

An improved calculation method of inductance and capacitances in π 1 circuits for resonant power amplifiers*

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Abstract: In the paper an improved method of calculation of the inductance and capacitances in the π 1 circuit for Class A, AB, B, and C resonant power amplifiers is presented. This method is based on an assumption that the quality factor of the inductor is finite and the capacitors are lossless. The input parameters for calculations are the amplifier load resistance, the transistor load resistance, the quality factor of the inductor, the loaded quality factor of the designed circuit, and the operating frequency. The presented method allows reducing the required regulation range of π 1 circuits elements in built resonant amplifiers as compared to the traditional calculation methods assuming lossless capacitors and inductor. This advantage is important, in particular, for long- and medium-wave transistor power amplifiers, where capacitances in π 1 circuits are high comparing to typical trimming capacitors.

Key words: resonant-circuit design, impedance transformation, inductor quality factor, loaded quality factor, Class A, Class AB, Class B, Class C

1. Introduction

In most transmitters for radio-communication and broadcasting systems a resonant amplifier is used as a final stage of h.f. (high frequency) signal amplifying set. In h.f. resonant power amplifiers the output network (Fig. 1) (or both, the input and the output networks) is selective, which reduces harmonic distortions of the output signal. In Class A, AB, B, and C resonant amplifiers the loaded output network at the input terminals should act as a parallel-resonant circuit. Hence, at the operating frequency f_o , the resistive load of the active element (R_D) is obtained (i.e resonance condition is satisfied [1]) and harmonics of the active-element

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current are short-circuited. As a result, the voltage across the active element is sinusoidal although its output current is distorted.

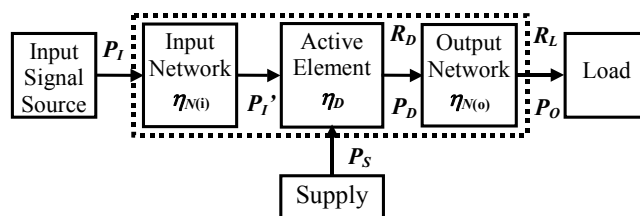


Fig. 1. Block diagram of the resonant power amplifier; P_I – input power, P_I' – active-element drive power, P_D – drain (or collector) power, P_O – output power, P_S – supply power

Secondly, the output network must transform the load resistance R_L (usually 50Ω) into the value R_D required by the active element¹, i.e. the impedance-matching condition must be satisfied [1]. The third function of the output network is filtering the active-element current and voltage in order to obtain sufficiently low-distortion sinusoidal output signal on the load resistance R_L .

The design and construction of the input and output network of h.f. power amplifiers are a difficult task. First of all, applied capacitors and inductors, as well as their interconnections must be designed for sufficiently high voltage and current. Therefore, these elements are large (and expensive) and the interconnections should be sufficiently long and thick. Secondly, applied capacitors and inductors must be linear in a wide range of voltage and current to avoid non-linear distortions of the transmitted signal.

Thirdly, efficiency of the output network ($\eta_{N(o)}$) should be as high as possible so as not to decrease the overall efficiency and the power gain of the amplifier. Besides, increasing efficiency of the output network reduces power dissipated in this network, which allows simplifying the cooling system of the amplifier. It is especially important in high-power amplifiers. For these reasons, loss resistance of inductors and capacitors applied in the input and output network should be as low as possible. Preferably, these elements should be approximately lossless.

Fourthly, frequency responses of the built network must have the assumed shape in a required band. This band is especially wide in Class-AB, Class-B, and Class-C amplifiers, in which current of the active element is a non-sinusoidal wideband signal containing many harmonics. Hence, for these amplifiers the frequency response of the built input or output network can be distorted by self resonances of applied capacitors and inductors. Thus, it is desirable to apply only capacitors and inductors with sufficiently high frequency of the first self resonance.

The considerations presented above show that the capacitors and inductors used in the constructed input and output networks of the power amplifier should be approximately the

¹ Principles of calculating the optimum load resistance of active elements in h.f. resonant power amplifiers are presented e.g. in Kazimierzczuk M.K., *RF Power Amplifiers*, John Wiley & Sons (2008).

ideal capacitances and inductances for all frequencies of the active-element-current spectrum. Therefore, in the basic schemes of input and output networks of the h. f. power amplifiers non-ideality of inductors and capacitors is neglected.

It should be noted that h. f. power inductors applied in the output and input networks of the resonant power amplifiers are usually custom-built by amplifier constructors. In contrast, the h. f. power capacitors are usually made by specialized manufacturers.

2. The $\pi 1$ circuit

The $\pi 1$ circuit consists of two capacitances C_1 , C_2 , and one inductance L (Fig. 2). The $\pi 1$ circuit is often used as an output network in Class A, AB, B, and C resonant power amplifiers because it can fulfil all required functions. Firstly, input impedance Z_D of the loaded $\pi 1$ circuit can have the parallel resonance when the current of C_2 is compensated by reactive component of the current of L . Besides, for sufficiently high frequencies, the input impedance Z_D is determined mainly by C_2 . Therefore harmonics of the active-element current i_d can be approximately short-circuited ensuring sinusoidal voltage across the active element v_{ds} even for single-ended Class-B, Class-AB and Class-C amplifiers.

