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ANALYTICAL METHOD OF A CALCULATION OF A TORQUE RIPPLE A TWO-PHASE PMSM SUPPLIED BY A PWM CONTROLLED INVERTER

METODA OBLICZEŃ ANALITYCZNYCH PULSACJI MOMENTU W DWUFAZOWYM SILNIKU SYNCHRONICZNYM Z MAGNESAMI TRWAŁYMI ZASILANYM Z PRZEKSZTAŁTNIKA PWM

Abstract: The paper deal with steady-state analysis of a two-phase permanent magnet synchronous motor (PMSM) drive with a voltage-source inverter is presented. A complex Fourier series approach is used to predict the inverter output voltage and line current waveform. Assuming the output inverter's voltage is controlled by a pulse-width modulation (PWM). The permanent magnet synchronous motor model is obtained from its analogy to the fixed-excited synchronous motor. From the induced voltage and line current waveforms the electromagnetic torque ripple waveform is calculated.

1. Introduction

The permanent magnet synchronous motors are very attractive engines for domestic and industrial application due to several advantages.

Water leakage problems and low efficiency of an induction motor are the main reasons why another concept of the water pump was developed, in which the rotor of the motor is immersed into pumping water. In this solution the rotor of the motor is exposed to chemically aggressive water. A squirrel-cage rotor cannot be used, but ferrite permanent magnet rotor seems to be a good choice. The permanent magnet rotors are chemically inert, which makes them suitable for applications in aggressive environments.

Strontium-ferrite magnets have high specific electric resistance, so they do not thermal problems due to eddy-current losses. Their low residual flux density permits the construction with high air-gape that is needed for small pump applications, where rotor is exposed to water or another in most cases aggressive liquid.

Small water pumps with rated power about 100 W have a large application in a central heating, and other house application (washing machines, dishwashers and other). They operate in most of cases with single-phase ac motor supplied either directly from mains or through converter, when speed control is needed.

Use two-phase motor has any several advantages. The stator winding has most simple form. Three shifted coil winding form one phase. The stator windings can be configured in either a serial or parallel two-phase system.

Normally, the windings are identical. The windings which form one phase are connected to induce opposite magnetic polarity.

Figure 1 shows the construction disposition of a two-phase synchronous motor having permanent magnet on the rotor.

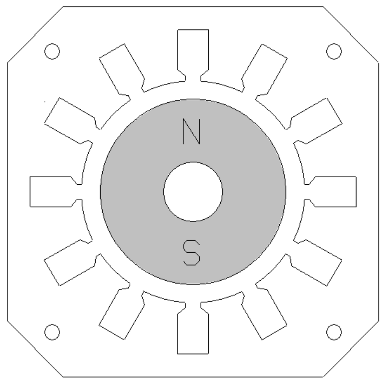

Fig. 1. Disposition of the two-phase permanent magnet synchronous motor

2. Mathematical model of permanent magnet synchronous motor

The analytical description of the two-phase synchronous motor with permanent magnet rotor is simpler than that of the three-phase one. The stator windings are physically orthogonal and thus magnetically decoupled. The mathematical description bears strong resemblance to its single-phase counterpart. Figure 2 shows the per-phase equivalent circuit of the motor.

Let us assume that the reluctance torque is negligible. This one depends on the air-gap between the poles.

Assuming, that all coils are identical and the magnetic circuit is symmetrical, instantaneous value of the electrical input power is determined as:

$$p = u_1 i_1 + u_2 i_2 \qquad (1)$$

This one consists of three parts:

$$p = p_j + p_m + p_e \qquad (2)$$

$p_j = R\left(i_1^2 + i_2^2\right)$ –presents the copper losses in the stator coils;

$p_m = L\left(i_1 \frac{di_1}{dt} + i_2 \frac{di_2}{dt}\right)$ – presents magnetic reactive power;

$p_e = e_1 i_1 + e_2 i_2$ – presents electrical output power;

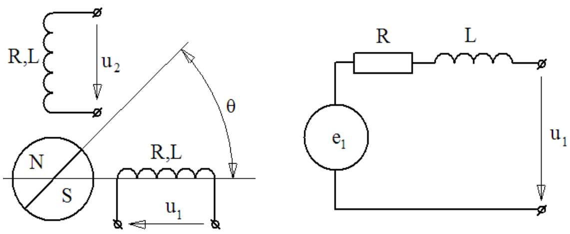

Fig. 2. Equivalent circuit

For each of the coils the voltage equations are valid:

$$\begin{cases} u_1 = Ri_1 + L\omega \dfrac{di_1}{d\theta} + e_1 \\ u_2 = Ri_2 + L\omega \dfrac{di_2}{d\theta} + e_2 \end{cases} \qquad (3)$$

Here θ is the angle of rotor position; e_1, e_2 phase electromotive forces (emf); R is the armature residence; L is the synchronous inductance.

