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Mathematical modeling of air duct heater using the finite difference method

M.M. Sarafraz[1*], S.M. Peyghambarzadeh[2], A. Marahel[3]

[1] Islamic Azad University, Department of Chemical Engineering, Mahshahr branch, Mahshahr, Iran
[2] Islamic Azad University, Department of Chemical Engineering, Mahshahr branch, Mahshahr, Iran, e-mail: Peyghambarzadeh@gmail.com
[3] Kurdistan University, Chemical Engineering College, Iran, e+mail: arashmarahel180@gmail.com
[*] Corresponding author: e-mail: Mohamadmohsensarafraz@gmail.com

In this research, mathematical modeling of a duct heater has been performed using energy conservation law, Stefan-Boltzman law in thermal radiation, Fourier's law in conduction heat transfer, and Newton's law of cooling in convection heat transfer. The duct was divided to some elements with equal length. Each element has been studied separately and air physical properties in each element have been used based on its temperature. The derived equations have been solved using the finite difference method and consequently air temperature, internal and external temperatures of the wall, internal and external convection heat transfer coefficients, and the quantity of heat transferred have been calculated in each element and effects of the variation of heat transfer parameters have been surveyed. The results of modelling presented in this paper can be used for the design and optimization of heat exchangers.

Keywords: Duct heater, Modeling, Variable physical properties, Finite difference, Log mean temperature difference.

INTRODUCTION

The importance of optimization and design of heat exchangers in industrial fields is known to engineers. Today's industrial designers are making a wide effort to achieve new economic, precise, optimized and high yield methods. The procedures of designing the heat exchangers are different depending on the heat exchangers applications. The finite difference is one of the confident, easy and useful methods for designing and solving heat transfer problems[1]. In addition, using the numerical method, including the variable physical properties for the design of heat exchangers enhances the quality and precision of the calculation steps. In simple and ordinary methods, the physical properties are estimated in T_f (film temperature which is defined as an arithmetic average of input and output temperatures of an element) or caloric temperature for more convenience[2]. It is important to consider this fact that physical properties are changing through the length of heat exchanger and it is not reasonable to estimate the physical properties in average or constant caloric temperature[3–7]. In this study, the numerical method is applied to analyse the heat transfer performance of a duct heater. Duct heaters are mainly used for transferring the heated or even cooled air, particularly in air condition systems and central heater of buildings and industries. Subsequently, the length of the heat exchanger has been divided into some intervals and furthermore, the physical properties of each segment have been estimated based on the film temperature independently. This numerical method (finite difference) has made new perspectives for accurate design of heat exchangers which leads to increase the operation yield and improves the economics factor simultaneously[8].

MODELING

For mathematical modeling, an air duct heater has been proposed for heating the environment. For simplicity, the shape of the duct heater is considered as cylinder and convection and radiation heat transfers to the environment have been considered from each element. The schematic of the considered geometry and a segment of channel with the length of Δx and cylinder side perimeter *P* are shown in Fig. 1.

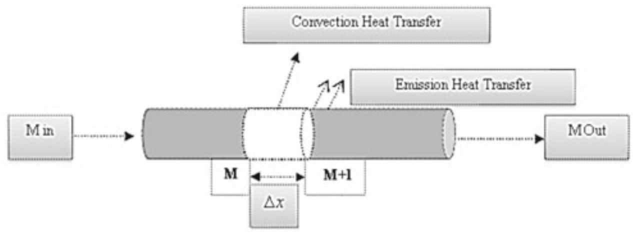

Figure 1. Representation of duct heater and a cylindrical element

By writing the total energy balance equation for the considered segment of the channel, Eq. (1) is obtained:

$$\dot{m}_a c_p T_{m,a} = h_i P Dx (T_{m,a} - T_{mw,i}) + \dot{m}_a c_p T_{m+1,a} \quad (1)$$

Where $\dot{m}_a$ is air mass flow rate and hi is defined as an internal heat transfer coefficient which is related to Reynolds number according to many empirical correlations existing in this field. In this study, hi has been estimated using Eqs. (2) and (3) for the laminar and turbulent flow, respectively[9]:

$$Nu_d = \frac{h_i d}{k} = 3.66 + \frac{0.0668(d/l)\mathrm{Re}_d \mathrm{Pr}}{1 + 0.04\left[(d/l)\mathrm{Re}_d \mathrm{Pr}\right]^{2/3}} \quad (2)$$

$$Nu_d = \frac{h_i d}{k} = 0.0214\left(\mathrm{Re}_d^{0.8} - 100\mathrm{Pr}\right)^{0.4} \quad (3)$$

