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COMPLEX FOURIER SERIES MATHEMATICAL MODEL OF A UNIVERSAL MOTOR SUPPLIED BY AN IGBT TRANSISTOR

MODEL MATEMATYCZNY SILNIKA UNIWERSALNEGO ZASILANEGO POPRZEZ TRANZYSTOR IGBT PRZY WYKORZYSTANIU ZESPOLONYCH SZEREGÓW FOURIERA

Abstract: The present contribution shows an analytical method of the calculus of the torque ripple and current waveforms of a universal motor supplied by an IGBT chopper. The chopper output voltage waveform is formulated by the Fourier series. The armature reaction of the motor is included in the calculus. The motor performance is computed using the circuit parameters determined by measurements. The calculated current waveforms are compared with the measured ones.

1. Introduction

Despite their disadvantages, universal motor belong to the most used electric machines in home appliances as well as workshop hand tools. Thanks to their excellent regulation properties they are employed as drive motors of washing machines, professional mixers or mills. Their versatility is given by the fact that they can be supplied by both direct and alternative voltages. In both cases the motor speed is controlled by the value of the supply voltage.

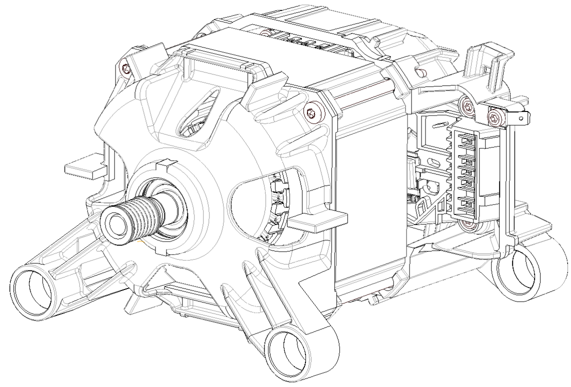

Fig. 1. Construction of universal motor used for washing machine drive

Main advantages of the universal motor include:

- Economical and smooth speed regulation in wide range.
- Easy start, large acceleration torque and small acceleration current.
- Good power factor.

Small universal motors are produced without compensating and commutation winding. The stator contains mostly two definitive poles with exciting winding. The number of the commutator brushes is equal to the number of exiting winding poles. Since magnetic field of the machine is alternating, the magnetic circuit is laminated. The stator and the rotor are series connected.

2. Mathematical model of the motor

Mathematical analysis of a universal motor is based on the voltage equations, with certain simplifying assumption [1]:

- Mechanical losses and also losses in iron are neglected.
- Commutation influence is neglected. A perfect commutating armature is assumed.
- Mutual inductance is supposed to be constant. Saturation effect is neglected.

Fig.2 depicts the equivalent circuit of a two-pole universal motor. The armature has two brushes on the diameter of the commutator and it is shifted by an angle α_a towards the axis of the exciting magnetic flux. The rotor rotates with mechanical angular velocity ω_m .

Suppose that the motor is supplied by a variable voltage u . In such a case the following voltage equation can be written:

$$u = Ri + L\frac{di}{dt} + u_i \qquad (1)$$

For the internal armature voltage , the folowing general equation :

$$u_i = M\frac{di}{dt}\sin\alpha_a - M\omega_m\cos\alpha_a \qquad (2)$$

M is a mutual inductance between stator and rotor

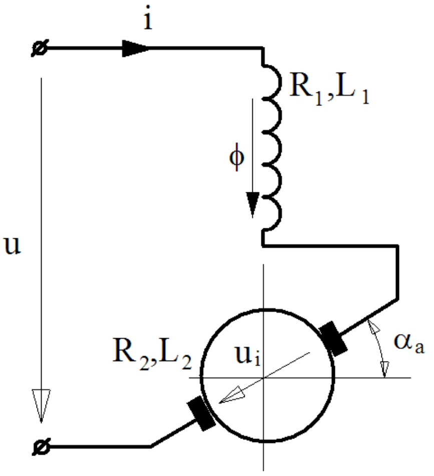

Fig. 2. Equivalent circuit of a universal motor

The instantaneous value of the induced electromagnetic torque is given by the formula:

$$m = Mi^2\cos\alpha_a \qquad (3)$$

3. Mathematical model of supply chopper

Fig. 3 shows the IGBT controlled universal motor drive [3], [4]. Suppose that the supply mains voltage is purely sinusoidal. At the output of the diode bridge rectifier one obtains the absolute value of the main supply voltage.

