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TORSIONAL VIBRATIONS IN THE ELECTRIC DRIVE SYSTEM WITH THE STEPPING MOTOR

ABSTRACT *In the paper there is studied dynamic interaction between electromechanical parts of the precise micro-drive system driven by the hybrid stepping motor which is a source of torsional vibrations. Since in such a case a possibly exact rotational motion of the mechanism must be assured, it is necessary to use sufficiently accurate models of the drive system and the electric motor, where dynamic electromechanical coupling effects are going to be taken into consideration. So that models are presented. From the computational results it follows that there is observed a significant influence of the electromechanical coupling on the dynamic behavior of the drive system driven by the stepping motor.*

Keywords: *stepping motor, driver system, mathematical model, torsional vibrations.*

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1. INTRODUCTION

Modern electric drive systems operate in the dynamic states which result from the modes of supplying the windings of the electric motors. The consequence of the transient states of the electric currents in the motor windings is the angular course of the electromagnetic torque different from the desirable constant or sinusoidal one. Then, the variable electromagnetic torque more or less significantly influence motion of the mechanical part of the drive system. Thus, the mechanical part starts exhibit the propensity to torsional vibrations [6, 7].

The problem of the torsional vibrations occurs in a particularly severe way in the drive systems driven by the stepping motors [1, 2, 3].

The stepping motors are the electromagnetic devices which can convert electrical pulses into precisely defined increments of its rotor position. These electrical pulses are changing the voltage signal system that supplies the coils that create the motor windings. The frequency of electrical pulses is called the stepping rate, stepping frequency, commutation frequency or input frequency. The stepping frequency is forced external and it is not dependent on the state of the motor.

Drive systems with stepping motors are designed primarily for use in the precise step-by-step positioning that is realized in open-loop mode. The stepping motor’s rotor reaches following stable equilibrium positions with variable acceleration or performs oscillations around the temporary static equilibrium position. During angular transition between following stable equilibrium positions the oscillations of the angular speed occur. The proximate cause of these oscillations is the variation of the electromagnetic torque which occurs during the discrete supplying the motor windings when the energy excess, over the value that is need to carry out the useful work, is supplied into the drive system. The oscillations of the angular speed generate more or less significant variable components of the induced voltages which are the source of the distortion of the electromagnetic torque angular course and thus additional vibrations of the entire mechanical part of the drive system. The phenomenon that the electrical part and the mechanical part has an effect on each other is called the electromechanical coupling and it plays an important role in transient states the electric drive systems.

The stepping motors are commonly applied in drive systems of various levels of structural and kinematical complexity. In this paper there is performed an analysis of transient electro-mechanical torsional vibrations of the drive system in rotational motion. This system is driven by the stepping motor with a permanent magnet (for instance hybrid stepping motor).

The main investigated problem can be formulated in the following way: how the torsional vibrations of the drive system driven by the stepping motor depend on the mechanical structure complexity?

2. MODELLING THE DRIVE SYSTEM WITH THE HYBRID STEPPING MOTOR

Currently, various advanced physical and mathematical models both of the stepping motors and the mechanical drive systems are applied in an engineering practice. Nevertheless, sufficiently reliable and not very complicated models seem to the most advantageous for computational studies of the coupled electromechanical effects.

In this paper the linear model of the stepping motor is used with the assumed constant values of electric and mechanical parameters as well as with the characteristic of the static electromagnetic torque versus rotor angular displacement of a sinusoidal shape for constant values of the winding currents [3, 6].

The mathematical model of the considered here two-phase hybrid stepping motor reduces to the following equations:

- the equations for the electric circuits:

$$L_0 \frac{di_1}{dt} - \frac{d\gamma}{dt}\left[K_E \sin\left(Z_r \gamma\right)\right] + R\, i_1 = u_1\left(t\right);$$

$$L_0 \frac{di_2}{dt} + \frac{d\gamma}{dt}\left[K_E \cos\left(Z_r \gamma\right)\right] + R\, i_2 = u_2\left(t\right) \tag{1}$$

- the functional description of the electromagnetic torque:

$$T_e = -K_T \cdot i_1\left(t\right) \cdot \sin\left(Z_r \gamma\right) + K_T \cdot i_2\left(t\right) \cdot \cos\left(Z_r \gamma\right) \tag{2}$$

