 4\pi/3) + q' \cos(\omega_\sigma t - 4\pi/3)]/3.$$

It should be noted that, as in the simpler case of the transfer matrix (1), matrix (2) gives the possibility to change the frequency and magnitude of the output voltage in MRFC. Studies performed in [7] have shown that in the case of control strategies with the sum of two different pulsations (2) the input current in the first phase has the form:

$$I_1(t) = E\left\{\frac{q}{Z}\cos(\omega t + \delta) + \frac{q'}{Z_S}\cos(\omega_S t + \delta_S) + \right.$$

$$\left. + \frac{q}{Z}\cos[(\omega + \omega_L - \omega_S)t + \delta] + \frac{q'}{Z_S}\cos[(\omega + \omega_S - \omega_L)t + \delta_S]\right\},$$

where $Z = Z(\omega_L)$ is a load impedance for the pulsation ω_L, $Z_S = Z(\omega_S)$ is a load impedance for the pulsation ω_S, E is amplitude of the input voltage, δ is a phase shift between the output voltage and current for the pulsation ω_L, δ_S is a phase shift between output voltage and currents for the pulsation ω_S. Input currents in the second and third phases are shifted with respect to each other by the angle of $2\pi/3$:

$$I_2(t) = I_1(t + 2\pi/3),\ \ I_3(t) = I_1(t + 4\pi/3).$$

So, currents in such a MRFC have additional pulsations $\omega + \omega_L - \omega_m$ and $\omega + \omega_\sigma - \omega_m$. In that case one can not use Fourier series to solve differential equations, since additional pulsations might be incommensurable. The expansion of differential equations of one variable of time in the differential equation in two variables of time allows to state the problem in form when double Fourier series could be used. Then the Galerkin method is applied to find the solution of the extended differential equations. Using of weight functions in the trigonometric form permits to find a steady-state solution as double Fourier series without a calculation of transient processes.

3. Mathematical model

Let's consider the three-phase MRFC with the buck-boost topology shown in Figure 3. Processes in such a converter are described by nonstationary differential equations with periodic coefficients

$$\frac{d\mathbf{X}(t)}{dt} = \mathbf{A}(t)\mathbf{X}(t) + \mathbf{B}(t), \tag{3}$$

where $\mathbf{X}(t)$ is a vector of state variables and $\mathbf{A}(t)$, $\mathbf{B}(t)$ are a matrix and a vector of the converter.

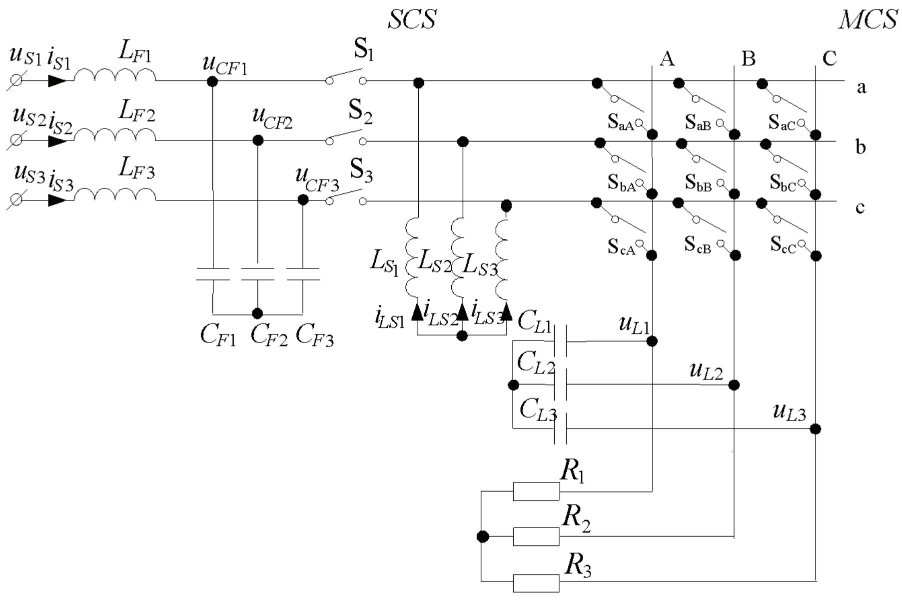

Fig. 3. MRFC based on buck-boost topology

On the basis of the averaged state-space method [8] processes in MRFC are described by the Equation (3) in which the vector of state variables $\mathbf{X}(t)$, vector $\mathbf{B}(t)$ and matrix $\mathbf{A}(t)$ have the form.