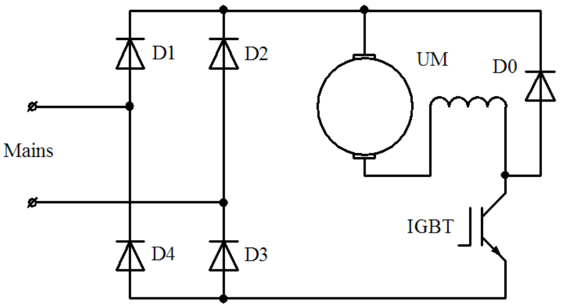

Fig. 3. IGBT universal motor drive

This supply voltage is chopping up with defined chopping frequency and duty cycle. Two parameters define the control of the supply voltage:

- *Coefficient of the modulation* m - equal to the ratio of the modulation and reference frequency.
- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the supply voltage amplitude [3].

The supply output voltage consists of the series of particular impulsions with amplitude equal to the instantaneous value of the rectifier output voltage.

Chopper output voltage can be expressed in a form of the complex Fourier series:

$$u = U_m\sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}c_{kn}e^{-j2k\theta} \qquad (4)$$

With Fourier coefficient:

$$c_{kn} = \frac{U_m}{\pi\left(1-4k^2\right)}\Big[e^{-j2k\alpha_n}\left(j2k\sin\alpha_n + \cos\alpha_n\right) - e^{-j2k\beta_n}\left(j2k\sin\beta_n + \cos\beta_n\right)\Big] \qquad (5)$$

Switch on and switch off angles a given by a formula:

$$\alpha_n = \left(n-1\right)\frac{\pi}{m}, \qquad \beta_n = \left(n-1+r\right)\frac{\pi}{m} \qquad (6)$$

With $n = 1,2,\ldots,m$

Fig.4 depict the waveform of the chopper output voltage for $m = 5$ and $r = 0{,}5$.

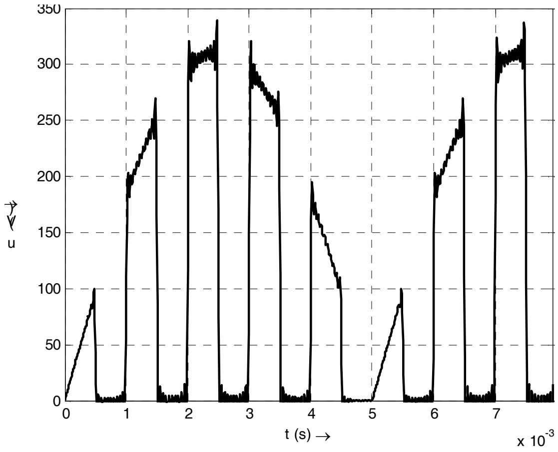

Fig. 4. The chopper output voltage

4. Motor current and electromagnetic torque calculation

To determine the motor current waveform, we need to solve equation *(1)*. After substituting *(2)* and *(4)* into *(1)* we obtain.

$$U_m\sum_{k=-\infty}^{\infty}\sum_{n=1}^{m}c_{kn}e^{-j2k\theta} = \sum_{k=-\infty}^{\infty}\Big[\left(R + M\omega_m\sin\alpha_a\right)i + \left(L - M\cos\alpha_a\right)j\omega ki\Big] \qquad (7)$$

The differential equation *(7)* has the following analytical solution:

$$i = U_m \sum_{k=-\infty}^{\infty} \frac{\sum_{n=1}^{m} c_{kn} e^{-j2k\theta}}{\left(R + M\omega_m \sin\alpha_a\right) + j\omega k\left(L - M\cos\alpha_a\right)} \qquad (8)$$

Instantaneous value of intern electromagnetic torque is given by a formula *(3)*.

