

## Thermal effects on magnetic hysteresis modeling

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(Received: 25.04.2011, revised: 17.09.2011)

**Abstract:** A temperature dependent model is necessary for the generation of hysteresis loops of ferromagnetic materials. In this study, a physical model based on the Jiles-Atherton model has been developed to study the effect of temperature on the magnetic hysteresis loop. The thermal effects were included through a model of behavior depending on the temperature parameters  $M_s$  and  $k$  of the Jiles-Atherton model. The temperature-dependent Jiles-Atherton model was validated through measurements made on ferrite material (3F3). The results have been found to be in good agreement with the model.

**Key words:** Jiles-Atherton model, magnetic hysteresis, temperature, modeling

### 1. Introduction

Magnetic hysteresis is encountered in the operation of most electrical engineering devices (motors, transformers, magnetic recording, and components for power electronics). The development of a model can accurately describe this cycle and its variation depending on the operating regime that is still an issue that inspires researchers.

Amongst these models, we find the Jiles-Atherton model which is based on energy considerations, that is on the movement of Bloch walls within the material. Its inconvenience lies in the identification of parameters and especially in case of temperature change. All machines are electric seat Joule losses, hysteresis and eddy currents, making these devices heat up. For instance, the temperature can be very high in case of heating induction furthermore, the integration of the temperature in the hysteresis model is imperative for a reliable estimate of losses related to this phenomenon.

Taking into account the variation of the five parameters model depending on the temperature during the integration of this model in a computer code by finite element field is a formidable task, especially in the study of magnetic electromagnetic device. In this paper, we propose the introduction of temperature effect in the Jiles-Atherton model through two parameters only, the  $M_s$  and  $k$  by taking into account the hysteresis.

## 2. The Jiles-Atherton model

The original J-A model presented earlier [1], gives the magnetization  $M$  versus the external magnetic field  $H$ . This model is based on the magnetic material response without hysteresis losses. This is the anhysteretic behavior which  $M_{an}$  curve that can be described with a modified Langevin equation:

$$M_{an} = M_s \left( \coth\left(\frac{H_e}{a}\right) - \frac{a}{H_e} \right), \quad (1)$$

where  $H_e = H + \alpha M$  is the effective field experienced by the domains:  $H$  is the external applied field and  $a$  mean field parameter representing inter-domain coupling. The anhysteretic magnetization represents the effects of moment rotation within domains but does not take into account losses induced by domain wall motions. Then, by considering rigid and planar domain walls, the energy dissipated through pinning sites during a domain wall displacement is calculated [1, 2]. The expression of the magnetization energy is obtained under the assumption of a uniform distribution of pinning sites. The magnetization energy is assumed to be the difference between the energy which would be obtained in the anhysteretic case minus the energy due to the losses induced by domain wall motions. Consequently, after some algebraic operations, the differential susceptibility of the irreversible magnetization  $M_{irr}$  can be written as

$$\frac{dM_{irr}}{dH_e} = \frac{M_{an} - M_{irr}}{k\delta}, \quad (2)$$

where the constant  $k$  is linked to the average pinning site energy [2]. The parameter  $\delta$  takes the value +1 when  $dH/dT > 0$  and  $dH/dT < 0$  with respect to the force which opposes variations of magnetization. However, during the magnetization process, domain walls do not only jump from one pinning site to another: they are flexible and bend when being held on pinning sites. Domain wall bending is associated to reversible changes in the magnetization process. Then, by some physical energy assumptions on the domain wall bending, the obtained reversible magnetization is linearly dependent on  $M_{an} - M_{irr}$  [1]:

$$M_{rev} = c(M_{an} - M_{irr}), \quad (3)$$

where the reversibility coefficient  $c$  belongs to the interval [0-1]. Assuming that the total magnetization is the sum of the reversible and irreversible components, we have the following expression:

$$M = M_{rev} + M_{irr} \quad (4)$$

with  $M_{irr}$  and  $M_{rev}$  defined by Eqs. (2) and (3). Using Eqs. (4) and (3) we can write

$$M = M_{irr} + c(M_{an} - M_{irr}). \quad (5)$$

Then, by differentiating this equation with respect to  $H$ ; the total differential susceptibility of the system is given by the following expression which has already been presented elsewhere [4]:

$$\frac{dM}{dH} = \frac{(1-c)\frac{dM_{irr}}{dH_e} + c\frac{dM_{an}}{dH_e}}{1 - \alpha c\frac{dM_{an}}{dH_e} - \alpha(1-c)\frac{dM_{irr}}{dH_e}}. \quad (6)$$