The phase-torque is proportional to a product of the phase current and phase emf, and disproportional to the rotor speed. Total motor torque is given by the sum of the phase-torques.

$$m = m_1 + m_2 = p\frac{i_1 e_1 + i_2 e_2}{\omega_m} \qquad (4)$$

The electromotive forces induced in the stator coils can be expressed as a sinusoidal or co-sinusoidal functions, which are lagged by the supply phase voltages by angle γ. Angle γ depends on the loading torque of the machine.

$$\begin{cases} e_1 = E\dfrac{e^{j(\theta-\gamma)} - e^{-j(\theta-\gamma)}}{2j} \\ e_2 = E\dfrac{e^{j(\theta-\gamma)} + e^{-j(\theta-\gamma)}}{2} \end{cases} \qquad (5)$$

Here E is the maximal value of the induced electromotive force and γ is the power angle.

3. Mathematical model of the supply converter

For inverter's operation study at steady state we consider following idealized conditions:

- Power switch, that means the switch can handle unlimited current and blocks unlimited voltage.
- The voltage drop across the switch and leakage current through switch are zero.
- The switch is turned on and off with no rise and fall times.
- Sufficiently good size capacity of the input voltage capacitors divider, to can suppose converter input DC voltage to by constant.

This assumption helps us to analyze a power circuit and helps us to build a mathematical model for the inverter at steady state. Figure 3 shows two-phase converter circuit layout.

The output voltage level of the inverter can be controlled by a reduction of the DC source voltage. Another form of the voltage control is by a notching, where the transistors in the inverter circuit are turned on and off so as to produce zero periods of equal length.

An improvement to the notched waveform is to vary the on and off periods such that the on-periods are longest at the peak of the wave. This form of control is known as pulse-width modulation (PWM).

It can be observed that area of each pulse corresponds approximately to the area under the sine-wave between the adjacent mid-points of the off-periods. The pulse width modulated wave has much lower order harmonic content than the other waveforms.

If the desired reference voltage is sine-wave, two parameters define the control:

Coefficient of the modulation m - equal to the ratio of the modulation and reference frequency.

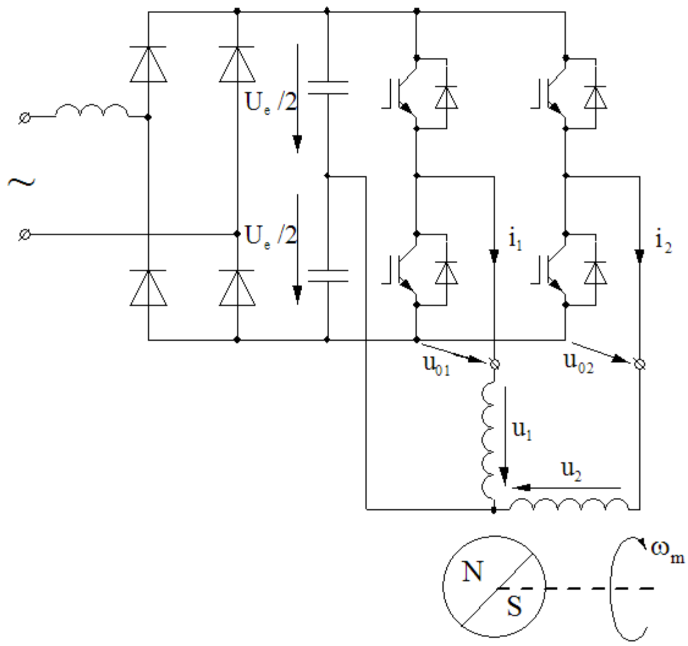

Fig. 3. Supply converter circuit layout

- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the DC supply voltage.

Generally the synchronic modulation is used. In synchronic modulation the modulation frequency is an integer multiple of the reference sine-wave.
Generally to control the inverter numeric control device is used. The turn on (α) and turn off (β) angles are calculated by the discredit of the reference sine-wave. That means the reference sine-wave is by a values discreet replaced. If the coefficient of modulation m is sufficiently great, the difference between real values and discrete values is negligible.