5. Computation results and their comparison with experimental data

In order to compare the results obtained from the mathematical models of the machine and chopper with measured values, the following parameters of the motor were used:

Machine power	$800W / 230V - 50Hz$
Stator resistance	$1{,}61\,\Omega$
Stator inductance	$71{,}4\ mH$
Rotor resistance	$3{,}04\,\Omega$
Rotor inductance	$20{,}7\ mH$
Mutual inductance	$63{,}1\ mH$

Using the equations of the previous sections, the waveforms of the supply voltage, motor current and electromagnetic torque were calculated.

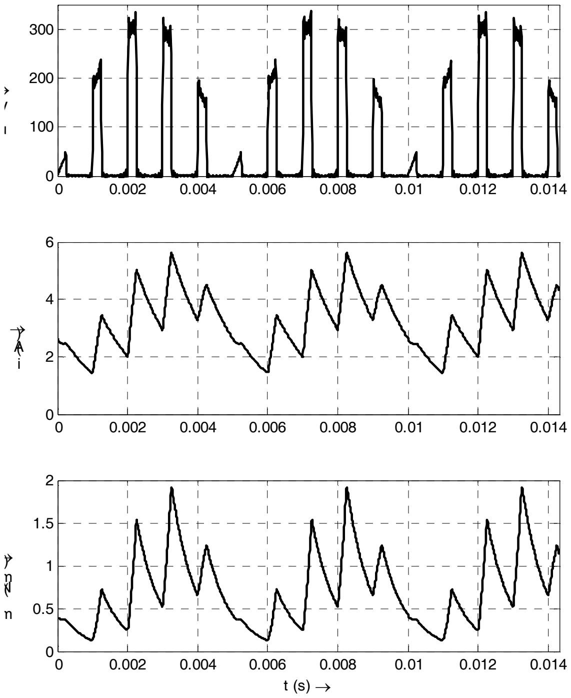

Fig. 5. Calculated motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 500 Hz

The motor worked at speed $6000\ rpm$ and was loaded by a constant torque $0{,}7\ Nm$.

The chopper was set to work with $25\,\%$ of duty cycles. Based on the bibliography [2], was angle of armature reaction α_a assumed to be 15°.

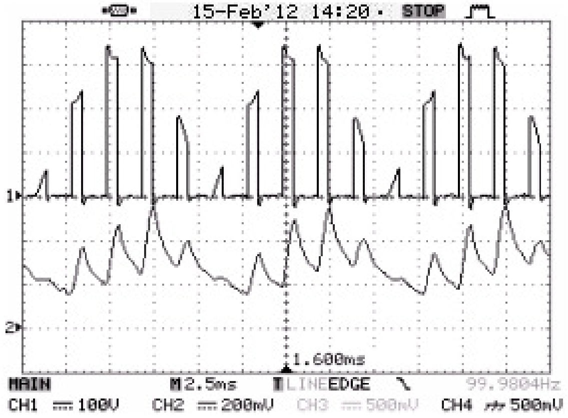

Fig. 6. Measured motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 500 Hz (U:100V/div, I:2A/div)

Fig.5 and Fig.6 show the calculated and measured quantities at steady state at the supply voltage chopped with a frequency $500\ Hz$.

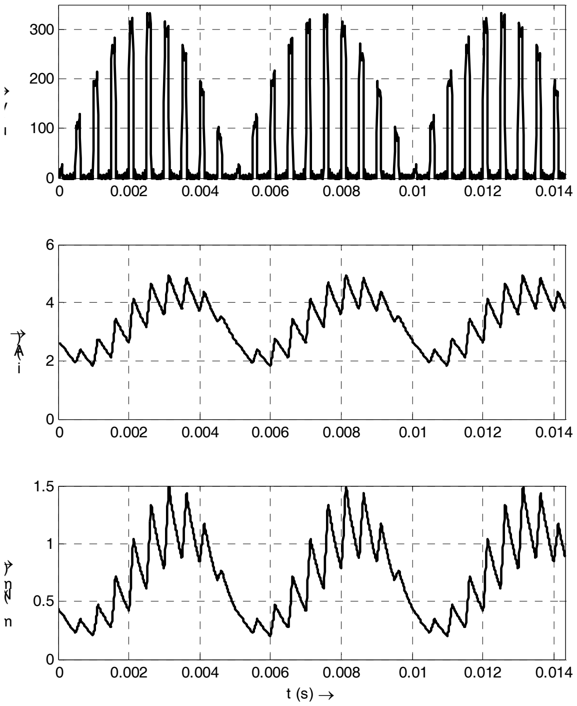

Fig. 7. Calculated motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 1 kHz

Fig.7 and Fig.8 show the same quantities for chopping frequency $1\ kHz$.