- the functional description of the control voltage signals:

$$u_1\left(t\right) = U\ sign\left[\cos\left(0.5 \cdot \pi \cdot f_k \cdot t\right)\right];$$

$$u_2\left(t\right) = U\ sign\left[\sin\left(0.5 \cdot \pi \cdot f_k \cdot t\right)\right] \tag{3}$$

where: u denotes the terminal phase winding voltage, U is the rated voltage, i denotes the exciting current in the phase windings, γ is the electrical angular position of the rotor with respect of the stator, K_E denotes the voltage constant, K_T is the torque constant, L_0 denotes the constant component of phase inductance, R is the phase resistance, Z_r denotes the number of the rotor teeth, f_k is the stepping frequency (pulses per second) and t denotes time.

In the case of current mode supply it is convenient to modify the stepping motor mathematical model in the following way:

- the functional description of the electromagnetic torque:

$$T_e = -K_T \cdot i_1(t) \cdot \sin(Z_r \gamma) + K_T \cdot i_2(t) \cdot \cos(Z_r \gamma) \tag{4}$$

- and the functional description of the control current signals:

$$i_1(t) = \frac{U}{R} sign\left[\cos(0.5 \cdot \pi \cdot f_k \cdot t)\right]; \quad i_2(t) = \frac{U}{R} sign\left[\sin(0.5 \cdot \pi \cdot f_k \cdot t)\right] \tag{5}$$

The mechanical part of the drive systems driven by the stepping motors can be substituted by discrete models consisting of numerous rigid bodies mutually connected by the mass-less visco-elastic torsional springs. These rigid bodies represent rotational inertia of the successive real object components and the torsional springs express their twisting flexibility. In such models a number of rigid bodies usually correspond to the number of considered degrees of freedom [7, 8].

In a general case of the multi-degree of freedom torsional train representing the mechanical system motion equations of the corresponding discrete model can be expressed in the following matrix form:

$$\mathbf{J}\ \ddot{\gamma}(t) + \mathbf{D}\ \dot{\gamma}(t) + \mathbf{K}\ \gamma(t) = \mathbf{T} \tag{6}$$

where $\boldsymbol{J}$ is the mass matrix containing mass moments of inertia of the rigid bodies, $\boldsymbol{D}$ denotes the matrix of damping coefficients, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{T}$ denotes the external torque vector and $\ddot{\gamma}(t), \dot{\gamma}(t), \gamma(t)$ are the column matrices of angular accelerations, speeds and displacements. For example, in the case of the three-degree of freedom model consisting of three rigid bodies of mass moments of inertia J_1, J_2, J_3, respectively connected by two visco-elastic springs of torsional stiffness K_{12}, K_{23}, relative (i.e. rigid disk to disk) damping coefficients D_{12}, D_{23} and absolute (i.e. rigid disk to surrounding) damping coefficients D_1, D_2, D_3, to which the external electromagnetic torque T_e and the retarding torque T_L are imposed, the mentioned above matrices have the forms:

$$\mathbf{J} = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} D_1 + D_{12} & -D_{12} & 0 \\ -D_{12} & D_{12} + D_2 + D_{23} & -D_{23} \\ 0 & -D_{23} & D_{23} + D_3 \end{bmatrix} \tag{7}$$

$$\mathbf{K} = \begin{bmatrix} K_{12} & -K_{12} & 0 \\ -K_{12} & K_{12} + K_{23} & -K_{23} \\ 0 & -K_{23} & K_{23} \end{bmatrix} \quad \boldsymbol{\gamma}(t) = \begin{bmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \gamma_3(t) \end{bmatrix} \quad \dot{\boldsymbol{\gamma}}(t) = \begin{bmatrix} \dot{\gamma}_1(t) \\ \dot{\gamma}_2(t) \\ \dot{\gamma}_3(t) \end{bmatrix} \quad \ddot{\boldsymbol{\gamma}}(t) = \begin{bmatrix} \ddot{\gamma}_1(t) \\ \ddot{\gamma}_2(t) \\ \ddot{\gamma}_3(t) \end{bmatrix} \quad \mathbf{T} = \begin{bmatrix} T_e \\ 0 \\ -T_L \end{bmatrix}$$