$$\mathbf{A}(t) =$$

$$= \begin{pmatrix}
\frac{-R_{F1}}{L_{F1}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F1}} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-R_{F2}}{L_{F2}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F2}} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{-R_{F3}}{L_{F3}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{L_{F3}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-R_{S1}}{L_{S1}} & 0 & 0 & \frac{1-D_S}{L_{S1}} & 0 & 0 & \frac{d_{aA}}{L_{S1}} & \frac{d_{aB}}{L_{S1}} & \frac{d_{aC}}{L_{S1}} \\
0 & 0 & 0 & 0 & \frac{-R_{S2}}{L_{S2}} & 0 & 0 & \frac{1-D_S}{L_{S2}} & 0 & \frac{d_{bA}}{L_{S2}} & \frac{d_{bB}}{L_{S2}} & \frac{ds_{bC}}{L_{S2}} \\
0 & 0 & 0 & 0 & 0 & \frac{-R_{S3}}{L_{S3}} & 0 & 0 & \frac{1-D_S}{L_{S3}} & \frac{d_{cA}}{L_{S3}} & \frac{d_{cB}}{L_{S3}} & \frac{d_{cC}}{L_{S3}} \\
\frac{1}{C_{F1}} & 0 & 0 & \frac{D_S-1}{C_{F1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{C_{F2}} & 0 & 0 & \frac{D_S-1}{C_{F2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{C_{F3}} & 0 & 0 & \frac{D_S-1}{C_{F3}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{-d_{aA}}{C_{L1}} & \frac{-d_{bA}}{C_{L1}} & \frac{-d_{cA}}{C_{L1}} & 0 & 0 & 0 & \frac{-1}{R_1 C_{L1}} & 0 & 0 \\
0 & 0 & 0 & \frac{-d_{aB}}{C_{L2}} & \frac{-d_{bB}}{C_{L2}} & \frac{-d_{cB}}{C_{L2}} & 0 & 0 & 0 & 0 & \frac{-1}{R_2 C_{L2}} & 0 \\
0 & 0 & 0 & \frac{-d_{aC}}{C_{L3}} & \frac{-d_{bC}}{C_{L3}} & \frac{-d_{cC}}{C_{L3}} & 0 & 0 & 0 & 0 & 0 & \frac{-1}{R_3 C_{L3}}
\end{pmatrix}$$

$$\mathbf{X}(t) = [i_{S1},\ i_{S2},\ i_{S3}, i_{LS1},\ i_{LS2},\ i_{LS3}, u_{CF1},\ u_{CF2}, u_{CF3}, u_{L1}, u_{L2}, u_{L3}]^T,$$

$$\mathbf{B}^T(t) = \left[\frac{U_1\cos(\omega t)}{L_{F1}}, \frac{U_2\cos(\omega t + 2\pi/3)}{L_{F2}},\ \frac{U_3\cos(\omega t + 4\pi/3)}{L_{F3}}, 0, 0, 0, 0, 0, 0, 0, 0, 0\right].$$

4. Method of calculation

Analysis of steady-state processes in MRFC is based on solving nonstationary differential Equations (3). The proposed method uses the Galerkin's method and the double Fourier series [9]. The method is also based on the extension of the Equation (3) for two variables of time. This extension is founded on the assumption that the control signals have different pulsations and there periods are incommensurable.

Approximation of the solution of (3) by the Galerkin's method consists in introducing the residuum defined as

$$\mathbf{R}_{\mathbf{X}(t)} = \frac{d\mathbf{X}(t)}{dt} - \mathbf{A}(t)\mathbf{X}(t) - \mathbf{B}(t), \tag{4}$$

on the interval $0 \le t \le T$, where $T = 2\pi/\omega$. In the residue (4) the vector of state variables $\mathbf{X}(t)$ is replaced by a new vector written in the form of a series with respect to weight functions. Next, one calculates integrals of the product of the residue (4) and weight functions. As a result we obtain a linear system of equations, whose coefficient are solution of equation (3) [10].