The phase's branches are controlled to create the output voltages:

$$\begin{cases} u_{01} = \dfrac{U_e}{2} + r\dfrac{U_e}{2}.sin\,\theta \\ u_{02} = \dfrac{U_e}{2} + r\dfrac{U_e}{2}.cos\,\theta \end{cases} \qquad (6)$$

U_e is a DC inverter's input voltage value.

To calculate a turn on (α) and turn off (β) angles we compare the DC impulse area with the requested voltage area, as depict on the Fig. 4.

For the left and right crosshatched areas the following equations are valid.

$$\begin{cases} \displaystyle\int_{\frac{2\pi}{m}\left(n-\frac{1}{2}\right)}^{n\frac{2\pi}{m}} \left(\frac{U_e}{2} + r\frac{U_e}{2}sin\,\theta\right)d\theta = U_e\left(n\frac{2\pi}{m} - \alpha_{01n}\right) \\ \displaystyle\int_{n\frac{2\pi}{m}}^{\frac{2\pi}{m}\left(n+\frac{1}{2}\right)} \left(\frac{U_e}{2} + r\frac{U_e}{2}sin\,\theta\right)d\theta = U_e\left(\beta_{01n} - n\frac{2\pi}{m}\right) \end{cases} \qquad (7)$$

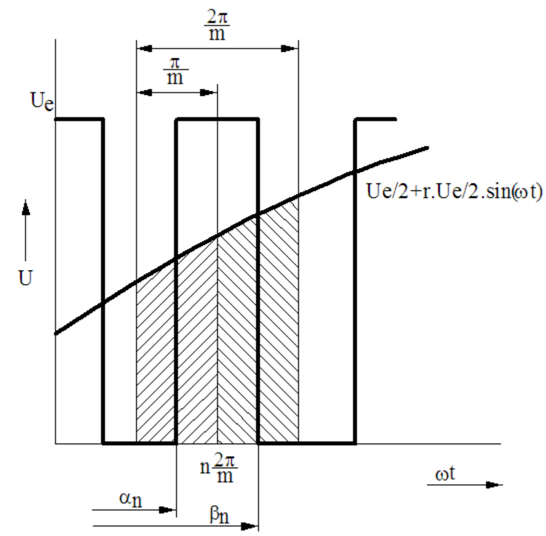

Fig. 4. Comparison the voltages areas

After the calculus we obtain for the first transistors branch:

$$\begin{cases} \alpha_{01n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) + \frac{r}{2}\left[cos\, n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n-1)\right] \\ \beta_{01n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) + \frac{r}{2}\left[cos\, n\frac{2\pi}{m} - cos\frac{\pi}{m}(2n+1)\right] \end{cases} \tag{8}$$

Similarly for the second transistors branch:

$$\begin{cases} \alpha_{02n} = \frac{\pi}{m}\left(2n - \frac{1}{2}\right) - \frac{r}{2}\left[sin\, n\frac{2\pi}{m} - sin\frac{\pi}{m}(2n-1)\right] \\ \beta_{02n} = \frac{\pi}{m}\left(2n + \frac{1}{2}\right) - \frac{r}{2}\left[sin\, n\frac{2\pi}{m} - sin\frac{\pi}{m}(2n+1)\right] \end{cases} \tag{9}$$

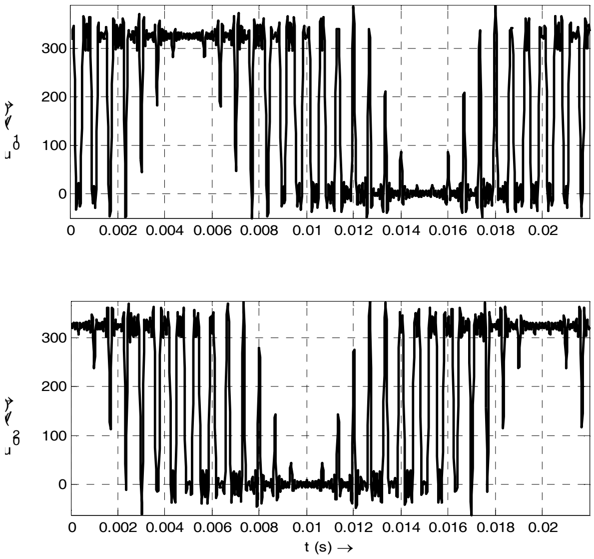

Fig. 5. The waveforms of the branch's voltages

The inverter's output voltage of the first branch can be mathematically expressed as a complex Fourier series of the form:

$$u_{01} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} \tag{10}$$

$$\begin{cases} c_{01n} = \frac{1}{j2k\pi}\left(e^{-jk\alpha_{01n}} - e^{-jk\beta_{01n}}\right) & for \quad k \neq 0 \\ c_{01n} = \frac{\beta_{01n} - \alpha_{01n}}{2\pi} & for \quad k = 0 \end{cases}$$