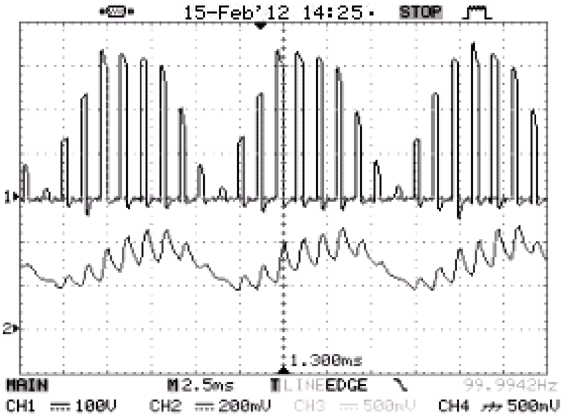

Fig. 8. Measured motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 1 kHz (U:100V/div, I:2A/div)

Fig.9 and Fig.10 show the calculated and measured quantities at steady state at the supply voltage chopped with a frequency 2 kHz.

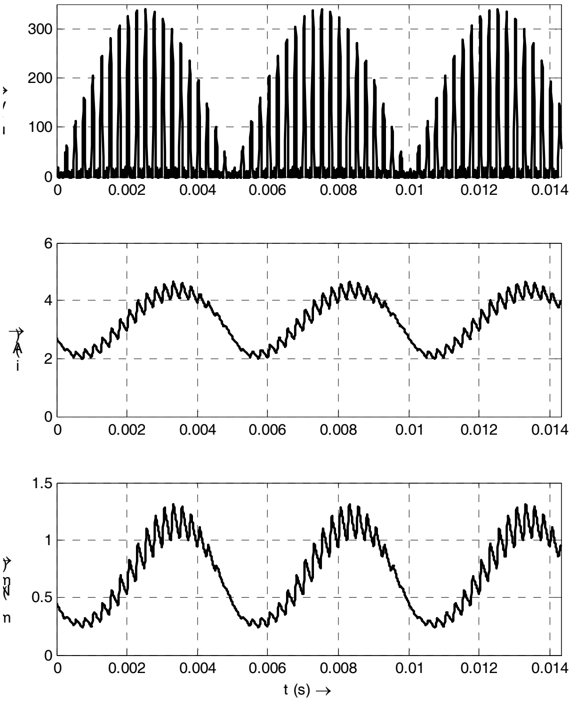

Fig. 9. Calculated motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 2 kHz

6. Conclusion

The paper describes the analytical method for modelling the behaviour of a universal motor supplied by a single phase voltage bridge rectifier and chopped by an IGBT transistor at steady state. The method is based on the complex Fourier series for the description of the motor supply voltage.

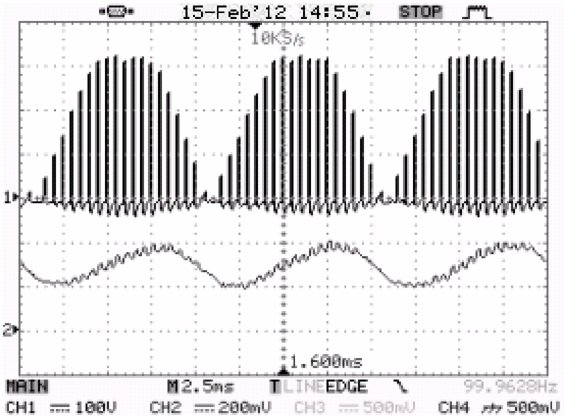

Fig. 10. Measured motor quantities for 6000 rpm, T=0,7 Nm, 25% duty cycle, chopping frequency 2 kHz (U:100V/div, I:2A/div)

The voltage, motor current and electromagnetic torque is calculated for different operation regimes of the motor.

In order to verify the calculated waveforms a motor supplied by an IGBT chopper was measured. The calculated and experimental results show that the method represents a reliable tool for calculating current and instantaneous torque of the universal motor.

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