The MRFC is controlled by signals with different pulsation. In this case it is reasonable to introduce a second variable of time τ corresponding to the second pulsation [11]. Then Equation (3) with two independent variables of time t and τ takes the form:

$$\frac{\partial \mathbf{X}(t,\tau)}{\partial t} + \frac{\partial \mathbf{X}(t,\tau)}{\partial \tau} = \mathbf{A}(t,\tau)\mathbf{X}(t,\tau) + \mathbf{B}(t). \tag{5}$$

In the system (5) matrix $\mathbf{A}(t,\ \tau)$ corresponds to the matrix $\mathbf{A}(t)$, in which the variables of time at different pulsations are different. Then the residue (4) should be transformed into the expression with two variables of time:

$$\mathbf{R}_{\mathbf{X}(t,\tau)} = \frac{\partial \mathbf{X}(t,\tau)}{\partial t} + \frac{\partial \mathbf{X}(t,\tau)}{\partial \tau} - \mathbf{A}(t,\tau)\mathbf{X}(t,\tau) - \mathbf{B}(t) \tag{6}$$

on the intervals $0 \le t \le T$, $0 \le \tau \le \Theta$, $\Theta = 2\pi/\tilde{\omega}_L$ In the expression (6) Θ defines a period of the load voltage, and T a period of the supply voltage.

In order to find a steady-state solution of (5) the weight functions are defined (7) in the trigonometric form:

$$\begin{aligned} \phi_{nk}(t,\tau) &= \sin(n\omega t)\sin(k\omega_L \tau), \\ \psi_{nk}(t,\tau) &= \sin(n\omega t)\cos(k\omega_L \tau), \\ \theta_{nk}(t,\tau) &= \cos(n\omega t)\sin(k\omega_L \tau), \\ \xi_{nk}(t,\tau) &= \cos(n\omega t)\cos(k\omega_L \tau)\,, \end{aligned} \tag{7}$$

where $n = 0, 1, 2, \ldots, k = 0, 1, 2, \ldots$

The solution describing the steady-state process in the space of two variables of time is periodic, so approximation of this solution can be obtained by means of periodic functions. For this purpose, we replace the vector $\mathbf{X}(t,\tau)$ by a vector $\tilde{\mathbf{X}}(t,\tau)$, whose components are described in the form of double Fourier series (8):

$$\begin{aligned} \tilde{x}^{(z)}(t,\tau) = \sum_{k=0}^{N}\sum_{n=0}^{N} [a_{kn}^{(z)}\phi_{kn}(t,\tau) + \\ + b_{kn}^{(z)}\psi_{kn}(t,\tau) + c_{kn}^{(z)}\theta_{kn}(t,\tau) + d_{kn}^{(z)}\xi_{kn}(t,\tau)], \end{aligned} \tag{8}$$

where

$$a_{kn}^{(z)}, b_{kn}^{(z)}, c_{kn}^{(z)}, d_{kn}^{(z)} \tag{9}$$

are coefficients of double Fourier series for the voltages and currents and z = 1, ..., 12 correspond to the elements of the vector $\mathbf{X}(t, \tau)$. The Galerkin method is based on determining the coefficients (9) so that the residue (6) with the new vector of state variables $\widetilde{\mathbf{X}}(t, \tau)$ is as small as possible. For this purpose, we multiply the residue (6) by the weight functions (7) and integrate the obtained expressions on the intervals $0 \le t \le T$, $0 \le \tau \le \Theta$:

$$\begin{aligned} &\int_0^T\int_0^\Theta \phi_{kn}(t,\tau)\mathbf{R}_{\widetilde{\mathbf{X}}(t,\tau)}d\tau dt = 0, \quad \int_0^T\int_0^\Theta \psi_{kn}(t,\tau)\mathbf{R}_{\widetilde{\mathbf{X}}(t,\tau)}d\tau dt = 0, \\ &\int_0^T\int_0^\Theta \theta_{kn}(t,\tau)\mathbf{R}_{\widetilde{\mathbf{X}}(t,\tau)}d\tau dt = 0, \quad \int_0^T\int_0^\Theta \xi_{kn}(t,\tau)\mathbf{R}_{\widetilde{\mathbf{X}}(t,\tau)}d\tau dt = 0. \end{aligned} \tag{10}$$

Having calculated the integrals (10), we obtain a linear system of equations. The solution of these equations determines the value of coefficients of double Fourier series (9).

