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## NUMERICAL SIMULATION OF ELECTROWETTING USING ELECTRIC BODY FORCE DENSITY AND SURFACE TENSION

**ABSTRACT** *The electrowetting is a phenomenon in which the shape of a droplet is deformed by external electric field. The most of electrowetting studies have been presented by researchers from the mechanical viewpoint. In this paper, we present a numerical method to calculate the droplet shape by taking into account the effects of external electric field, surface tension and gravity. The numerical analysis for shape calculation is formulated by using the equilibrium condition of hydrostatical pressure in the coupled system of external electric field and surface tension in the presence of gravity. The model is numerically implemented and coupled using a standard finite element procedure. The proposed method is numerically tested and validated in a shaping problem of water droplet placed above a conductor coated by dielectric material in external electric field. The electrowetting phenomenon was successfully modeled and analyzed by the proposed approach.*

**Keywords:** *simulation, force density, surface tension, electric field, gravity, modelling calculating, electrowetting, droplet shape, body force.*

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## 1. INTRODUCTION

The electrowetting is a phenomenon in which the shape of a droplet is deformed by an external electric field. Various applications, such as lap-on-a-chip, electronic display, and adjustable lenses are based on the electrowetting phenomenon. In recent papers, they assert that the shape of a droplet is initially determined by the contact angle, and then the contact angle is changed by applying a voltage between two electrodes [1, 2].

It is commonly understood that the contact angle, with no applied voltage, is only determined by the triple line as shown in Figure 1. The angle can be calculated using the following relation (1),

$$\gamma_{LG} \cos\theta = \gamma_{SG} - \gamma_{SL} \quad (1)$$

where  $\gamma$  is the surface tension coefficient (e.g.,  $\gamma_{LG}$  at a liquid/gas interface) and  $\theta$  is the contact angle. When a voltage is applied, the relation (1) is modified to (2) to consider the influence of the electric field [1],

$$\gamma_{LG} \cos\theta = \gamma_{SG} - \gamma_{SL} + \frac{1}{2} CV^2 \quad (2)$$

where  $C$  is the capacitance and  $V$  is the applied voltage. It was asserted that Maxwell stress can be used as the electric pressure [3].

From the mechanical viewpoint [4-7], the shape of a droplet is determined by the contact angle. And the change of the contact angle by the voltage is explained to be due to the change of material property by the voltage. However, this paper aims to analyze the electrowetting from the electrical viewpoint using the distributed electric force, which is generated by electric field distribution. That is, we couple the static fluid system with the electric system to form a coupled system equation for calculation of the droplet shape [8].



**Fig. 1. Schematic view of the forces at the triple line (Solid, Liquid, Gas):**  
a) Hydrophilic contact,  $\theta < 90^\circ$ ; b) Hydrophobic contact,  $\theta > 90^\circ$

We present a numerical method to calculate the droplet shape by taking into account the effects of external electric field, surface tension and gravity. The different physical phenomena influence each other and they constitute a coupled system. So, their transient interaction between them is very complicated and difficult to analyze and calculate. Thus, in this paper, the numerical analysis for shape calculation is formulated by using the equilibrium condition of hydrostatic pressure in the coupled system of external electric field and surface tension in the presence of gravity.

The model is numerically implemented and coupled using a standard finite element procedure. The proposed method is numerically tested and validated in a shaping problem of a water droplet placed above a conductor coated by dielectric material in an external electric field.

## 2. PROPOSED APPROACH

From the hydrostatic force equilibrium, the pressure on the surface can be expressed as follows,

$$p_i = \int_{L_0}^{L_i} (\rho \mathbf{g} + \mathbf{f}_e) \cdot d\mathbf{l} + p_0 + p_{LS} \quad i = 1, 2, \dots, n \quad (3)$$

where  $p$  is the isotropic mechanical scalar pressure,  $i$  is the index for indicating different positions on the surface,  $n$  is the number of control positions,  $\rho$  is the mass density of the liquid,  $\mathbf{g}$  is the acceleration vector of gravity,  $\mathbf{f}_e$  is the Kelvin force density in the liquid,  $L_0$  is an arbitrary fixed position on the free surface of the liquid,  $p_0$  is the atmospheric pressure,  $L$  is a position on the droplet surface and  $p_{LS}$  is an additional pressure by surface curvature at position  $L$ .