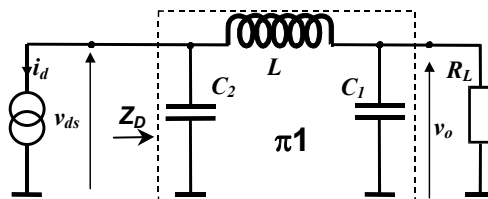


Fig. 2. The $\pi 1$ circuit as an output network in the Class A, AB, B or C resonant power amplifier

Secondly, the $\pi 1$ circuit can transform the load resistance R_L into the required value R_D for all possible situations ($R_L < R_D$, $R_L = R_D$, $R_L > R_D$). Thirdly, the $\pi 1$ circuit has a low-pass-filter structure, therefore the harmonic distortions of the amplifier output voltage $v_o(t)$ are significantly lower than the harmonic distortions of the active-element voltage $v_{ds}(t)$. It is very important because globally applicable standards for spurious emissions of the radio transmitters are restrictive [2]. Additionally, the output capacitance of the active element is a part of the C_2 capacitance of the $\pi 1$ circuit. Hence, the value of C_2 cannot be arbitrarily small. The $\pi 1$ circuit can also be applied as an input network of the resonant power amplifier [3].

3. Traditional method of calculation of the $\pi 1$ circuit elements

The traditional method of calculation of $\pi 1$ circuit element values is based on the simplifying assumption that both capacitors are ideal capacitances and the inductor is an ideal inductance [1, 4-10]. Under this assumption there are only three parameters of the $\pi 1$ circuit elements: C_1 , C_2 , and L . Therefore to calculate required reactances of C_1 , C_2 , L only three

input parameters are needed. Two of them are: the load resistance R_L of the amplifier, and the optimum load resistance of the active element R_D (which is the input resistance of the $\pi 1$ circuit at the operating frequency). The third input parameter is usually the loaded quality factor Q_L of the $\pi 1$ circuit at the operating frequency. Unfortunately, in basic papers concerning π circuit [4-7] the definitions of Q_L are not the same. Assuming classic definition of the quality factor of the loaded LC circuit (at the resonant frequency) [1]:

$$Q_L = 2\pi \frac{\text{Energy Stored in Inductances and Capacitances}}{\text{Energy Dissipated per Cycle}} \quad (1)$$

we obtain the following formulas for C_1, C_2, L (at a given operating frequency f_0) [8-10]:

$$C_1 = \frac{Q_L - \sqrt{\frac{R_D}{R_L}} \sqrt{(Q_L)^2 + 2 - \frac{R_D}{R_L} - \frac{R_L}{R_D}}}{2\pi f_0 (R_L - R_D)}, \quad (2)$$

$$C_2 = \frac{Q_L - \sqrt{\frac{R_L}{R_D}} \sqrt{(Q_L)^2 + 2 - \frac{R_D}{R_L} - \frac{R_L}{R_D}}}{2\pi f_0 (R_D - R_L)}, \quad (3)$$

$$L = \frac{Q_L \cdot R_L}{2\pi f_0 [1 + (B_1 R_L)^2]} \quad \text{or} \quad L = \frac{Q_L \cdot R_D}{2\pi f_0 [1 + (B_2 R_D)^2]} \quad (4)$$

where $B_1 = 2\pi f_0 C_1$, $B_2 = 2\pi f_0 C_2$.

The above formulas for C_1, C_2 are correct if $R_D \neq R_L$ and if the loaded quality factor is sufficiently high $Q_L > \sqrt{R_D / R_L} - 1$ for $R_D > R_L$ or $Q_L > \sqrt{R_L / R_D} - 1$ for $R_D < R_L$ [4, 7]. For $R_D = R_L$ these capacitances are equal to [10]:

$$C_1 = C_2 = \frac{Q_L}{4\pi f_0 R_D}. \quad (5)$$

The value of Q_L results from required frequency response of the $\pi 1$ circuit (and the designed amplifier) [9].

4. The idea of the improved method

The idealizing assumptions that capacitors are linear, lossless, frequency-independent capacitances, and the inductor is the linear, lossless, frequency-independent inductance can be fulfilled only approximately even by the high-performance elements. Therefore, measured frequency responses of the built $\pi 1$ circuit, which has been designed using the traditional method (Formulas 2-5), can differ from characteristics calculated for the basic circuit (Fig. 2) at the project level. In particular, measured resonant frequency and resonant resistance of the built $\pi 1$ circuit usually differ from the assumed values f_0 and R_D . Hence, the labour-consuming tuning by adjusting values of C_1, C_2, L is necessary.

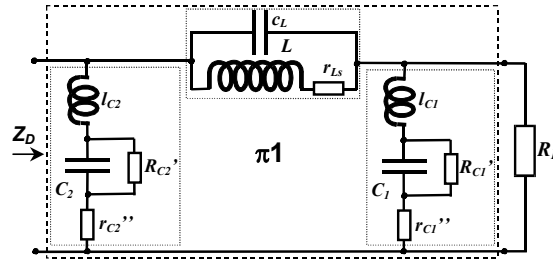


Fig. 3. Equivalent scheme of physical π l circuit in a wide frequency range considering the first self resonance and losses of the inductor and capacitors

An improved design method of the π l circuits should consider more parameters of capacitors and inductors than their capacitance and inductance. In the general case, the number of parameters of equivalent circuit describing these elements in a wide band is very high because they are a distributed-constant circuits. Fortunately, in order to maintain the assumed low level of spurious emissions of the amplifier the inductor and each capacitor in the π l circuit should operate below their first-self-resonance frequency:

$$\begin{aligned}
 f_{SR(C1)} &> f_{i \max}, \\
 f_{SR(C2)} &> f_{i \max}, \\
 f_{SR(L)} &> f_{i \max},
 \end{aligned}
 \tag{6}$$

where $f_{SR(C1)}, f_{SR(C2)}$ are frequencies of the first self resonance (series) of the capacitors, $f_{SR(L)}$ is frequency of the first resonance (parallel) of the inductor, $f_{i \max}$ is the highest frequency in spectrum of the active-element current. Hence, the π l circuit for the resonant power amplifier can be well described by the equivalent circuit presented in Fig. 3. Unfortunately, frequency of the self resonance is not a published parameter of h.f. capacitors. It is also very difficult to design and build an inductor with a given inductance and a given frequency of self resonance. Therefore, the equivalent circuit presented in Fig. 3 cannot be used in a first step of design of the π l circuit.