The proposed method allows obtaining an analytic solution in the form of the linear system of equations. The solution of this system is realized after setting numerical values of circuit parameters.

5. Calculation and simulation test results

The steady-state solution for the MRFC is obtained by using Mathematica, assuming that the system is symmetrical and taking into account the following parameters of the circuit:

$$\begin{gathered} R_{S1} = R_{S2} = R_{S3} = R_{F1} = R_{F2} = R_{F3} = 0.1\ \Omega, \\ L_{S1} = L_{S2} = L_{S3} = L_{F1} = L_{F2} = L_{F3} = 100\ \text{mH}, \\ C_{F1} = C_{F2} = C_{F3} = C_{L1} = C_{L2} = C_{L3} = 100\ \mu\text{F}, \\ R_1 = R_2 = R_3 = 10\ \Omega,\ U_1 = U_2 = U_3 = 50\ \text{V}, \\ \omega_L = 2\pi / \Theta = 400\ \text{rad/s},\ \omega = 2\pi/\text{T} = 250\ \text{rad/s}, \\ D_S = 0.3,\ q = 0.4,\ q' = 0.15. \end{gathered}$$

The graphs of steady-state processes of the current in the inductor L_{F1} and voltage across the capacitor C_{L1} for two periods $0 \le t \le 2T$, $0 \le \tau \le 2\Theta$ are shown in Figures 4 and 5 respectively. Double Fourier series coefficients are calculated for $N = 4$.

As one can see the calculated processes are periodic in the space of two variable of time.

To verify the obtained results the steady-state process at $t = \tau$ is calculated. The obtained results are compared with results obtained by the numerical method embedded in Mathematica. For this purpose, NDSolve standard function is used. The time waveforms of the current in the inductor L_{F1} and voltage across the capacitor C_{L1} are presented and compared for different N in Figures 6 and 7.