Physical properties required in the above mentioned equations have been estimated at $T_{m,a}$ indicating the air bulk temperature. Energy balance on the wall of duct channel is defined as Eq. (4):

$$Q_{conv.,i} = Q_{rad.,o} + Q_{conv.,o} \quad (4)$$

By substitution of the Newton's cooling law and Stefan-Boltzmann radiation law into Eq. (4), the following equation is obtained:

$$h_i (T_{m,a} - T_{m,w,i}) = h_o (T_{m,w,o} - T_\infty) + \sigma\varepsilon (T_{m,w,o}^4 - T_\infty^4) \quad (5)$$

where $T_{m,w,o}$ and $T_{m,w,i}$ are outside and inside wall temperatures, respectively. Likewise, conduction heat transfer from the wall of duct heater can be obtained from Eq. (6):

$$Q_{cond} = k_m \frac{T_{m,w,i} - T_{m,w,o}}{\Delta x_m} \quad (6)$$

The outside heat transfer coefficient for the outside duct wall that is exposed to stagnant air with environment temperature value is obtained by Eq. (7)[2]:

$$h_o = 0.27\left(\frac{T_{m,w,o} - T_\infty}{d}\right)^{1/4} \quad (7)$$

By substitution of Eq. (7) into Eq. (5), the following equation has been obtained:

$$h_i(T_{m,a} - T_{m,w,i}) = \frac{0.27}{d^{1/4}}(T_{m,w,o} - T_\infty)^{5/4} + \sigma\varepsilon(T_{m,w,o}^4 - T_\infty^4) \quad (8)$$

Note that Eq. (9) must be used for calculating and estimating the outside temperature of each segment, which leads to Eq. (9):

$$T_{m+1,a} = (1 - \frac{h_i P \Delta x}{\dot{m}_a c_p})_m T_{m,a} + (\frac{h_i P \Delta x}{\dot{m}_a c_p})_m T_{m,w} \quad (9)$$

Briefly speaking, the following algorithm is used in a computer based programming for the calculation of heat transfer rate:

1) An element length (Δx) is selected. It should be mentioned that the accuracy of this method strongly depends on the number of segments or the selected length of each segment. Smaller length of elements will increase the accuracy of the results.

2) Calculation will start with the air inlet position which is kept as (x = 0) and all the physical properties for start point must have been estimated in $T_{bulk\text{-}inlet}$.

3) Calculate the Reynolds number; select the proper Eqs. (2) or (3) to find h_i.

4) Assume a value for $T_{m,w,i}$.

5) Solve the Eqs. (10) and (11) simultaneously:

$$q_{conv,i} = q_{cond} \quad (10)$$

$$h_i\left(T_{m,a} - T_{m,w,i}\right) = k_m \frac{T_{m,w,i} - T_{m,w,o}}{\Delta x_m} \quad (11)$$

Solve Eq. (11) for estimating the value of $T_{m,w,o}$ as follow:

$$T_{m,w,o} = T_{m,w,i} - \left[\left(\frac{h_i \Delta x_m}{k_m}\right)\left(T_{m,a} - T_{m,w,i}\right)\right] \quad (12)$$

Computed values of $T_{m,w,i}$ and $T_{m,w,o}$ from Eqs. (10) and (11) are subsequently substituted into Eq. (7). If it is not consistent, return to step 4 for swapping the assumption.

6) Solve Eq. (9) for $T_{m+1,a}$. This value will be considered as the air inlet temperature in the next segment.

7) Calculation steps (1–6) are subsequently repeated for the next segments until the end of the duct heater. However, heat flux in each segment can be calculated using Eq. (13):

$$q = h_i\left(T_{m,a} - T_{m,w,i}\right) = k_m \frac{T_{m,w,i} - T_{m,w,o}}{\Delta x_m} = h_c\left(T_{m,w,o} - T_\infty\right) + \sigma\varepsilon\left(T_{m,w,o}^4 - T_\infty^4\right) \quad (13)$$

In Table 1, the geometry of the considered channel has been exhibited. For accuracy and simplifying in calculation steps, MATLAB computer based programming has been used.