The J-A theory has already been extended to incorporate magneto-elastic effects [5] and anisotropy [6]. The previous works on extending the model to include thermal effects [7, 8, 12, 13] were based on the fitting of model parameters to experimental data. This work focuses on expressing the microstructural hysteresis parameters as functions of temperature and compares the resulting temperature-dependent J-A model to experimental curves.

### 3. Temperature dependence in J-A model

Thermal effects can be incorporated into the model through the temperature dependence of hysteresis parameters in (1)-(2): spontaneous magnetization  $M_s$  and parameter  $k$ .

#### 3.1. Spontaneous magnetization $M_s$

The temperature dependence of spontaneous magnetization  $M_s$  can be expressed using Weiss theory of ferromagnetism [9]

$$M_s(T) = M_s^{Ta} \left( 1 - \exp\left(\frac{T - T_c}{\tau_{M_s}}\right) \right), \quad (7)$$

where  $M_s^{Ta}$  is the value of spontaneous magnetization at room temperature,  $T_c$  is the Curie temperature, and  $\tau_{M_s}$  is the constant defined on the experimental curve of spontaneous magnetization with temperature [9].

#### 3.2. Parameter $k$

In soft magnetic materials, the parameter  $k$  can be approximated to coercivity ( $k \approx H_c$ ) [2].

Due to the exponential decay of coercive field in a ferromagnetic material [10], the domain wall parameter  $k$  should vary exponentially with temperature according to the equation

$$k(T) = k^{Ta} \exp\left(\frac{-\tau_{H_c} T}{T_c}\right), \quad (8)$$

where  $k^{Ta}$  is the pinning factor at room temperature and  $\tau_{H_c}$  is the constant defined on the experimental curve of coercive field with temperature.

#### 3.3. General identification procedure

The hysteresis parameters governing the magnetization processes  $M_s$ ,  $k$ ,  $a$ ,  $c$  and  $\alpha$  can be identified from the magnetic properties such as initial susceptibility  $\chi_{in}$ , anhysteretic susceptibility  $\chi_{an}$ , coercivity  $H_c$ , and remanence  $M_r$ . The identification procedures are well docu-

mented in [11] and are sufficient for modeling purposes. The measured spontaneous magnetization  $M_s$  and parameter  $k$  as a function of temperature was used to estimate the  $\tau_{M_s}$  and  $\tau_{H_c}$ , by fitting the analytical model shown in (7) and (8). This procedure will describe hysteretic behavior at any temperature up to Curie point.

### 3.4. Example calculations

For example, after identifying the necessary hysteresis parameters,  $M_s(Ta)$ ,  $k(Ta)$ ,  $a(Ta)$ ,  $c(Ta)$ , and  $\alpha(Ta)$ , the spontaneous magnetization versus temperature curve was used to identify  $\tau_{M_s}$  and  $T_c$  as shown in Figure 1. The spontaneous magnetization drops rapidly to zero at 220°C, indicating the existence of a transition from a magnetically ordered state to a magnetically disordered state.

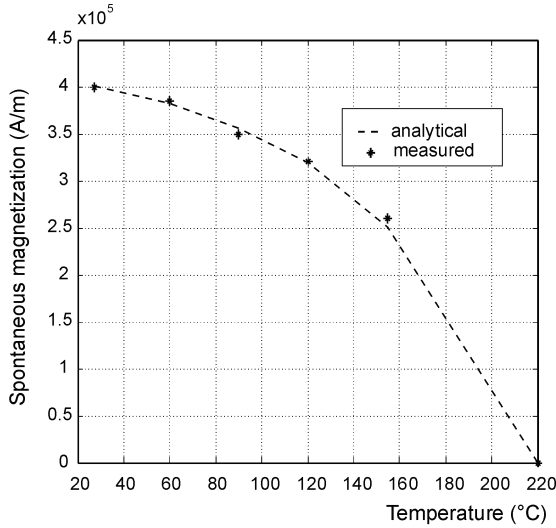


Fig. 1. Variation of spontaneous magnetization with temperature measured in a 3F3 material with Curie point at  $T_c = 220^\circ\text{C}$ . The measured data was fit to the analytical model shown in (7) to estimate  $\tau_{M_s}$ .