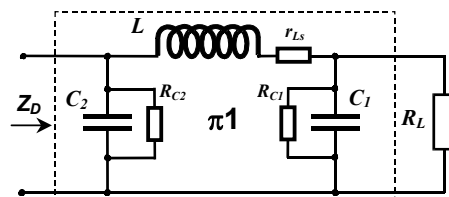


Fig. 4. Simplified equivalent scheme of physical π l circuit considering losses in inductor and capacitors

On the other hand, manufacturers of h.f. capacitors publish information about their losses. Similarly, for the designed inductor we can estimate the quality factor [1]. Hence, the im-

proved method of design of the $\pi 1$ circuit could be based on the simplified equivalent circuit presented in Fig. 4.

However, in most cases, the quality factor of the inductor in the $\pi 1$ circuit constructed for a given operating frequency is significantly lower than quality factor of the applied capacitors. In other words, in $\pi 1$ circuits physical capacitors are usually much closer to the ideal capacitance than the physical inductors are to the ideal inductance. Hence, the assumed and the measured values of the parameters of $\pi 1$ circuits designed by the traditional method (Formulas 2-5) usually differ mainly due to loss resistance of the applied inductor. Thus, the traditional design method of the $\pi 1$ circuit should be improved first of all by considering the finite quality factor of applied inductor.

This paper presents a method of calculation of capacitance C_1 , C_2 , and inductance L of a $\pi 1$ circuit with the assumed finite quality factor of inductor and with lossless capacitors (Fig. 5). Values of C_1 , C_2 , L are calculated for given values of the following parameters:

- f_0 – operating frequency,
- R_L – load resistance,
- R_D – input resistance at the operating frequency,
- Q_L – loaded quality factor of the designed circuit,
- Q_I – quality factor of the applied inductor

$$Q_I = \frac{2\pi f_0 L}{r_{Ls}}, \quad (7)$$

where r_{Ls} is a series loss resistance of the inductor.

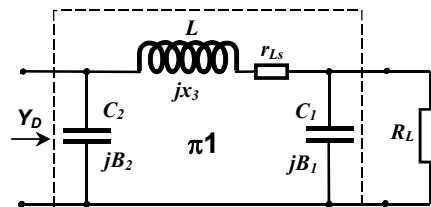


Fig. 5. Model of $\pi 1$ circuit assumed in the proposed improved calculation method

5. Formulas derivation

To derive formulas for calculating L , C_1 , C_2 of $\pi 1$ circuit as functions of R_L , R_D , Q_L , Q_I , and f_0 the formula of the $\pi 1$ circuit input admittance at the operating frequency $Y_D(f_0)$ is needed. Let us introduce the following notations:

$$\begin{aligned} B_1 &= 2\pi f_0 C_1, \\ B_2 &= 2\pi f_0 C_2, \\ x_3 &= 2\pi f_0 L. \end{aligned} \quad (8)$$

Hence, impedance components of the parallel two-terminal network C_1, R_L at the operating frequency f_0 are

$$\begin{aligned} r_1 &= \frac{R_L}{1 + (B_1 R_L)^2}, \\ x_1 &= \frac{B_1 R_L}{1 + (B_1 R_L)^2}. \end{aligned} \quad (9)$$

Thus, the input admittance of the loaded π 1 circuit with a lossy inductor at the operating frequency is equal to (Fig. 6):

$$Y_D(f_0) = jB_2 + \frac{1}{jx_3 + r_{Ls} + r_1 + jx_1}. \quad (10)$$

Applying Equations (9) the admittance $Y_D(f_0)$ may be presented as follows [11]:

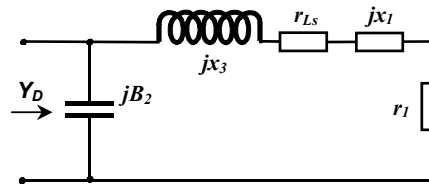


Fig. 6. Equivalent scheme of the π 1 circuit with a lossy inductor

$$\begin{aligned} Y_D(f_0) &= jB_2 + \frac{1}{jx_3 + r_{Ls} + R_L \frac{1 - jB_1 R_L}{1 + (B_1 R_L)^2}} = \\ &= jB_2 + \frac{[1 + (B_1 R_L)^2] \{R_L + r_{Ls}(1 + (B_1 R_L)^2) - j[x_3(1 + (B_1 R_L)^2) - B_1(R_L)^2]\}}{\{R_L + r_{Ls}(1 + (B_1 R_L)^2)\} + \{x_3(1 + (B_1 R_L)^2) - B_1(R_L)^2\}^2}. \end{aligned} \quad (11)$$

The total resistance in series with the inductance L is $r_s = r_{Ls} + r_1$ (Fig. 6). Hence, the loaded quality factor of the π 1 circuit with a lossy inductor is described by the following formula:

$$Q_L = \frac{\omega_0 L}{r_1 + r_{Ls}} = x_3 \frac{1 + (B_1 R_L)^2}{R_L + r_{Ls}[1 + (B_1 R_L)^2]}. \quad (12)$$

Let us denote by Q_{L0} the loaded quality factor of the π 1 circuit in the situation when the applied inductor is lossless ($r_{Ls} = 0$):

$$Q_{L0} = \frac{\omega_0 L}{r_1} = x_3 \frac{1 + (B_1 R_L)^2}{R_L}. \quad (13)$$