Table 1. Geometry characteristics of the considered channel

Geometry properties	values
Bulk temperature of environment	310 K
Length of channel	150 m
Length of an element	15 m
Inlet air temperature	811 K
Duct diameter	0.457 m
Channel wall thickness	0.15 m

RESULTS AND DISCUSSIONS

Effect of various parameters

The effects of some important parameters such as flow rate of inlet air flow, internal diameter of the channel, and channel construction material on the heat transfer ability of the duct heater have been studied in separate following parts of this paper.

Flow rate of inlet air

The flow rate of the inlet air was changed in order to make two different flow regimes (namely laminar and turbulent flows). It has been particularly done for investigating on the values of heat transfer coefficients in two different regimes of air flow. Figs. 2–7 show the results obtained due to the air inlet variations.

As shown in Fig. 2, increasing the flow rate of inlet air, raises the temperature of the air through the effective length of channel. However, the temperature of the air is reduced with increasing the length of the duct. Table 2 typically represents the status of $T_{m,w,i}$ besides the $T_{m,w,o}$ with variation of air flow rate through the length of duct. Increasing the mass flow rate of the air raises the internal heat transfer coefficients of the air in the channel length. These data are given in Fig. 3.

Table 2. Effect of air flow rate on $T_{m,w,i}$ and $T_{m,w,o}$

Material / parameter	Emissivity [-]	Conduction coefficient [W.m^{-1}. K^{-1}]	Reference
Oxidized iron	0.26	67	[10,11,12]
Oxidized copper	0.78	367	[10,11,12]

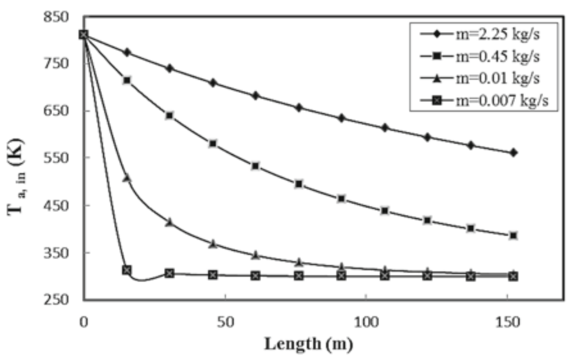

Figure 2. Effect of air flow rate variation on the air temperature in the channel length

Furthermore, the influence of air mass flow rate on the external heat transfer coefficient has been given in Fig. 4, which implies that increasing the mass flow rate of water, the external heat transfer coefficient is increased in contrast to this value is decreased through the length of channel as well as heat flux will be increased with raising the mass flow rate of air and will be decreased with length of duct which have been truly shown in Fig. 5.

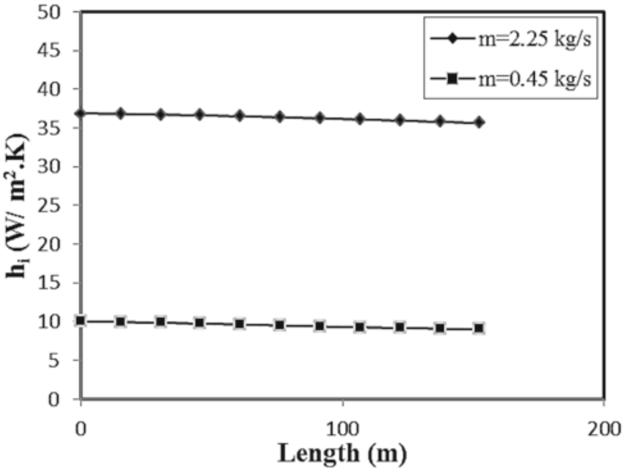

Figure 3. Effect of air flow rate on the internal heat transfer coefficient of the air