The parameter  $k$  versus temperature curve was used to identify  $\tau_{H_c}$ . The parameter  $k$  drops exponentially with temperature, indicating the exponential decrease of coercive field [10], as shown in Figure 4.

The temperature-dependent magnetization loops were calculated for a set of temperatures ranging from 25°C to 200°C, as shown in Figure 3. As the temperature increases towards the Curie point ( $T_c = 220^\circ\text{C}$ ), the hysteresis loop gradually flattens and after, the material becomes paramagnetic.

## 4. Comparison with experimental

Figure 2 illustrates the experimental setup for measuring magnetic hysteresis loops at different temperatures using ring sample. The sample is a torus with primary and secondary windings. Excitation is applied using an arbitrary function generator which allows us to

impose current or voltage on the primary winding. A computer remotely controls these devices. Using the Ampere and Faraday laws, the field  $H$  and the magnetic flux density  $B$  are calculated from the measurement of the current in the primary winding and the secondary winding voltage.

The sample under test was placed in the middle of an environment chamber, which could maintain a constant temperature over a wide range from 25°C to 160°C.

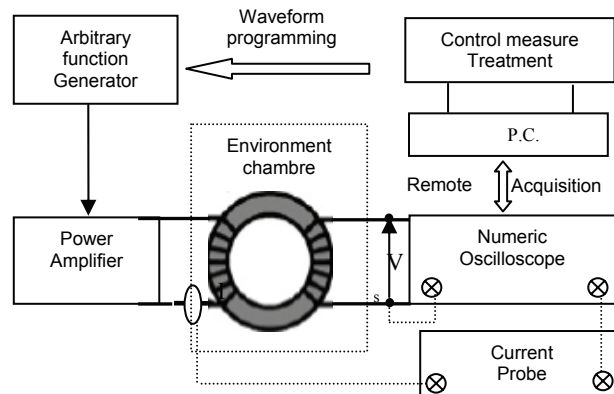


Fig. 2. System for measuring magnetic hysteresis loops

The temperature dependent J-A model was validated against experimental data of 3F3 material with Curie point at 220°C. The hysteresis loops were measured using the experimental setup of Figure 2 at various temperatures ranging from 27°C to 155°C and compared to those calculated numerically.

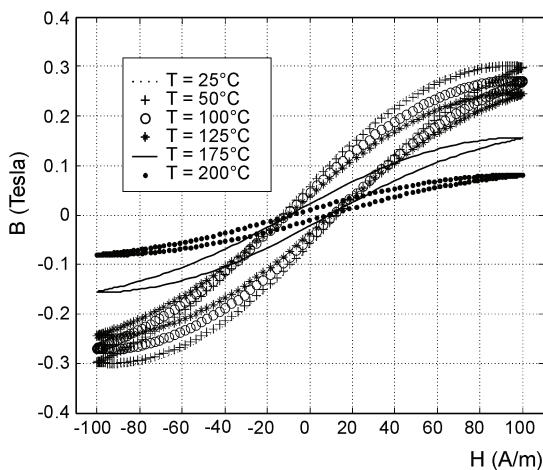


Fig. 3. Calculated temperature dependence of magnetic hysteresis loops in a 3F3 material with Curie point at  $T_c = 220^\circ\text{C}$ . The parameters used in the modeling are  $M_s(T)$ ,  $K(T)$ ,  $a = 33.5$ ,  $c = 0.45$ ,  $\alpha = 3.45 \times 10^{-6}$ ,  $\tau_{M_s} = 75$  and  $\tau_{H_c} = 0.6$

Figure 4 shows the dependence of coercive field of ferrite (3F3) on temperature. The coercivity decays exponentially with temperature as expected [10]. It is evident from the figure that the analytical model is in good agreement with the measurements.