Hence, the loaded quality factor of the π l circuit with a lossy inductor can be presented by

$$Q_L = \frac{1}{\frac{1}{Q_{L0}} + \frac{1}{Q_I}}. \quad (14)$$

Thus, the Formula (11) can be transformed to the following form

$$Y_D(f_o) = jB_2 + \frac{\left[1 + (B_1 R_L)^2\right] \left\{ R_L \frac{Q_{w0}}{Q_L} - j \left[Q_{w0} R_L - B_1 (R_L)^2 \right] \right\}}{\left(R_L \frac{Q_{w0}}{Q_L} \right)^2 + \left[Q_{w0} R_L - B_1 (R_L)^2 \right]^2}. \quad (15)$$

In the power amplifier input admittance of the loaded π l circuit at the operating frequency $Y_D(f_o)$ must satisfy two equations [1]:

1) resonance condition:

$$\text{Im}(Y_D(f_o)) = 0, \quad (16)$$

2) impedance-matching condition

$$\text{Re}(Y_D(f_o)) = \frac{1}{R_D}. \quad (17)$$

The Equations (16), (17) are two-equation system with two unknowns (B_1 , B_2). The impedance matching condition (17) is equivalent to the following equation with the unknown B_1 :

$$\frac{\left[1 + (B_1 R_L)^2\right] R_L \frac{Q_{L0}}{Q_L}}{\left(R_L \frac{Q_{L0}}{Q_L} \right)^2 + \left[Q_{L0} R_L - B_1 (R_L)^2 \right]^2} = \frac{1}{R_D}. \quad (18)$$

This equation can be transformed to

$$(B_1 R_L)^2 \left(1 - \frac{R_D}{R_L} \frac{Q_{w0}}{Q_L} \right) - 2 Q_{L0} (B_1 R_L) + \left(\frac{Q_{L0}}{Q_L} \right)^2 + (Q_{L0})^2 - \frac{R_D}{R_L} \frac{Q_{L0}}{Q_L} = 0. \quad (19)$$

For $Q_{L0} R_D \neq Q_L R_L$ Equation (19) has two solutions:

$$B_{11} = \frac{Q_{L0}}{Q_L R_L - Q_{L0} R_D} \left[Q_L - \sqrt{\frac{R_D}{R_L}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right], \quad (20)$$

$$B_{12} = \frac{Q_{L0}}{Q_L R_L - Q_{L0} R_D} \left[Q_L + \sqrt{\frac{R_D}{R_L}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right]. \quad (21)$$

In contrast, for $Q_{L0} R_D = Q_L R_L$ there is only one solution

$$B_1 = \frac{Q_L}{2R_D} \left[1 + \frac{1}{(Q_L)^2} - \frac{1}{(Q_{L0})^2} \right]. \quad (22)$$

The resonance condition (16) is equivalent to the following equation:

$$B_2 = - \frac{[1 + (B_1 R_L)^2] \cdot [Q_{L0} R_L - B_1 (R_L)^2]}{\left\{ R_L \frac{Q_{L0}}{Q_L} \right\}^2 + \{ Q_{L0} R_L - B_1 (R_L)^2 \}^2} = 0. \quad (23)$$

Substituting (18) into (23) yields

$$B_2 R_D + \frac{Q_L}{Q_{L0}} B_1 R_L = Q_L. \quad (24)$$

From (24), (20), and (21) two values of susceptance B_2 can be obtained for $Q_{L0} R_D \neq Q_L R_L$:

$$B_{21} = \frac{Q_L}{Q_{L0} R_D - Q_L R_L} \left[Q_{L0} - \sqrt{\frac{R_L}{R_D}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right], \quad (25)$$

$$B_{22} = \frac{Q_L}{Q_{L0} R_D - Q_L R_L} \left[Q_{L0} + \sqrt{\frac{R_L}{R_D}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right], \quad (26)$$

where B_{21} corresponds to B_{11} and B_{22} to B_{12} .

In contrast, for $Q_{L0} R_D = Q_L R_L$ from (22) and (24) we obtain

$$B_2 = \frac{Q_L}{2R_D} \left[1 + \frac{1}{(Q_{L0})^2} - \frac{1}{(Q_L)^2} \right]. \quad (27)$$

It should be noted that B_1 and B_2 are susceptances of capacitances (C_1 , C_2), therefore they must satisfy the conditions:

$$B_1 > 0, \quad B_2 > 0. \quad (28)$$

For $Q_{L0} R_D < Q_L R_L$ the susceptance B_{22} is negative therefore it must be rejected together with the corresponding susceptance B_{12} . Similarly, for $Q_{L0} R_D > Q_L R_L$ the susceptance B_{12} is negative and must be rejected together with the corresponding susceptance B_{22} . Hence, there can be only one solution of the equation system (16), (17). Thus, capacitances of the π l circuit are equal to:

$$C_1 = \frac{Q_{L0}}{2\pi f_O (Q_L R_L - Q_{L0} R_D)} \left[Q_L - \sqrt{\frac{R_D}{R_L}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right]. \quad (29)$$

$$C_2 = \frac{Q_L}{2\pi f_O \cdot (Q_{L0} R_D - Q_L R_L)} \left[Q_{L0} - \sqrt{\frac{R_L}{R_D}} \cdot \sqrt{Q_L Q_{L0} + \frac{Q_L}{Q_{L0}} + \frac{Q_{L0}}{Q_L} - \frac{R_L}{R_D} - \frac{R_D}{R_L}} \right]. \quad (30)$$

For $Q_{L0}R_D = Q_L R_L$ we obtain

$$C_1 = \frac{Q_L}{4\pi f_O R_D} \left[1 + \frac{1}{(Q_L)^2} - \frac{1}{(Q_{L0})^2} \right]. \quad (31)$$

$$C_2 = \frac{Q_L}{4\pi f_O R_D} \left[1 + \frac{1}{(Q_{L0})^2} - \frac{1}{(Q_L)^2} \right]. \quad (32)$$