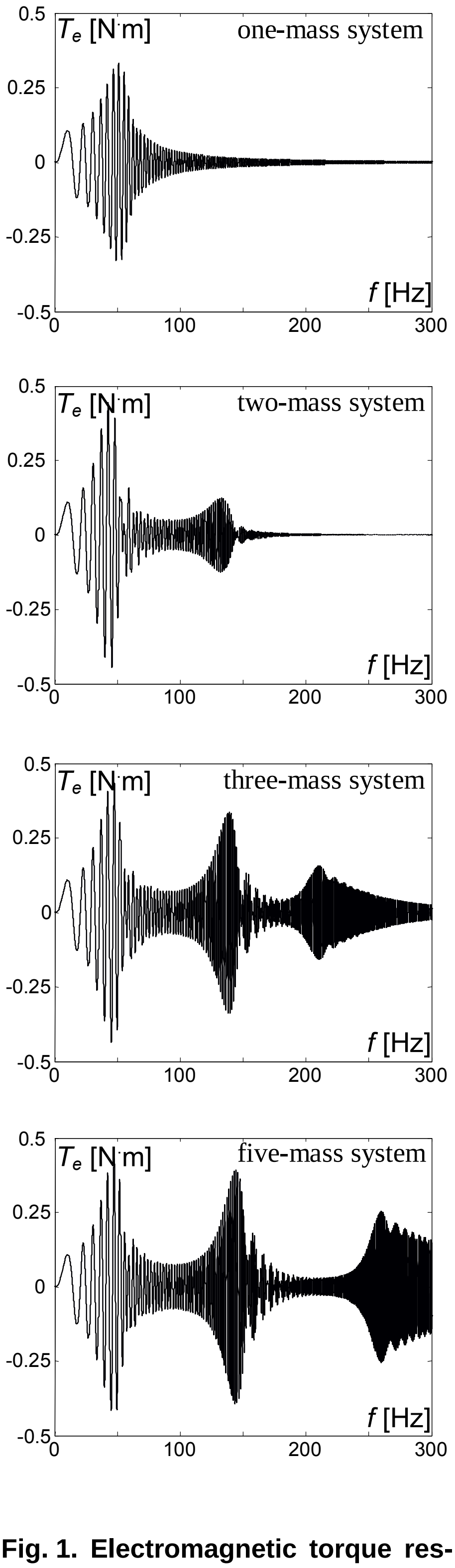
3. RESULTS OF SIMULATIONS

To examine how the torsional vibrations of the drive system driven by the stepping motor depend on the number of degrees of freedom of the mechanical model, numerous computer simulations based on the mathematical model represented by equations (1), (2), (3) and (6) have been performed. In addition to the absence of mechanical damping, the considered drive system under harmonic retarding torque T_L of various excitation frequencies f has been investigated. In such conditions particularly severe torsional vibrations are induced because of a lack of damping which naturally reduces vibration amplitudes. It is worth noting that the two-phase hybrid stepping motor operating in two--phase-on voltage mode is characterized by relatively high electromagnetic damping.

The simulations have been performed for the system driven by the stepping motor of the following parameters:
U = 3,85 V, R = 7,7 Ω, L_0 = 0,0217 H, $K_T = K_E$ = 0,554 Nm/A, Z_r = 50.

In the computational examples the following cases of mechanical drive system discretisation have been considered:

- the one-mass system with J_1 = 0.0003 kg·m^2,
- the two-mass system with J_1 = 0.0001 kg·m^2, J_2 = 0.0002 kg·m^2, K_{12} = 25 Nm/rad,
- the three-mass system with J_1 = 0.0001 kg·m^2, J_2 = 0.0001 kg·m^2, J_3 = 0.0001 kg·m^2, K_{12} = 50 Nm/rad, K_{23} = 50 Nm/rad and
- the five-mass system with J_1 = 0.0001kg·m^2, J_2 = 0.00005kg·m^2, J_3 = 0.00005kg·m^2, J_4 = 0.00005kg·m^2, J_5 = 0.00005kg·m^2, K_{12} = 100 Nm/rad, K_{23} = 100 Nm/rad, K_{34} = 100 Nm/rad, K_{45} = 100 Nm/rad.