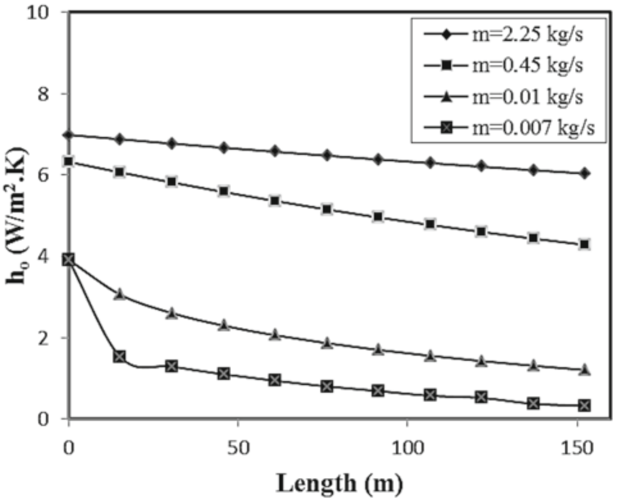

Figure 4. Effect of flow rate on external heat transfer coefficient

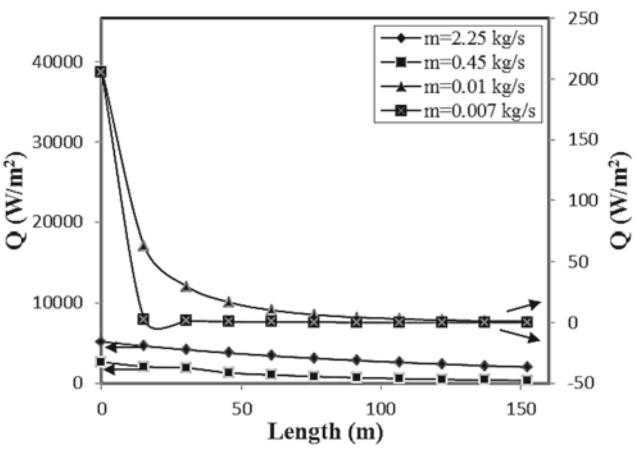

Figure 5. Effect of flow rate on heat flux

Effect of internal diameter of the channel

To investigate the effects of internal diameter on other parameters, different values of internal diameter have been employed at air flow rate 2.3kg.s^{-1}. For better understanding the results are given in the next coming figures. Air output temperature is decreased with increasing of the internal diameter and is decreased through the length of the channel, too. Figs. 6–9 present the influence of variations of diameter on: air temperature, internal heat transfer coefficient, external heat transfer coefficient and heat flux respectively.