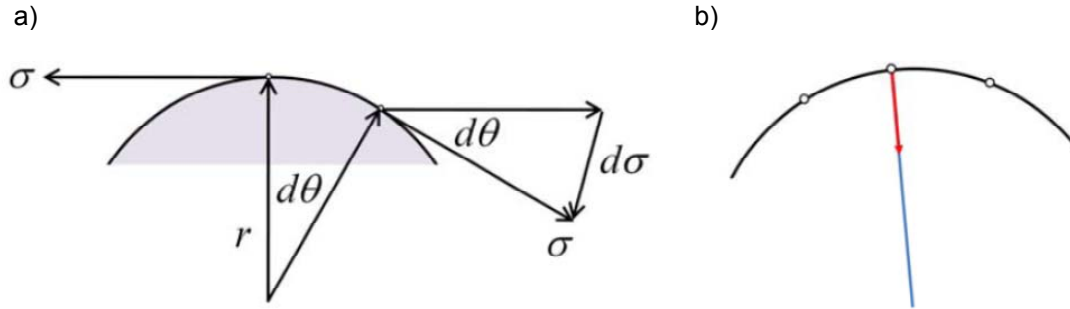
The line integration path should go through the inside of the fluid to consider the internal body force. It is noted that the path can be arbitrary because the pressure  $p_i$  is independent of the integration path. In the state of hydrostatical equilibrium, all the  $p_i$  should be the atmospheric pressure,  $p_0$ .

From [9], the formula of volume force density is written as

$$\mathbf{f}_e = -(\mathbf{P} \cdot \nabla) \nabla V = (\mathbf{P} \cdot \nabla) \mathbf{E} \quad (4)$$

where  $\mathbf{P}$  is polarization,  $\mathbf{E}$  is electric field intensity and  $V$  is the electric scalar potential.

The calculation technique of pressure using surface tension is shown in Figure 2a. Also, the points in Figure 2b represent the control points to draw the surface of a droplet. Pressure by surface tension is expressed as follows (5).



**Fig. 2. Schematic diagram to calculate (a) pressure by surface tension and (b) control points of droplet surface**

$$P_{st} = p_{LS} = \frac{2 \times \sigma d\theta}{rd\theta} = \frac{2\sigma}{r} \left( N/m^2 \right) \quad (5)$$

Where  $\sigma$  is the surface tension constant of water,  $r$  is the radius, determined by control points.

The model is numerically implemented and coupled using a standard finite element procedure. The proposed method is numerically tested and validated in a shaping problem of water droplet placed above a conductor coated by dielectric material in an external electric field.

### 3. NUMERICAL ANALYSIS

At the stage of the experiment, Figure 3 shows the dimensions and properties for a numerical model. An axis-symmetric 2D formulation was derived and a finite element analysis was performed.

From (4), the equation in cylindrical coordinates, multiplying it by  $r$  to avoid singularities at  $r = 0$ , becomes