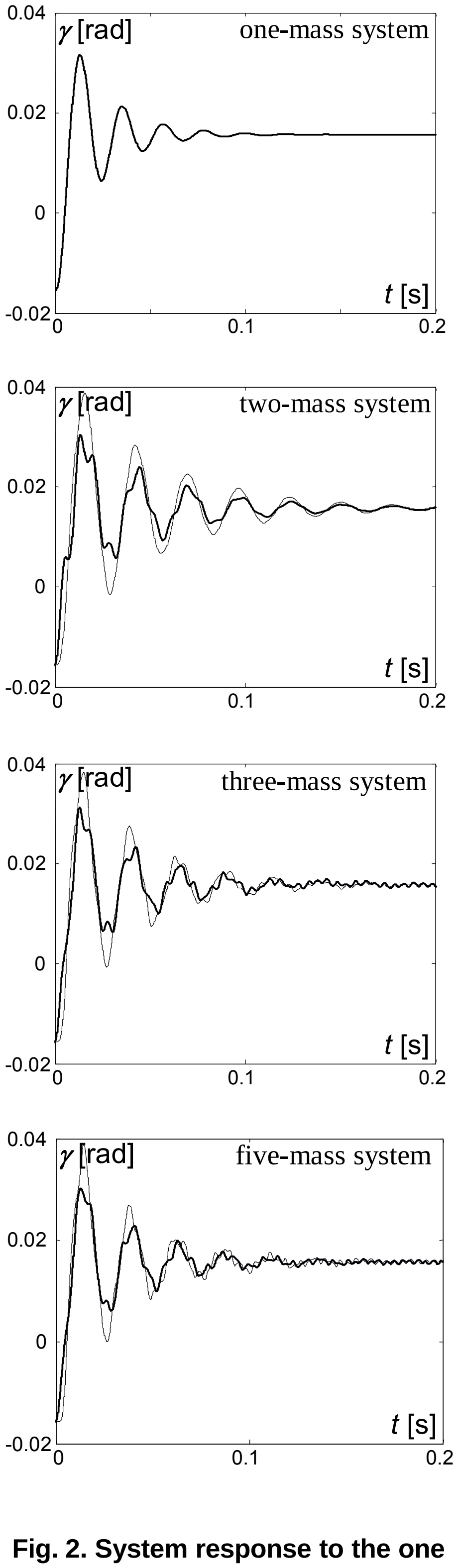


Fig. 1. Electromagnetic torque response of the system under harmonic loading torque characterized by the gradually increasing excitation frequency *f*

Fig. 2. System response to the one step input (thick line indicates the rotor position response and thin line shows the last rigid disk position response)

In order to study an influence of high-frequency or impact-type excitation induced by the mechanical system on the electromagnetic torque generated by the stepping motor, also backlash effects are taken into consideration. According to the above, in the three-mass model there are going to be investigated two cases of backlash usually occurring in gear stages or couplings in the drive system. In the first case the rotational clearance of the visco-elastic spring connecting the rigid disk representing the motor rotor with the middle rigid disk is assumed. In the second case there is considered the rotational clearance of the visco-elastic spring connecting the middle rigid disk with the last disk at the opposite side to the rotor.

From the numerical data assumed for the above computational examples it follows that in all cases the entire mass moment of inertia of the modeled drive system is identical and equal 0.0003 kg·m^2. Moreover, the resultant torsional stiffness of the entire mechanical drive system is also the same and equal 25 N·m/rad, regarded as a result of mutual connection in series all successive visco-elastic elements representing twisting flexibility of the torsional train.