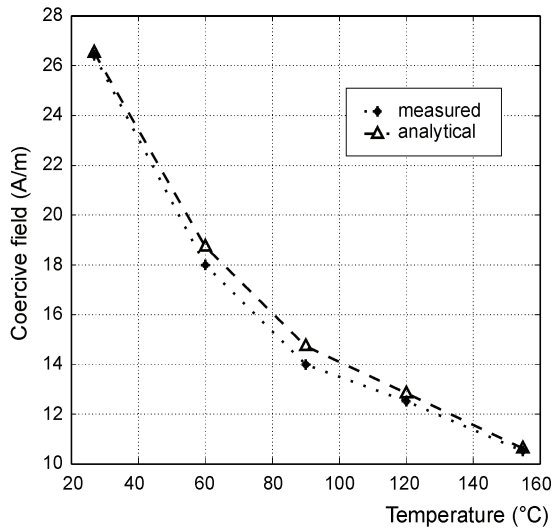


Fig. 4. Variation of coercive field with temperature in a 3F3 material with Curie point at  $T_c = 220^\circ\text{C}$ . The analytical model is in good agreement with the measurements

The measured and calculated hysteresis loops of 3F3 material at temperatures  $27^\circ\text{C}$ ,  $60^\circ\text{C}$ , and  $155^\circ\text{C}$  are compared in Figure 5. The hysteresis loop gradually flattens as the temperature approaches the Curie point. After the Curie point, the material becomes paramagnetic and does not exhibit hysteresis.

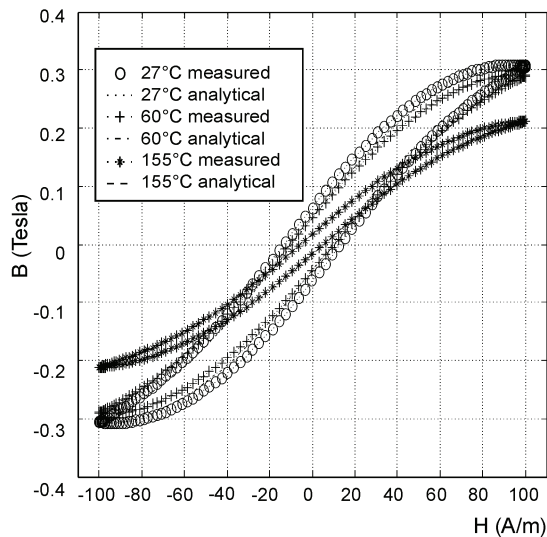


Fig. 5. Variation of magnetic hysteresis loops with temperature in a 3F3 material with Curie point at  $T_c = 220^\circ\text{C}$ . The analytical model is in good agreement with the measurements

In this approach we introduced the temperature effect in the physical model of J-A to generate the hysteresis loops depending of the temperature, using the thermal behavior of two parameters: the spontaneous magnetization and the parameter  $k$ . For the evolution of the parameter  $k$  depending of the temperature, we used the evolution of  $H_c$  as function of temperature, these two parameters ( $M_s$ ,  $H_c$ ) are easy to measure and their variation with temperature is remarkable, compared to other parameters. So we can directly determine the two constants  $\tau_{M_s}$  and  $\tau_{H_c}$ . The hysteresis loops simulated by the proposed model depending of the temperature are close to the hysteresis loops measured despite that the three parameters remain constant as a function of temperature.

This approach does not give very good results in case the three parameters ( $a$ ,  $c$  and  $\alpha$ ) are significantly related to temperature change.

The proposed method should be applicable the other types on magnetic materials that have the parameter  $k$  very close to the coercive field  $H_c$

## 5. Conclusions

Jiles-Atherton theory was developed. The thermal effects were included in the model of Jiles-Atherton using of thermal behavior of two parameters of Jiles-Atherton model: spontaneous magnetization and the parameter  $k$ . The identification procedure for the suggested model is to calculate the model parameters to room. In this work a model of hysteresis depending on the temperature based on an extension of the temperature, and determine the two constants  $\tau_{M_s}$  and  $\tau_{H_c}$ , starting from the evolution of  $M_s$  and  $H_c$  as a function of temperature. The model can even be integrated into a computer code field finite element to study the coupled magneto-thermal phenomena. The obtained results are approved.

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