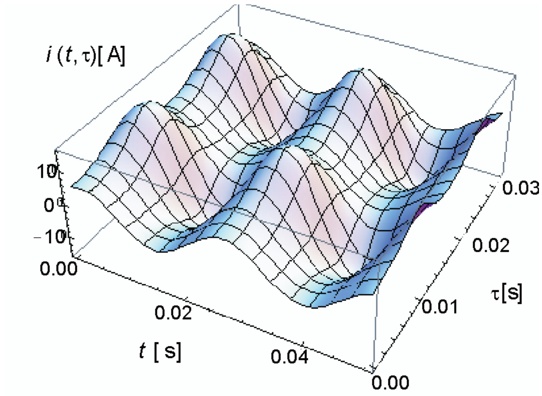

Fig. 4. The steady-state process of the current in the inductor L_{F1}

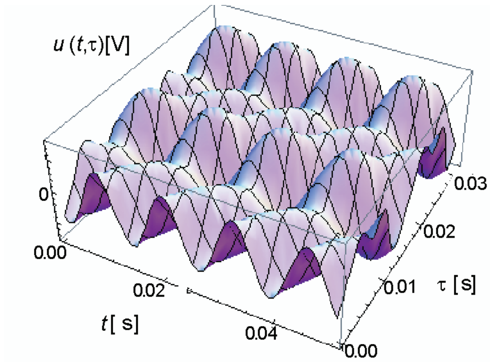

Fig. 5. The steady-state process of the voltage across the capacitor C_{L1}

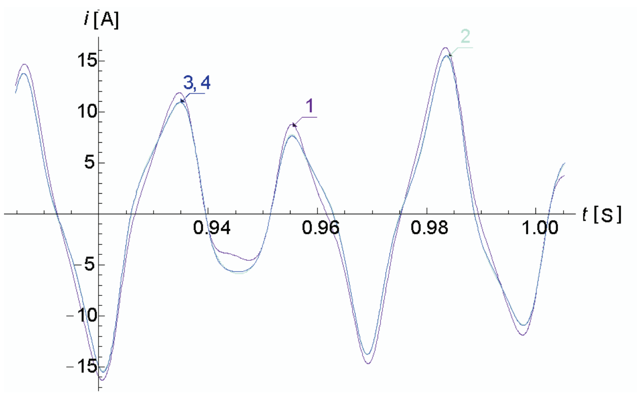

Fig. 6. The steady-state current in the inductor L_{F1}: 1) $N = 2$, 2) $N = 3$, 3) $N = 4$ – described method, 4) numerical method, curves 3 and 4 coincide

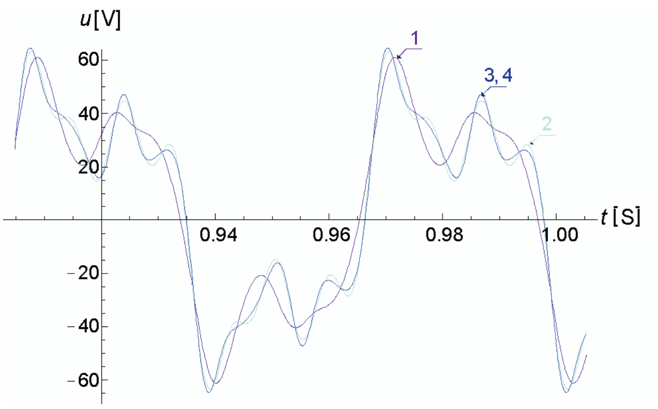

Fig. 7. The steady-state voltage across the capacitor C_{L1}: 1) $N = 2$, 2) $N = 3$, 3) $N = 4$ – described method, 4) numerical method, curves 3 and 4 coincide

One can see that the results obtained by the proposed method (curve 3) are completely coincide with results obtained by the numerical method (curve 4), in case that the number of approximation function $N = 4$.

The steady-state currents and voltages in three- phase of MFRC are presented correspondingly in Figures 8, 9 and in Figures 10, 11.

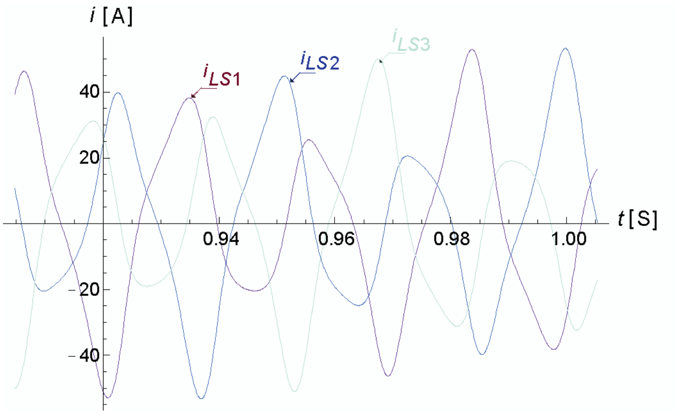

Fig. 8. The steady-state currents in the inductors L_{F1}, L_{F2}, L_{F3}

One can see that in the space of one variable of time calculated processes are not periodic. It should be noted that the proposed method allows to omit the calculation of a transient process. In this case computational errors could be reduced. In Figures 12 and 13 there are shown examples of waveforms of transient and steady state processes for the current in the inductor L_{F1} and voltage across the capacitor C_{L1} obtained by numerical and proposed methods.