The required reactance x_3 of inductance L can be obtained from the Equation (12) substituting

$$r_{Ls} = \frac{\omega_O L}{Q_I}$$

(Equation (7)). Hence, we obtain:

$$x_3 = \frac{R_L}{1 + (B_1 R_L)^2} \frac{Q_L Q_I}{Q_I - Q_L} = \frac{Q_{L0} R_L}{1 + (B_1 R_L)^2}. \quad (33)$$

Thus, in the $\pi 1$ circuit required inductance of the inductor is equal to:

$$L = \frac{Q_{L0} R_L}{2\pi f_O [1 + (B_1 R_L)^2]}, \quad (34)$$

where B_1 is given by Formula (20) or (22).

Similarly to the traditional method of calculation of the $\pi 1$ circuit elements, in the presented improved method the physically realizable solutions $C_1 \geq 0$, $C_2 \geq 0$ exist only for sufficiently high loaded quality factor Q_L and for sufficiently high quality factor of the applied inductor Q_I . Minimum values of Q_L and Q_I increase for high values of R_L/R_D and for low values of R_L/R_D .

6. Influence of the finite inductor quality factor on calculated values C_1 , C_2 , L

Values of C_1 , C_2 , L calculated for given values of R_L , R_D , and Q_I using presented Formulas (29), (30), and (34) (or (31), (32), and (34)) depend on the assumed value of quality factor Q_I of the employed inductor. In order to assess the impact of Q_I on C_1 , C_2 , L values of C_1 , C_2 , L were calculated for $\pi 1$ circuits with different impedance transformation ratio R_L/R_D and loaded quality factor Q_L . Results of these calculations are presented in the Table 1 for $\pi 1$ circuits with three typical values of transformation ratio:

- $R_L/R_D = 5$ ($\pi 1$ circuit for power amplifiers with transistors),
- $R_L/R_D = 1$ ($\pi 1$ circuit used as an additional filter),
- $R_L/R_D = 0.2$ ($\pi 1$ circuit for power amplifiers with vacuum tubes).

For each of these π circuit calculations were done for the inductors with the following values of quality factor: $Q_I = 50, 100, 150$. In the Table 1 there are also presented results of calculations for lossless inductors ($Q_I \rightarrow \infty$) obtained using Formulas (2), (3), (4), (5).

Table 1. Values of C_1, C_2, L of π circuits with different R_L/R_D and Q_L calculated for $Q_I = 50, 100, 150$, and ∞ ($R_L = 50 \Omega, f_0 = 1 \text{ MHz}$)

R_L/R_D		$Q_L = 5$				$Q_L = 10$			
		$Q_I = 50$	$Q_I = 100$	$Q_I = 150$	$Q_I \rightarrow \infty$	$Q_I = 50$	$Q_I = 100$	$Q_I = 150$	$Q_I \rightarrow \infty$
5	C_1 [nF]	12.652	12.096	11.920	11.586	26.862	24.347	23.615	22.282
	C_2 [nF]	22.642	22.128	21.964	21.646	51.706	49.595	48.953	47.746
	L [μ H]	2.6316	2.7131	2.7398	2.7924	1.3774	1.4860	1.5215	1.5915
1	C_1 [nF]	8.6426	8.2855	8.1732	7.9577	18.821	17.235	16.769	15.915
	C_2 [nF]	8.1371	8.0443	8.0115	7.9577	17.774	16.319	16.180	15.915
	L [μ H]	5.2806	5.3866	6.4209	7.4881	2.7660	2.9165	2.9652	3.0607
0.2	C_1 [nF]	4.7265	4.5203	4.4550	4.3293	11.128	10.271	10.017	9.5493
	C_2 [nF]	2.3323	2.3242	2.3218	2.3172	4.5857	4.5174	4.4963	4.4563
	L [μ H]	13.794	13.884	13.911	13.962	7.5234	7.7481	7.8196	7.9577

The presented results show, firstly, that the lower is the quality factor Q_I of the inductor used in the π circuit with given values of R_L, R_D, Q_L , the higher are the needed values of C_1, C_2 and the lower must be inductance L of this inductor. Secondly, the capacitance C_1 depends more on quality factor Q_I of the employed inductor than the capacitance C_2 . For example, if the inductor with $Q_I = 50$ is used instead of the lossless one, then C_1 must be increased by $8.6 \div 20.6\%$ while the required increase of C_2 is only $0.65 \div 8.6\%$. The inductance L of the employed inductor must be decreased by $3.8\text{-}13.5\%$.

However, it should be stressed that if a low-quality-factor inductor (e.g. $Q_I = 50$) is applied in the π circuit with relatively high loaded quality factor ($Q_L = 5 \div 10$) then the efficiency of this circuit is low ($80 \div 90\%$) according to the following formula [1]:

$$\eta_C = 1 - \frac{Q_L}{Q_I}. \quad (35)$$

Hence, if this π circuit is applied as the output network in the power amplifier then the overall efficiency of the amplifier is noticeably decreased.