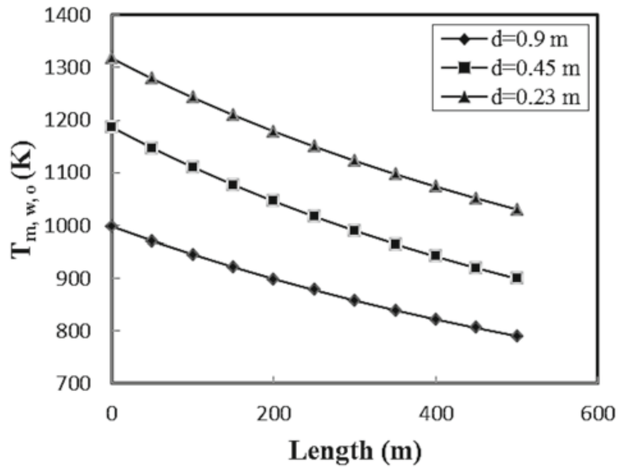

Figure 6. Effect of channel diameter on the exit air temperature

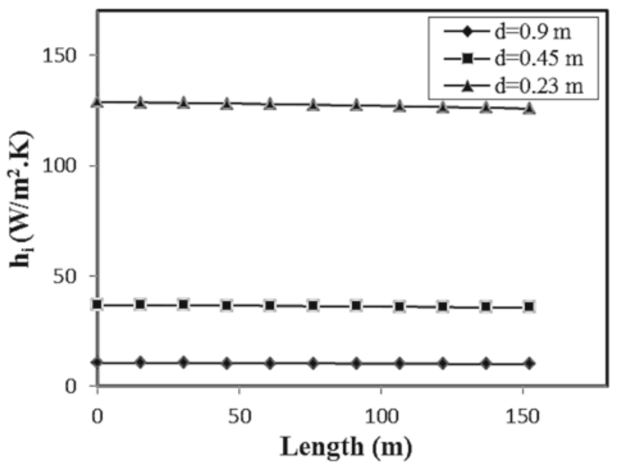

Figure 7. Effect of channel diameter on the internal heat transfer coefficient

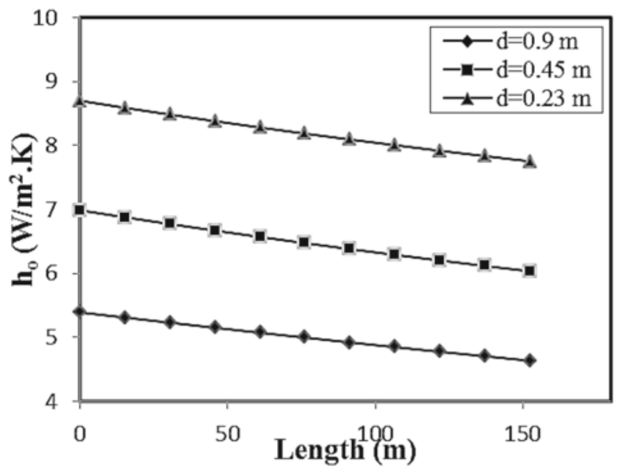

Figure 8. Effect of channel diameter on the external heat transfer coefficient

Channel material of construction

In this section, two channels with two different materials of construction are simulated in order to evaluate the effect of channel material on its heat transfer performance. The only properties which have influences on the heat transfer from this duct heater are emissivity and conductivity of the metals. Table 3 summarized the properties of the two channels which were used for the simulation in this paper.

As shown in Fig. 10, temperature drop in each element of the oxidized copper channel is less than that in the oxidized iron channel. The main reason for this

Table 3. Emissivity and conduction coefficient of selected materials

Material / parameter	Emissivity [-]	Conduction coefficient [W.m^{-1}. K^{-1}]	Reference
Oxidized iron	0.26	67	[10,11,12]
Oxidized copper	0.78	367	[10,11,12]

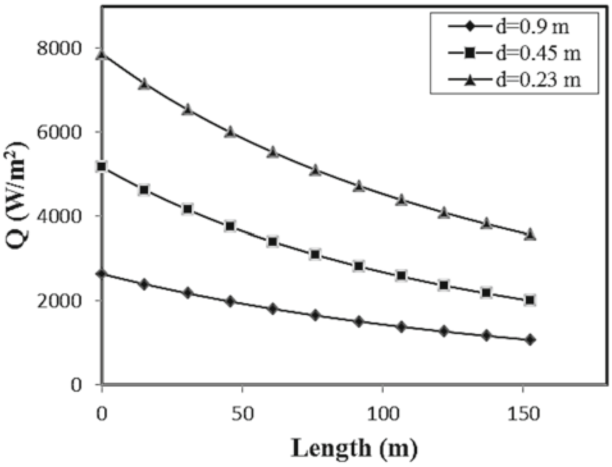

Figure 9. Effect of channel diameter on heat flux

phenomenon is smaller conduction resistance of the oxidized copper channel in comparison to the iron ones. Moreover, because of that, it is concluded that the internal and external wall temperature of copper channel is less than iron specimen due to the high conduction heat transfer coefficient of copper that enhances the rate of heat transfer from the internal wall towards outside. Additionally, higher emissivity of oxidized copper in compared to oxidized iron leads to more radiative heat transfer between the outside wall and ambient, subsequently the outside wall temperature is smaller in comparison with the oxidized iron channel.

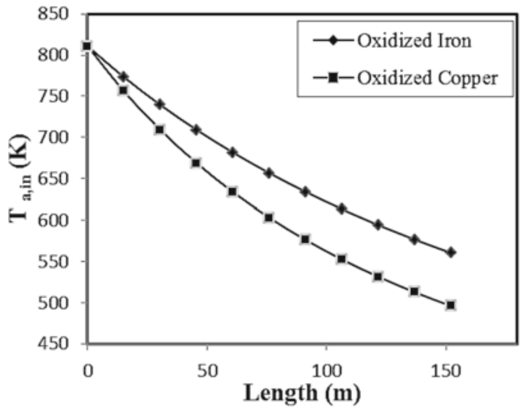

Figure 10. Effect of duct material of construction on the inlet air temperature

As shown in Figs. 11–12, the internal and external convection coefficient of oxidized copper in each element of the channel is higher than that of the oxidized iron channel. In fact, the outside wall temperature of the oxidized copper channel is smaller than for oxidized iron channel and according to Eq. (7) if the outside temperature of the channel decreases, the outside convection coefficient of the channel decreases, too. Through the channel, according to Eq. (2), the Nusselt number is proportional to the multiplication of the Reynolds number and the Prandtl number. Furthermore,