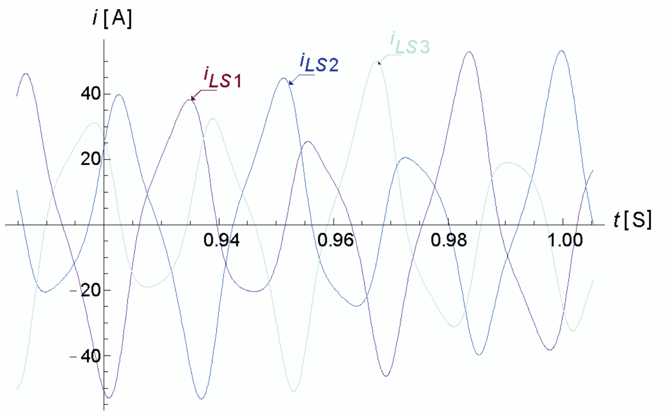

Fig. 9. The steady-state currents in the inductors L_{S1}, L_{S2}, L_{S3}

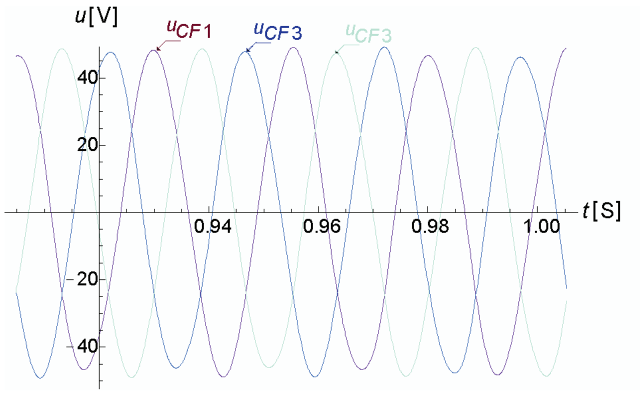

Fig. 10. The steady-state voltages across the capacitors C_{F1}, C_{F2}, C_{F3}

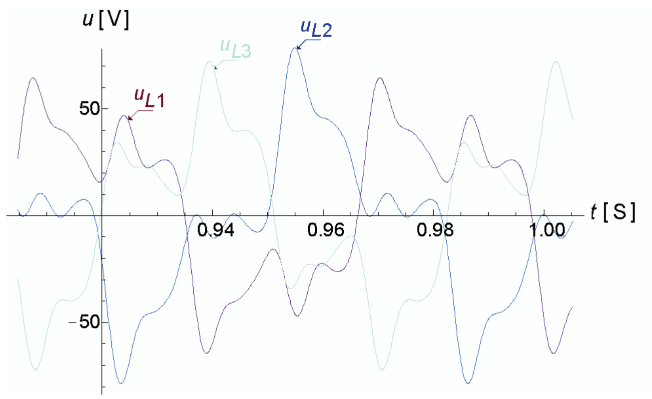

Fig. 11. The steady-state voltages across the capacitors C_{L1}, C_{L2}, C_{L3}

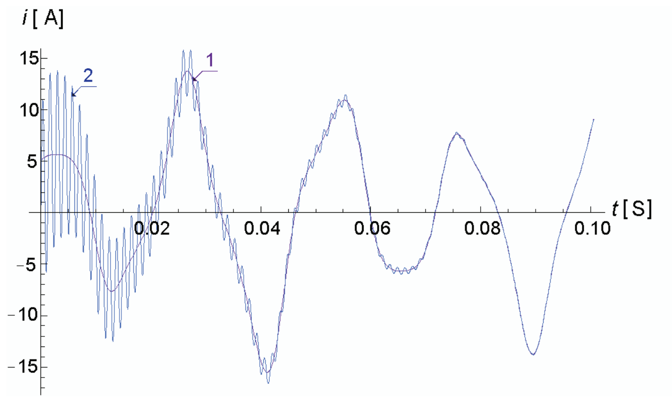

Fig. 12. The transient and steady-state currents in the inductor L_{F1}: 1) described method, 2) numerical method

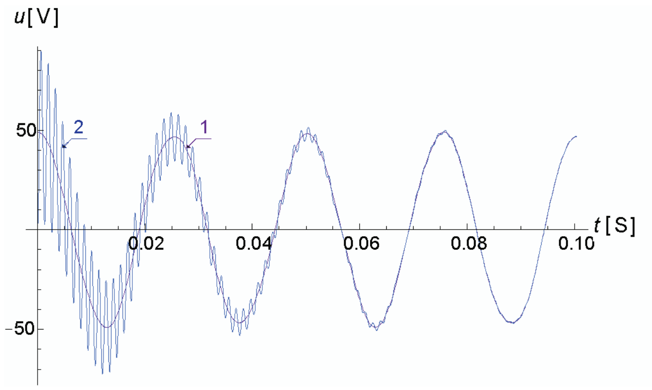

Fig. 13. The transient and steady state voltages across the capacitors C_{L1}: 1) described method, 2) numerical method

It should also be noted that unlike the method of ordinary Fourier series [10], where each variable is represented by one component, in the proposed method with double Fourier series each variable is represented by four components. In Figure 14 the coefficients of double Fourier series for the voltage u_{L1} for first, second, third and fourth harmonic are shown.