7. The second stage of calculations

Calculations of values of the capacitances C_1, C_2 , and the inductance L using derived Formulas (29), (30), and (34) (or (31), (32), and (34)) are only the first stage of design of the π circuit with given values of the load resistance R_L , the input resistance R_D , the loaded quality factor Q_L , the quality factor of the applied inductor Q_I , and the operating frequency f_0 . It re-

sults from the fact that designing and building an inductor with given values of the inductance L and the quality factor Q_I is a very difficult task (see [1, 11]). Hence, constructing the $\pi 1$ circuit with given values of C_1 , C_2 , L , and Q_I one usually can obtain required capacitances of both capacitors (using trimming capacitors or connecting in parallel a few different capacitors). In contrast, in the case of built inductor we can usually adjust only its inductance (e.g. by changing length of the coil) whereas its real quality factor Q_I^* usually differs from the assumed value ($Q_I^* \neq Q_I$). Hence, in the built $\pi 1$ circuit neither the resonance condition (16) nor the impedance-matching condition (17) are exactly satisfied. Thus, after building the inductor, adjusting its inductance, and measuring its quality factor (Q_I^*) the second stage of the $\pi 1$ circuit calculations is usually required.

In the second stage, for given inductance L and quality factor Q_I^* of the built inductor there are calculated capacitances C'_1 , C'_2 , ensuring the required values of R_L , R_D , and f_O . It should be noted that the loaded quality factor Q_L of the designed $\pi 1$ circuit cannot be assumed in these calculations. Hence, the presented Formulas (29), (30), and (34) (or (31), (32), and (34)) cannot be used in this stage.

To derive new formulas for C'_1 , C'_2 the $\pi 1$ circuit input admittance (11) must be transformed as follows:

$$Y_D(f_O) = jB'_2 + \frac{1}{jx_3 + r'_{Ls} + \frac{R_L}{1 + jB'_1 R_L}} =$$

$$= jB_2 + \frac{(1 + jB_1 R_L)[R_L + r'_{Ls} - x_3 B'_1 R_L - j(x_3 + r'_{Ls} B'_1 R_L)]}{(R_L + r'_{Ls} - x_3 B'_1 R_L)^2 + (x_3 + r'_{Ls} B'_1 R_L)^2}, \quad (36)$$

where $B'_1 = 2\pi f_O C'_1$, $B'_2 = 2\pi f_O C'_2$, $r'_{Ls} = x_3 / Q_I^*$ is a series resistance of the built inductor. Dividing numerator and denominator in (36) by x_3 and substituting $r'_{Ls} = x_3 / Q_I^*$ we obtain

$$Y_D(f_O) = jB_2 + \frac{\frac{1 + jB'_1 R_L}{x_3} \left[\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L - j \left(1 + \frac{1}{Q_I^*} B'_1 R_L \right) \right]}{\left(\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L \right)^2 + \left(1 + \frac{1}{Q_I^*} B'_1 R_L \right)^2}. \quad (37)$$

The impedance-matching condition (17) is equivalent to the following equation:

$$\frac{\frac{R_L}{x_3} + \frac{1}{Q_I^*} + \frac{1}{Q_I^*} (B'_1 R_L)^2}{\left(\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L \right)^2 + \left(1 + \frac{1}{Q_I^*} B'_1 R_L \right)^2} = \frac{x_3}{R_D}, \quad (38)$$

which can be transformed to the quadratic equation

$$(B'_1)^2 \left[x_3 + \frac{x_3}{(Q_I^*)^2} - \frac{R_D}{Q_I^*} \right] - 2B'_1 \left[\frac{R_D}{x_3 R_L} - \frac{R_D}{R_L^2 Q_I^*} + \frac{x_3}{R_L} + \frac{1}{x_3} \left(1 + \frac{x_3}{R_L Q_I^*} \right)^2 \right] = 0. \quad (39)$$

For

$$x_3 + \frac{x_3}{(Q_I^*)^2} - \frac{R_D}{Q_I^*} \neq 0$$

Equation (39) has two solutions:

$$B'_{1,2} = \frac{1 \mp \sqrt{1 - \left[x_3 + \frac{x_3}{(Q_I^*)^2} - \frac{R_D}{Q_I^*} \right] \left[\frac{x_3}{R_L^2} + \frac{1}{x_3} \left(1 + \frac{x_3}{R_L Q_I^*} \right)^2 - \frac{R_D}{x_3 R_L} - \frac{R_D}{R_L^2 Q_I^*} \right]}}{x_3 \left(1 + \frac{1}{(Q_I^*)^2} \right) - \frac{R_D}{Q_I^*}}. \quad (40)$$

The resonance condition (15) is equivalent to the following equation:

$$B'_2 = -\frac{1}{x_3} \frac{B'_1 R_L \left(\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L \right) - 1 - \frac{1}{Q_I^*} B'_1 R_L}{\left(\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L \right)^2 + \left(1 + \frac{1}{Q_I^*} B'_1 R_L \right)^2}. \quad (41)$$

Substituting (38) into (41) we obtain

$$B'_2 = -\frac{1}{R_D} \frac{B'_1 R_L \left(\frac{R_L}{x_3} + \frac{1}{Q_I^*} - B'_1 R_L \right) - 1 - \frac{1}{Q_I^*} B'_1 R_L}{\frac{R_L}{x_3} + \frac{1}{Q_I^*} + \frac{1}{Q_I^*} (B'_1 R_L)^2} = \frac{Q_I^*}{R_D} \frac{x_3 [1 + (B'_1 R_L)^2] - B'_1 (R_L)^2}{Q_I^* R_L + x_3 + x_3 (B'_1 R_L)^2}. \quad (42)$$

Choosing positive values of B'_1 and B'_2 we obtain required capacitances for designed πl circuit:

$$C'_1 = \frac{1 + \sqrt{1 - \left[x_3 + \frac{x_3}{(Q_I^*)^2} - \frac{R_D}{Q_I^*} \right] \left[\frac{x_3}{R_L^2} + \frac{1}{x_3} \left(1 + \frac{x_3}{R_L Q_I^*} \right)^2 - \frac{R_D}{x_3 R_L} - \frac{R_D}{R_L^2 Q_I^*} \right]}}{2\pi f_0 \left[x_3 \left(1 + \frac{1}{(Q_I^*)^2} \right) - \frac{R_D}{Q_I^*} \right]}, \quad (43)$$

$$C'_2 = \frac{Q_I^*}{2\pi f_0 R_D} \frac{x_3 [1 + (B'_1 R_L)^2] - B'_1 (R_L)^2}{Q_I^* R_L + x_3 [1 + (B'_1 R_L)^2]}. \quad (44)$$