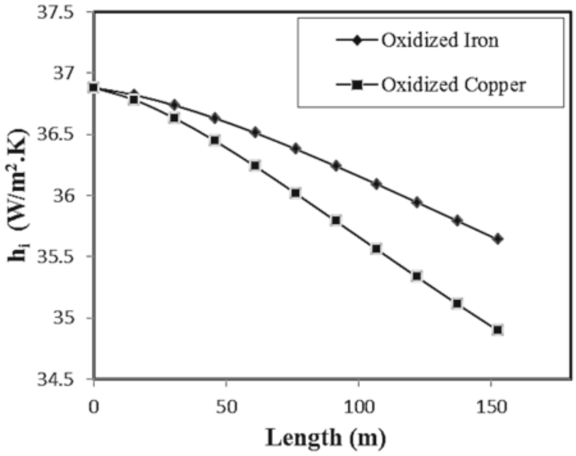

Figure 11. Effect of duct material of construction on the internal heat transfer coefficient

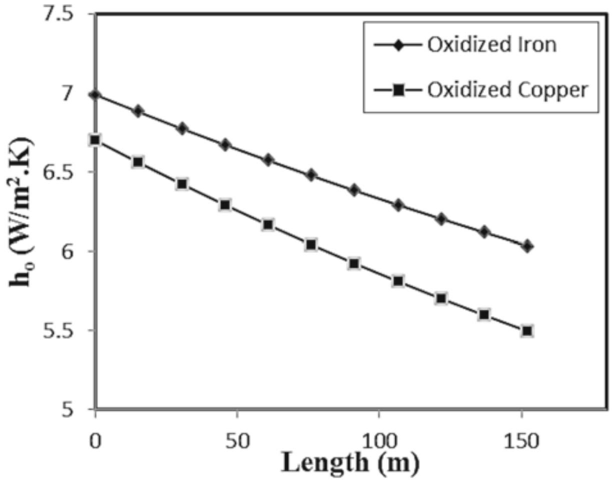

Figure 12. Effect of duct material of construction on the external heat transfer coefficient

reducing the temperature through the length of channel results in increasing the air density, subsequently leads to decreasing its viscosity, too. Accordingly, the Reynolds number will increase through the length of the channel. Meanwhile, the Prandtl number is decreased with decreasing the temperature. Therefore, the heat transfer coefficient through the channel is decreased. In addition, this influence is explicitly sensible for the oxidized copper channel in comparison to the oxidized iron channel and cause the reduction of heat transfer coefficient of the oxidized copper channel.

Also, heat flux in each element of the oxidized copper channel is higher in comparison to the oxidized iron channel. The oxidized copper emissivity and conduction coefficients are higher than for oxidized iron. The internal and external heat transfer coefficients of the oxidized iron are higher than the oxidized copper. At the outlet of these two channels, according to Figs. 11–12, heat transfer coefficients are closer to each other and the influence of emissivity and conduction coefficient lead to large values of transferred heat. But at the outlet of the channel, decrease of heat transfer coefficient in the oxidized copper is more than that of iron channel. It is the major reason of the identification of heat fluxes for two different materials of channels. The results of the obtained heat fluxes are given in Fig. 13.

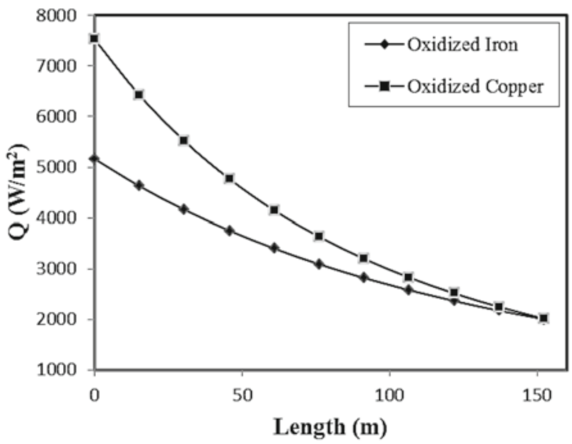

Figure 13. Effect of duct material of construction on heat flux

Comparison between LMTD and the finite difference method

Using Eq. (14), film temperature in each element (caloric temperature) has been calculated. The physical properties of the air have been estimated in this temperature. Also, log mean temperature difference has been obtained through Eq. (15):

$$T_f = \frac{1}{2}\left(T_{a,in} + T_{a,out}\right) \tag{14}$$

$$\Delta T_{lm} = \frac{(T_{a,in} - T_\infty) - (T_{a,out} - T_\infty)}{\ln \dfrac{T_{ain} - T_\infty}{T_{aout} - T_\infty}} \tag{15}$$

$$Q = U_i A_i \Delta T_{lm} \tag{16}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{\ln \dfrac{r_o}{r_i}}{2\pi kL} A_i + \frac{A_i}{A_o}\frac{1}{h_c} \tag{17}$$