In the computational examples the following simulations have been performed:

- for determination of the stepping motor electromagnetic torque response due to the retarding torque assumed as the harmonic input function T_L = 0.1 sin ($2\pi \cdot ft$) within the excitation frequency range 0÷300 Hz, see Figure 1;
- in order to determine the system angular displacement due to the one step input, see Figure 2;
- for determination of the angular displacement of the last rigid disk at the model opposite side to the rotor due to the stepping motor eight steps input with the constant stepping frequency f_k, see Figure 3;
- in order to compare the angular displacements of the last rigid disk at the model opposite side to the rotor for the one-mass and three-mass mechanical systems due to the eight steps input with the given f_k = 64 steps/sec stepping frequency, see Figure 4;
- for determination of the electromagnetic torque response of the system with backlash due to the retarding torque assumed as the harmonic input function T_L = 0.1 sin ($2\pi \cdot ft$) within the excitation frequency range 0÷300 Hz for the three-mass system, see Figure 5;
- in order to determine the angular displacement of the system with backlash due to the one step input, see Figure 6.

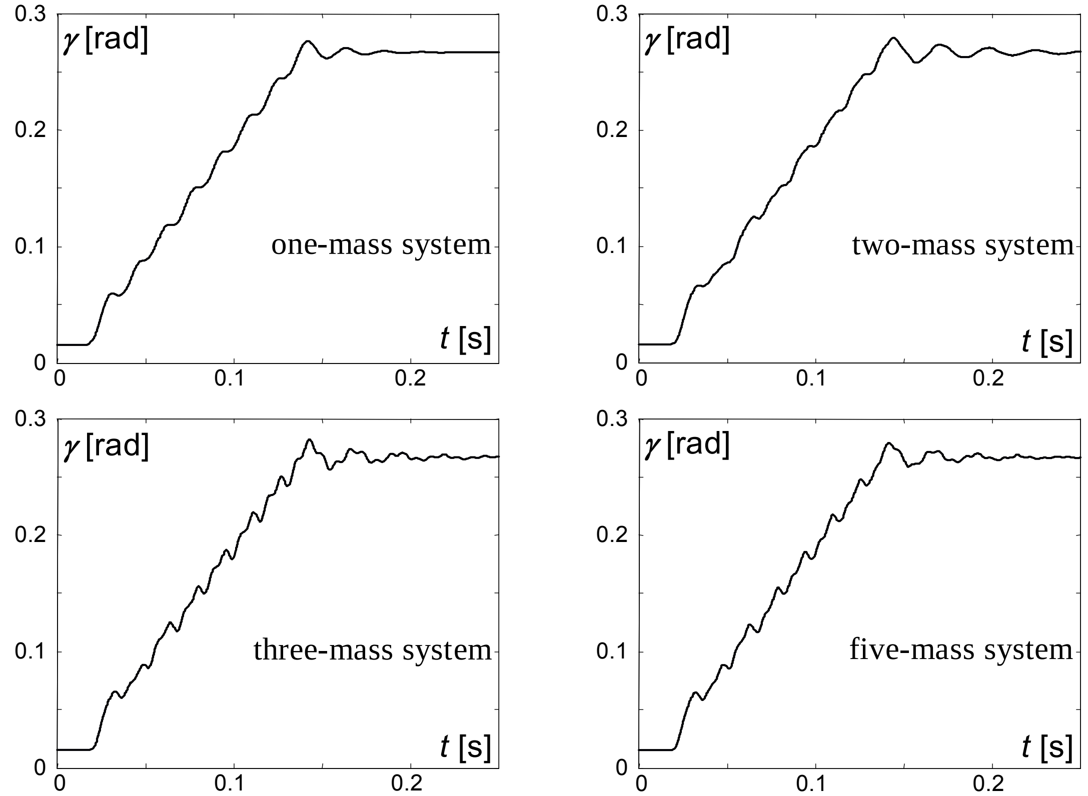
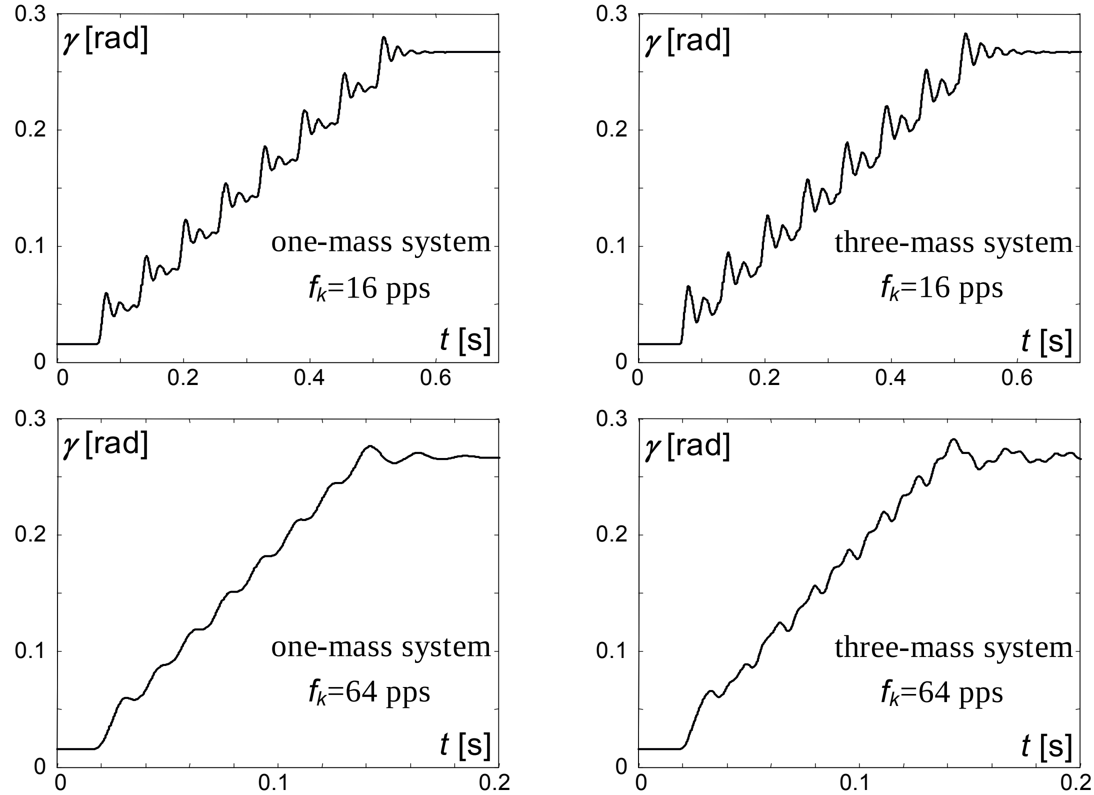