The loaded quality factor of the $\pi 1$ circuit with given inductor (L , Q_I^*) and capacitances C_1' , C_2' can be obtained from Equation (12)

$$Q_L = \frac{x_3 Q_I^* [1 + (B_1' R_L)^2]}{Q_I^* R_L + x_3 [1 + (B_1' R_L)^2]} \quad (45)$$

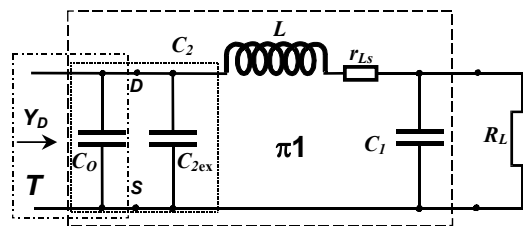


Fig. 7. Components of the capacitance C_2 in the $\pi 1$ circuit used as an output network in a resonant power amplifier

In the second stage of $\pi 1$ circuit calculation it must be considered that in the power amplifier this circuit is driven by an active element T with the output capacitance C_O . Hence, capacitance C_2 of the $\pi 1$ circuit is the sum of the active-element output capacitance C_O and the external capacitance C_{2ex} (Fig. 7). Thus, capacitance of the external capacitor is equal to

$$C_{2ex} = C_2' - C_O \quad (46)$$

8. Examples

As an example, consider the $\pi 1$ circuit with the following parameters: $R_L = 50 \Omega$, $R_D = 10 \Omega$, $f_o = 1 \text{ MHz}$, and $Q_L = 5$. Using the traditional calculation method (for idealized lossless inductor) we obtain: $L = 2.7924 \mu\text{H}$, $C_1 = 11.586 \text{ nF}$, $C_2 = 21.646 \text{ nF}$ (Formulas 2, 3, 4). If real inductors with finite quality factor ($Q_I = 180, 100$, and 60) are applied in this circuit, then the resonance condition is not satisfied at the operating frequency (Fig. 8). However, for the inductor with high quality factor $Q_I = 180$ (the large air-core coil) the angle of the input impedance at f_o is only -1.5° .

In contrast, using the derived formulas (29), (30), (34) for the inductor quality factor $Q_I = 180$ we obtain $L = 2.7486 \mu\text{H}$, $C_1 = 11.863 \text{ nF}$, $C_2 = 21.910 \text{ nF}$. Obviously, input impedance of this circuit satisfies the resonance condition and the impedance-matching condition at f_o (Fig. 9, curves no. 1). Efficiency of the $\pi 1$ circuit with this inductor is high $\eta_C \cong 97.2\%$ (Formula 35).

If in the $\pi 1$ circuit an inductor with required inductance ($L = 2.7486 \mu\text{H}$) and lower quality factor $Q_I = 100$ is applied, then the value of C_1 should be a little higher: $C_1' = 11.88 \text{ nF}$ (Formula 43). In contrast, the value of C_2 should be lower: $C_2' = 21.600 \text{ nF}$ (Formula 44). The loaded quality factor of this circuit ($Q_L = 4.9079$ – Formula 45) is a little lower than assumed

and the efficiency is decreased to $\eta_C \cong 95.1\%$. Applying a low-quality-factor inductor ($L = 2.7486 \mu\text{H}$) with $Q_I = 60$ (e.g. a ferrite-core coil) we obtain $C_1' = 11.918 \text{ nF}$, $C_2' = 21.142 \text{ nF}$, $Q_L = 4.7746$, $\eta_C \cong 91.7\%$. For these circuits the resonance condition and the impedance-matching condition are satisfied at f_O (Fig. 9, curves no. 2, 3).

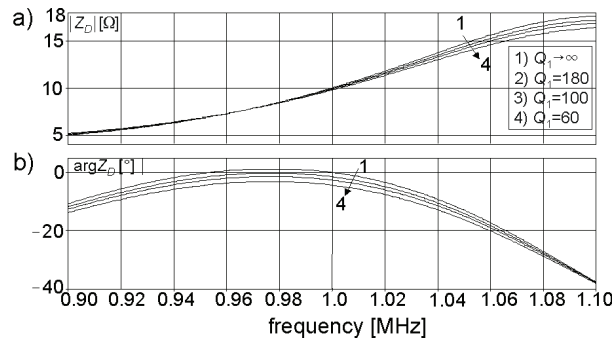


Fig. 8. Impact of the finite inductor quality factor on input impedance of the πl circuit designed using the traditional method (for loss-less inductor): a) magnitude (b) angle; ($R_D = 10 \Omega$, $R_L = 50 \Omega$; PSpice simulations)

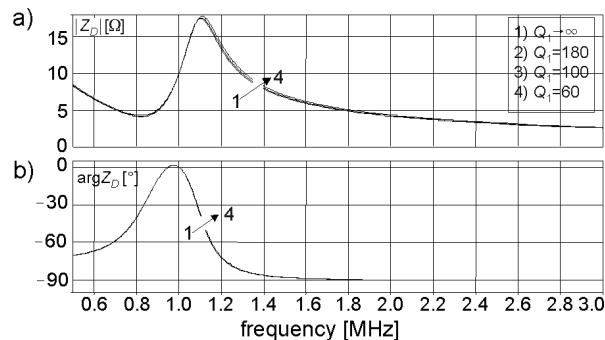


Fig. 9. Magnitude (a) and angle (b) of input impedance of the πl circuits designed using the derived formulas ($R_D = 10 \Omega$, $R_L = 50 \Omega$; PSpice simulations);