Equations (15–17) are used for calculating the overall heat transfer coefficient and the rate of heat, respectively. Table 4 gives a rough comparison between LMTD (Log Mean Temperature Difference method) and the finite difference mathematical method in estimating the heat transfer rate. As shown, there is maladjustment between the LMTD obtained results and the finite difference method. One of the major reasons of this difference refers to the estimating condition of variable physical properties. There is an important point that the physical properties are not constant throughout the heat exchanger which was mentioned before and this point must be considered. Since in our study, the variable physical properties are considered, the values of Reynolds are not constant. Therefore the range of Re and Pr are reported in Table 5.

CONCLUSIONS

In this research, the effect of some parameters such as mass flow rate, the internal diameter of the channel and the material quality of the channel have been directly or indirectly investigated on heat transfer values and parameters. The results show that, with increasing the flow rate values, air temperature will increase so do all the parameters, such as the inside wall temperature and the outside wall temperature and all the heat transfer coefficients. Also, reducing the channel diagonal leads to an increase of all the properties, such as heat transfer coefficient and air stream temperatures. Likewise the quality and the material of the skin channel composer, has an undeniable influence on heat transfer values, furthermore the heat flux will increase. The effective length of the duct channel has no effect on the internal heat transfer coefficient, too. Also, the materials which were selected to be used for designing of the air duct channel were tested separately for measuring the heat transfer coefficient.

NOMENCLATURES

$\dot{m}_a$	air mass flow rate [kg. s^{-1}]
T_f	film temperature [K]
$T_{a,in}$	inlet air temperature [K]
$T_{a,out}$	air outlet temperature [K]
U_i	total heat transfer coefficient [W.m^{-2}.K^{-1}]
r_i	inside radius [m]
r_O	external radius [m]
P	perimeter of an element [m]
C_P	specific heat of air [J.kg^{-1}.K^{-1}]
$T_{m,a}$	temperature of air in each element (segment) [K]
$T_{m,w,i}$	inside skin temperature in each element [K]
$T_{m,w,o}$	external skin temperature in each element [K]
h_i	internal heat transfer coefficient [W.m^{-2}.K^{-1}]
h_o	external heat transfer coefficient [W.m^{-2}.K^{-1}]
$\dot{m}_a$	inlet flow rate [kg. s^{-1}]
d	diameter of channel [m]
l	length of channel [m]
K	conduction heat transfer coefficient [W. m^{-1}. K^{-1}]
K_m	material conduction heat transfer coefficient [W. m^{-1}.K^{-1}]
$Q_{conv,i}$	convection heat transfer [W]
$Q_{Cond,P}$	external convection heat transfer [W]
Q_{Cond}	conduction heat transfer [W]
$Q_{rad,P}$	radiation heat transfer [W]

Greek Letters:

ε	emissivity [-]
Δx	length of segment [m]
Δx_m	thickness of channel [m]

Table 4. Comparison between the results obtained using LMTD and finite difference methods

Mass flow rate	Reynolds no. range	Prandtl no. range	regime	Q (heat transfer) finite difference method (kW)	Q(heat transfer) LMTD method (kW)
$\dot{m} = 2.25 kg/s$	169399 – 211045	0.68 – 0.692	Turbulent	791	674
$\dot{m} = 0.45 kg/s$	35190 – 61392	0.689 – 0.692	Turbulent	264	249
$\dot{m} = 0.01 kg/s$	782 – 1538	0.708 – 0.692	laminar	7.6	7.6
$\dot{m} = 0.007 kg/s$	547 – 1076	0.708 – 0.692	laminar	4.7	3.5

ΔT_{lm} log mean temperature difference [-]
σ Stephan Boltzmann constant [W.m-2.K-4]

Dimensionless numbers and parameters:

Re Reynolds number $= \frac{\rho . V.D}{\mu}$ [-]

Nu Nusselt Number $= \frac{h.x}{k}$ [-]

Pr Prandtl Number $= \frac{Cp.\mu}{k}$ [-]

Abbreviations:

LMTD Log Mean Temperature Difference [see Eq. 15]

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