Fig. 3. Angular displacement of the last rigid disk at the model opposite side to the rotor due to the eight steps input with the stepping frequency f_k = 64 steps/sec



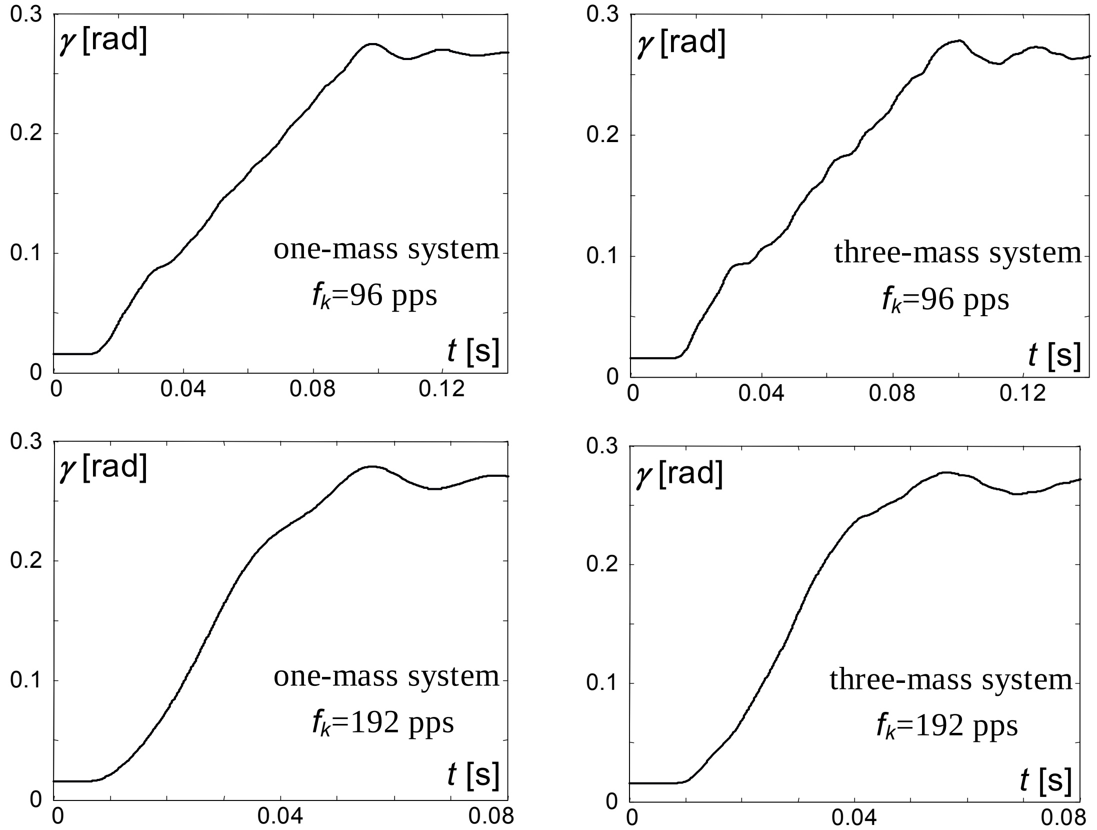


Fig. 4. Angular displacements of the last rigid disk at the model opposite side to the rotor due to the eight steps input for various values of the stepping frequency f_k and for a comparison of responses obtained for the one- and three-mass systems

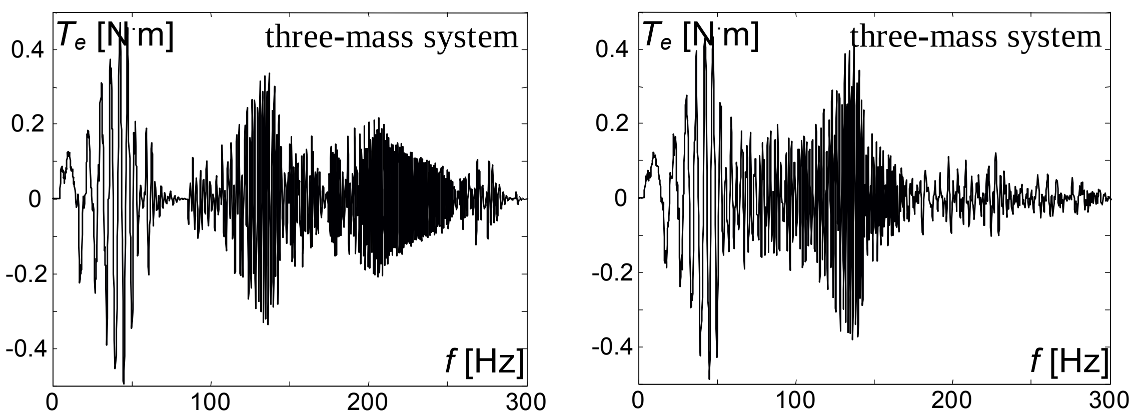


Fig. 5. Electromagnetic torque response of the system under harmonic loading torque characterized by the gradually increasing excitation frequency f for the system with the 0.005 rad backlash (left figure shows the response with the backlash between the rotor and the middle rigid disk and right figure presents the response with the backlash between the middle and the last rigid disk)