1: $Q_I = 180$, $Q_L = 5$, $L = 2.7486 \mu\text{H}$, $C_1 = 11.863 \text{ nF}$, $C_2 = 21.910 \text{ nF}$, ((29), (30), (34));

2: $L = 2.7486 \mu\text{H}$, $Q_I = 100$, $C_1 = 11.886 \text{ nF}$, $C_2 = 21.600 \text{ nF}$, $Q_L = 4.9079$, ((43), (44));

3: $L = 2.7486 \mu\text{H}$, $C_1 = 11.918 \text{ nF}$, $C_2 = 21.142 \text{ nF}$, $Q_I = 60$, $Q_L = 4.7746$, ((43), (44))

9. Experimental results

For the πl circuit analysed in Chapter 8 ($R_L = 50 \Omega$, $R_D = 10 \Omega$, $Q_L \approx 5$, $f_O = 1 \text{ MHz}$) the low-volume inductor with $L \approx 2.8 \mu\text{H}$ was built and tested. It is a single-layer air-core solenoid (diameter: 9.8 mm, axial length: 33 mm, number of turns: 33, wire diameter: 0.7 mm). Its inductance has been adjusted to 2.79 μH (at 1 MHz) by regulating the axial length. Measured at 1 MHz quality factor of this inductor is low: $Q_I \cong 56$ at 1 MHz.

This inductor was applied to an experimental π 1 circuit with capacitances calculated by the traditional calculation method (for idealized loss-less inductor). As C_1 , C_2 sets of ceramic SMD (1206) NP0 capacitors connected in parallel with 68 pF trimming capacitor were used (C_1 : $3 \cdot 3.3 \text{ nF} + 1.5 \text{ nF} + 150 \text{ pF}$ adjusted to 11.59 nF, C_2 : $6 \cdot 3.3 \text{ nF} + 1.8 \text{ nF}$ adjusted to 21.65 nF at 1 MHz). For this capacitors parasitic resistances and inductances may be approximately neglected in a frequency range up to 10 MHz. The capacitors, the coil, and the 50 Ω /10 W load resistor were mounted on a double-sided printed circuit board with ground plane.

Measured at 1 MHz input impedance of this π 1 circuit was $|Z_D| \cong 9.7 \Omega$, $\arg Z_D \cong -4.5^\circ$. Varying length of the inductor from 30 mm to 36 mm (i.e. varying its inductance between approximately 2.5 μH and 3.1 μH) we did not satisfy the resonance and impedance-matching conditions. However, the required value $|Z_D| = 10 \Omega$ was obtained but the resonance condition was not satisfied ($\arg Z_D = -5.2^\circ$). In contrast, by minimizing $|\arg Z_D|$ we obtained $\arg Z_D \cong -2.2^\circ$, $|Z_D| \cong 7.4 \Omega$.

In the second experiment inductance of the inductor in the built π 1 circuit was adjusted to 2.75 μH whereas capacitances were approximately adjusted to values calculated for $L = 2.7486 \mu\text{H}$ and $Q_I = 56$ using derived Formulas (43), (44):

$$\begin{aligned} C_1 &\cong 11.92 \text{ nF}, \\ C_2 &\cong 21.06 \text{ nF}. \end{aligned} \tag{47}$$

After a small correction of the inductor length there were obtained the assumed values of the built- π 1 circuit input impedance(at 1 MHz):

$$\begin{aligned} |Z_D| &\cong 10.0 \Omega \\ \arg Z_D &\cong 0. \end{aligned} \tag{48}$$

10. Conclusions

The presented calculation procedure can facilitate design and construction of the π 1 circuit for resonant power Class A, Class-AB, Class-B, and Class-C amplifiers. First of all, tuning of the built π 1 circuit is simplified because this procedure allows calculating the required values of C_1 , C_2 and L more precisely. Hence, the range of adjustments of C_1 , C_2 and L necessary to satisfy the resonant condition (16) and the impedance matching condition (17) in the built circuit is significantly reduced, particularly in the case of low-quality-factor inductors. This fact allows reducing cost of the experimental optimisation of the built π 1 circuit and the complete resonant power amplifier.

Reduction of the required trimming range of C_1 , C_2 and L in the built π 1 circuit is also important because of typical tuning method of the π 1 circuits for power amplifiers. Due to nonlinearity of the transistor output capacitance C_O (Fig. 7) such circuits cannot be exactly tuned using an impedance meter. Hence, in practice, the π 1 circuit is usually tuned in the working amplifier (loaded, supplied and driven) by varying C_1 , C_2 , L to maximize output

power of the amplifier [12]. Unfortunately, for a wide range of C_1 , C_2 , L trimming it is likely that the obtained input resistance of π l circuit (after tuning) differs considerably from the required value R_D , i.e. that the impedance-matching condition (17) is not satisfied. In this situation important properties of the amplifier (e.g. non-linear distortions or efficiency) are made worse.

It should be noted that there are some resonant power amplifiers, for which the simplified equivalent scheme of π l circuit applied in the presented calculation procedure (Fig. 5) can cause significant errors. For example, these are amplifiers with low load resistance of the active element ($R_D = 1.5 - 3 \Omega$) and high loaded quality factor ($Q_L = 5 - 10$), in which impedance of C_2 must be very low (below 1Ω). In this situation parasitic series inductance of the capacitor C_2 usually cannot be neglected (e.g. [11]). Hence, it is justified to continue research into resonant circuits for power amplifiers and development of design methods.

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