Similarly for the second branch:

$$u_{02} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} \tag{11}$$

$$\begin{cases} c_{02n} = \frac{1}{j2k\pi}\left(e^{-jk\alpha_{02n}} - e^{-jk\beta_{02n}}\right) & for \quad k \neq 0 \\ c_{02n} = \frac{\beta_{02n} - \alpha_{02n}}{2\pi} & for \quad k = 0 \end{cases}$$

Figure 5 shows the calculated waveforms of the branch's voltages. As seen the branch voltages are unipolar.

Based on the voltage equation, the phase voltages are given as a different between branch voltages and voltage of the capacitor divider:

$$\begin{cases} u_1 = u_{01} - \frac{U_e}{2} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} - \frac{U_e}{2} \\ u_2 = u_{02} - \frac{U_e}{2} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} - \frac{U_e}{2} \end{cases} \tag{12}$$

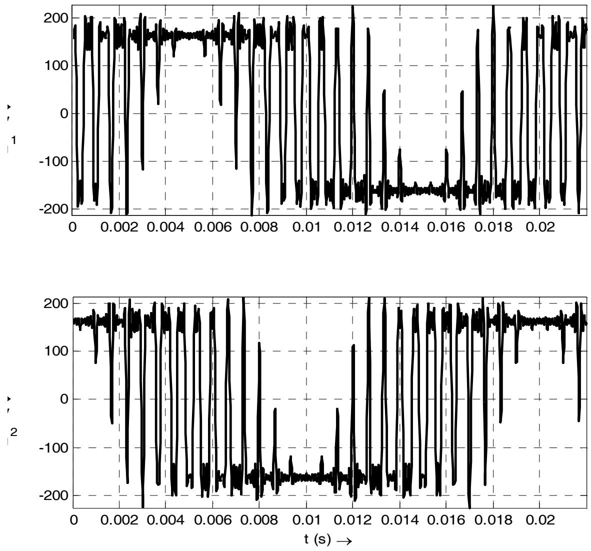

Fig. 6. The waveform of the phase's voltages

In the Figure 6 are shown the phases voltages waveforms. As seen the phase voltages are bipolar.

4. Motor currents calculation

For linear given circuits (Fig.2) the principle of superposition is valid. The phase current is the different of converter current and current of emf. Current of the first phase is given:

$$\begin{cases} i_1 = U_e \sum_{k=-\infty}^{\infty} \sum_{m=1}^{p} c_{01n} \frac{e^{jk\theta}}{R + jk\omega L} - \frac{U_e}{2R} \\ -\frac{E}{2j}\left(\frac{e^{j(\theta-\gamma)}}{R + j\omega L} - \frac{-e^{-j(\theta-\gamma)}}{R - j\omega L}\right) \end{cases} \tag{13}$$

Similarly for the second phase current:

$$\begin{cases} i_2 = U_e \sum_{k=-\infty}^{\infty} \sum_{m=1}^{p} c_{02n} \frac{e^{jk\theta}}{R + jk\omega L} - \frac{U_e}{2R} \\ -\frac{E}{2}\left(\frac{e^{j(\theta-\gamma)}}{R + j\omega L} + \frac{-e^{-j(\theta-\gamma)}}{R - j\omega L}\right) \end{cases} \tag{14}$$

In the Figure 7 are given the calculated of the phase inverter current output waveforms. The currents are calculated for $R = 2\Omega$; $L = 100mH$; amplitude of emf $E = 120V$ and power angle $\gamma = 36^{\circ}$.

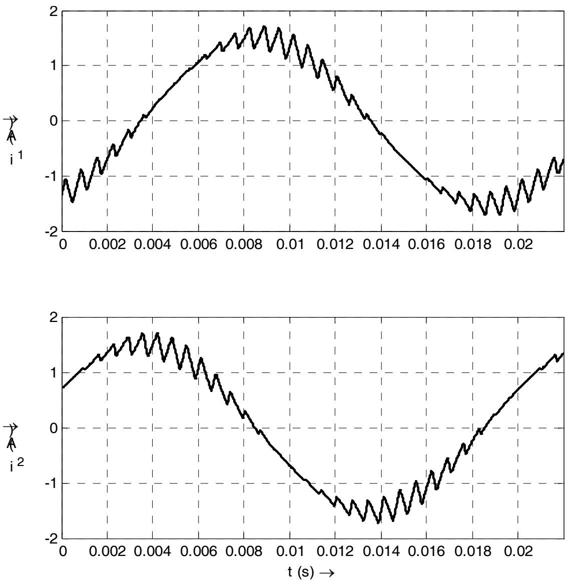

Fig. 7. Output current waveforms

5. Electromagnetic torque calculation

Assume two-pole permanent magnet synchronous machine. Instantaneous value of internal electromagnetic torque each of phases is given by the equations:

$$\begin{cases} m_1 = \dfrac{i_1 e_1}{\omega_m} \\ m_2 = \dfrac{i_2 e_2}{\omega_m} \end{cases} \tag{15}$$

Resultant electromagnetic torque is given by a sum of the phase torque.

$$m = m_1 + m_2 = \frac{i_1 e_1 + i_2 e_2}{\omega_m} \tag{16}$$

In the Figure 8 are given the waveforms form of internal electromagtnetic torque of each of phases and resultant electromagnetic torque for coefficient of modulation $m = 30$. In the Figure 9 are given the torque waveforms for coeficient of modulation $m = 50$. As seen, the resultant torque ripple depends on coefficient of modulation. The lower the coefficient of modulation, the torque ripple is higher. But increasing of coefficient of modulation evoke increasing of transistors switching losses.

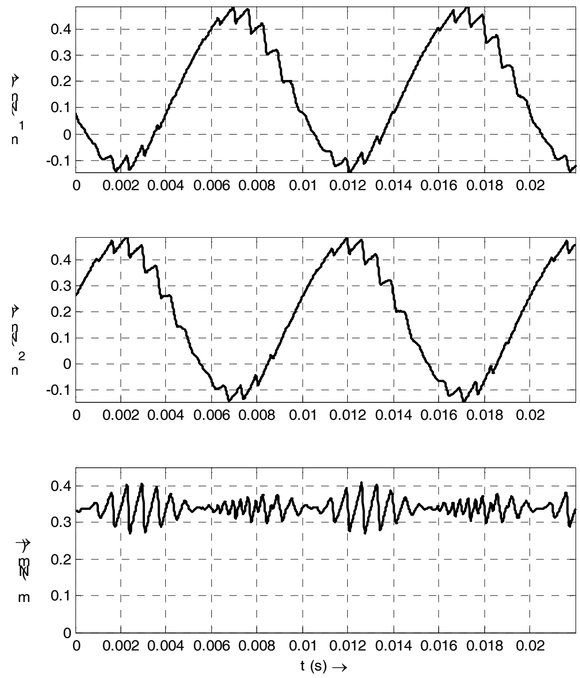

Fig. 8. Electromagnetic torque waveforms (m=30)

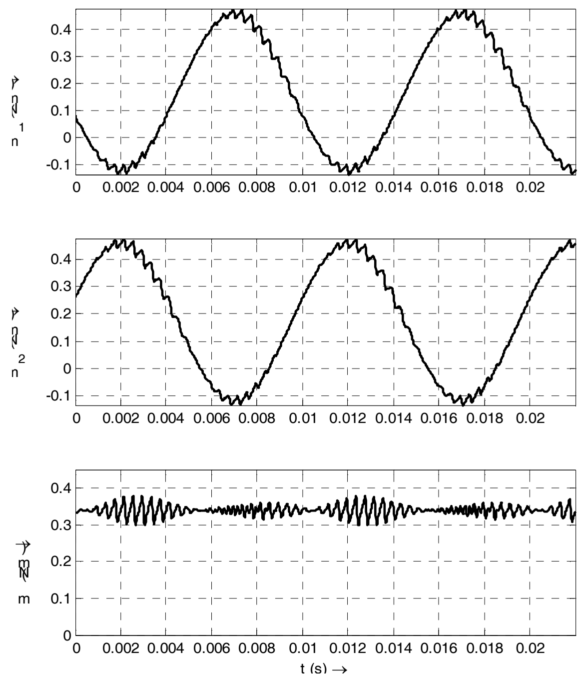

Fig. 9. Electromagnetic torque waveform (m=50)

Conclusion

A steady-state analysis of a two-phase permanent magnet synchronous motor drive with voltage-source inverter and PWM of the input voltage has been presented. An analytical expression has using complex Fourier series for the phase currents and electromagnetic torque of the motor has been obtained. The output electromagnetic torque ripples waveform has been calculated.

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