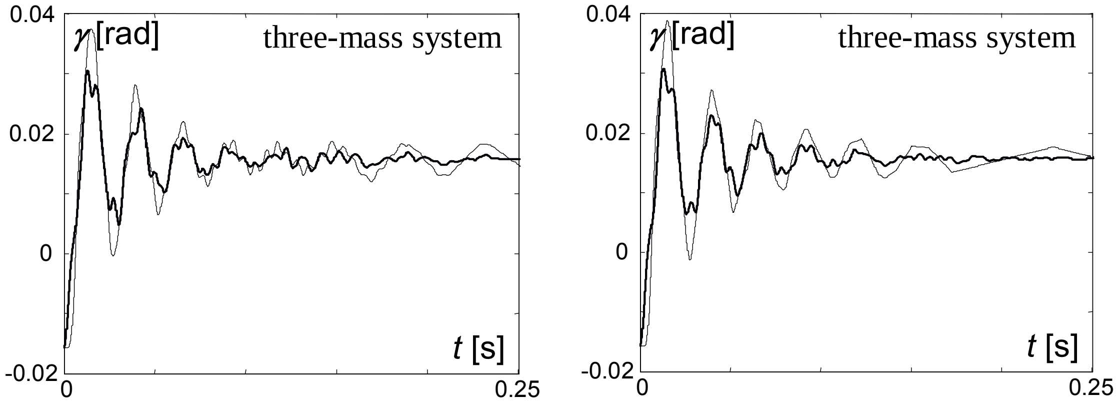


Fig. 6. The response of the system with the 0.002 rad backlash due to the one step input (the thick line presents the rotor angular displacement and the thin line shows the angular displacement of the last rigid disk; left figure shows the response for the backlash between the rotor and the middle rigid disk and the right figure presents the response for the backlash between the middle and the last rigid disk)

4. CONCLUSION

In the paper torsional vibrations of the drive system driven by the hybrid stepping motor have been investigated. The main problem studied here was the interaction between electromechanical parts in the electric drive system with the stepping motor. The authors searched for an answer on the question: how the torsional vibrations of that drive system depend on the mechanical structure complexity? From the results of carried out dynamic investigations the following conclusions can be drawn:

- An application of the only one-mass drive train model in the case of the mechanical system driven by the stepping motor very often can be insufficient.
- Since the real drive systems of machines and mechanisms are always characterized by natural inertial-visco-elastic properties appropriately distributed according to their usually complex structure and geometry, for computational analyses proper mechanical models should be used in order to represent in a reliable way their dynamic and static properties. This target can be achieved by an application of multi-degree of freedom discrete models consisting of rigid disks mutually connected by mass-less visco-elastic springs in the form of torsional trains or by means of the more advanced finite element or discrete-continuous structural modeling techniques. In many cases in such models some backlash and other local non-linear effects should be taken into consideration.

- In the case of current mode of motor winding supplying there is no observed the influence of the mechanical system dynamic behavior on the electromagnetic torque. Thus, the stepping motor does not yield electromagnetic damping.
- The results of simulations and the conclusions presented above can be modified for other drive systems driven by the stepping motors of different electromechanical parameters.

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DRGANIA SKRĘTNE W SYSTEMIE NAPĘDOWYM Z SILNIKIEM SKOKOWYM

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STRESZCZENIE *Współczesne elektryczne systemy napędowe pracują prawie wyłącznie w stanach dynamicznych, co wynika ze sposobu zasilania uzwojeń silnika elektrycznego. Konsekwencją stanów dynamicznych w uzwojeniach silnika jest kształt funkcji momentu elektromagnetycznego znacznie odbiegający od przebiegu pożądanego: stałego* *lub sinusoidalnego w zależności od typu silnika. Zmienny moment elektromagnetyczny wywołuje drgania części mechanicznej, które to drgania zależą nie tylko od momentu wymuszającego, ale także od konfiguracji i parametrów części mechanicznej oraz od wartości i zmienności momentu obciążenia. Problem drgań występuje szczególnie jaskrawo w systemach napędowych z silnikiem skokowym, dla których dyskretne zasilanie pasm uzwojenia wywołuje nierównomierność przebiegu momentu elektromagnetycznego. Ważnym czynnikiem wpływającym na właściwości systemu napędowego jest sprzężenie elektromechaniczne, które objawia się w następujący sposób: nierównomierność momentu elektromagnetycznego generowanego przez silnik wywołuje nierównomierność prędkości wirowania wirnika, co z kolei oddziałuje na przebieg momentu elektromagnetycznego poprzez nierównomierność przebiegu wartości chwilowych napięć indukowanych. W pracy podano model matematyczny silnika skokowego i zastosowano wielomasowy model części mechanicznej systemu napędowego składający się z odpowiedniego połączenia brył sztywnych i elastyczno-tłumiących elementów bezmasowych łączących te bryły. Zastosowany model dobrze nadaje się do badania wpływu struktury i parametrów systemu napędowego na drgania skrętne elementów mechanicznych.*

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