It follows from Figure 14. that the distribution of harmonics is asymmetrical.

6. Conclusions

The presented analytical-numerical method is used to determine the processes in three-phase MRFC's, described by nonstationary periodic differential equations with different pulsations.

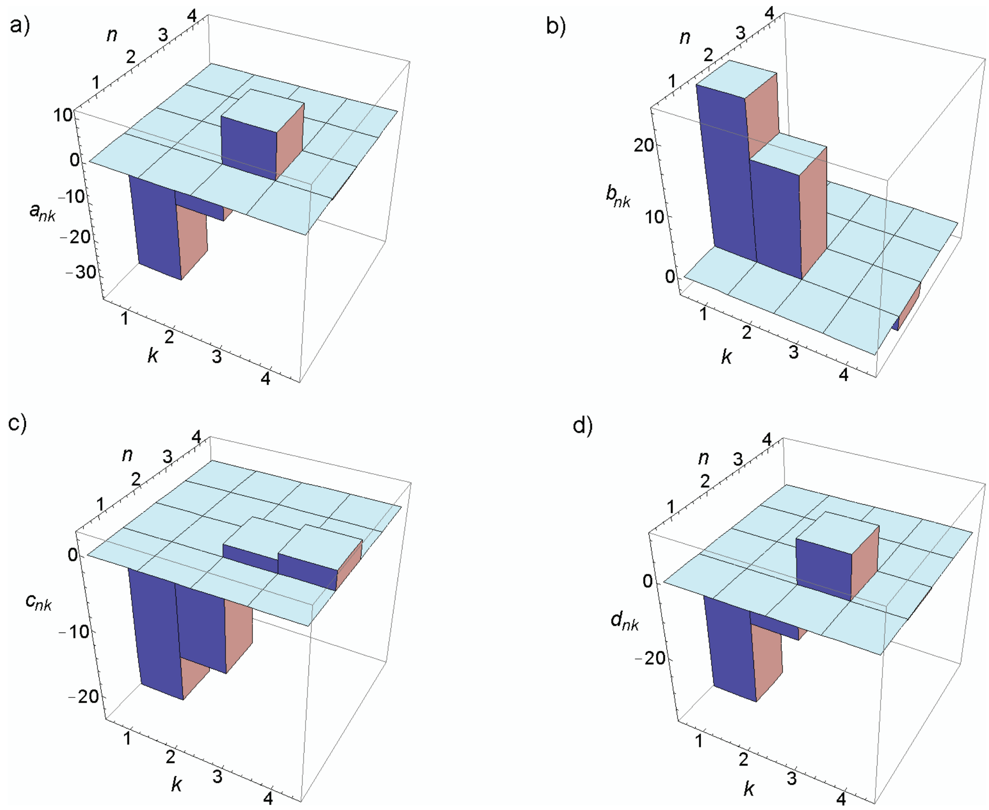

Fig. 14. Fourier series coefficient a) a_{nk}, b) b_{nk}, c) c_{nk}, d) d_{nk} for voltage u_{L1}, $n, k = \{1, 2, 3, 4\}$

In order to find a steady-state solution, differential equations are extended by additional, independent variables of time, corresponding to different pulsations. The solution is obtained by the Galerkin method in form of double Fourier series.

The accuracy of the method depends on the number of approximation functions. Comparative calculations in the MRFC with buck-boost topology have been performed, using the proposed method and numerical method. The obtained results confirm the correspondence between the proposed and numerical methods.

The calculations show that in the space of one variable of time the steady-state processes are not periodic but in space of two variable of time the steady-state processes are periodic. The proposed method allows calculating steady-state processes without the calculation of transient processes.

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