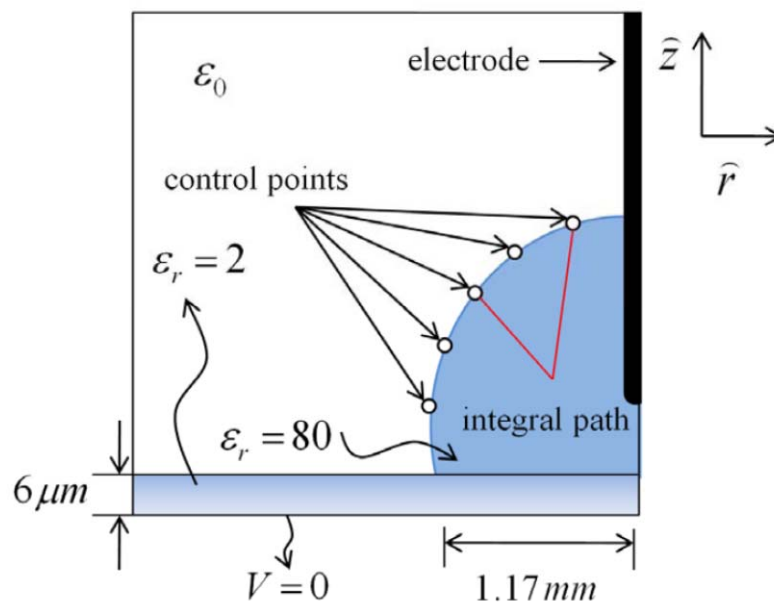
$$\mathbf{f}_e = -\frac{1}{r} (\mathbf{P} \cdot \nabla) r \mathbf{E}_0 \quad (6)$$

where  $r$  denotes the radial directional components and  $\mathbf{E}_0$  is the external electric field intensity.

$\mathbf{E}_0$  is expressed as follows [8],

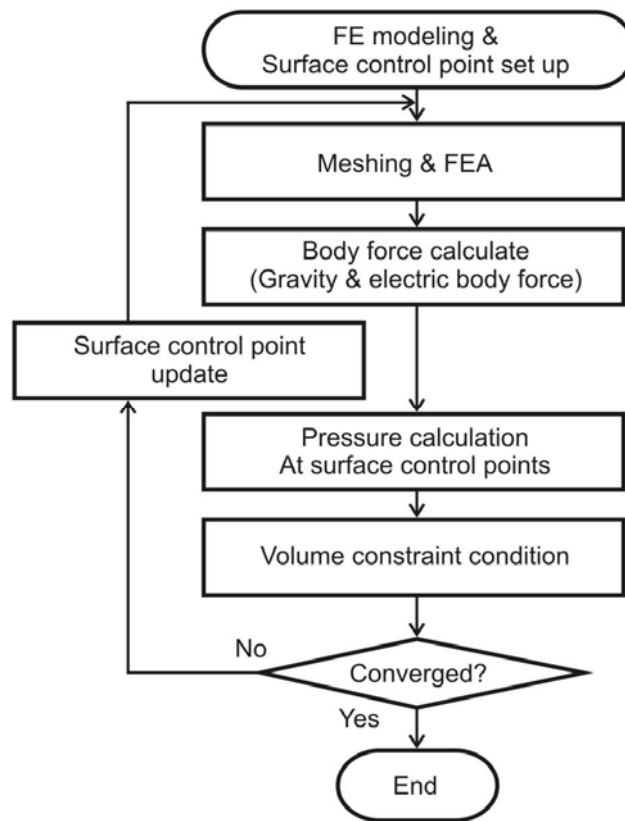
$$\mathbf{E}_0 = \frac{1 + \varepsilon_r}{2} \mathbf{E} \quad (7)$$

The concept of an external field was first introduced by Kelvin, as follows in [9]. The electromagnetic external field can be also found in [10-12].



**Fig. 3. Numerical model at the experimental stage. Axis-symmetric 2D formulation and finite element analysis were performed. The line integration path should go through the inside of the fluid. The line integration path is used to calculate the pressure on the droplet surface. 30 points on the surface were used for pressure calculation**

The droplet shape is detected with the pressure equilibrium condition at control points. The surface tension on the droplet through the radius and hysteresis phenomenon is ignored. The volume of the droplet is  $5 \mu\text{l}$ , and the Teflon coating thickness is  $6 \mu\text{m}$ . The integration path to the control points is shown in Figure 4.



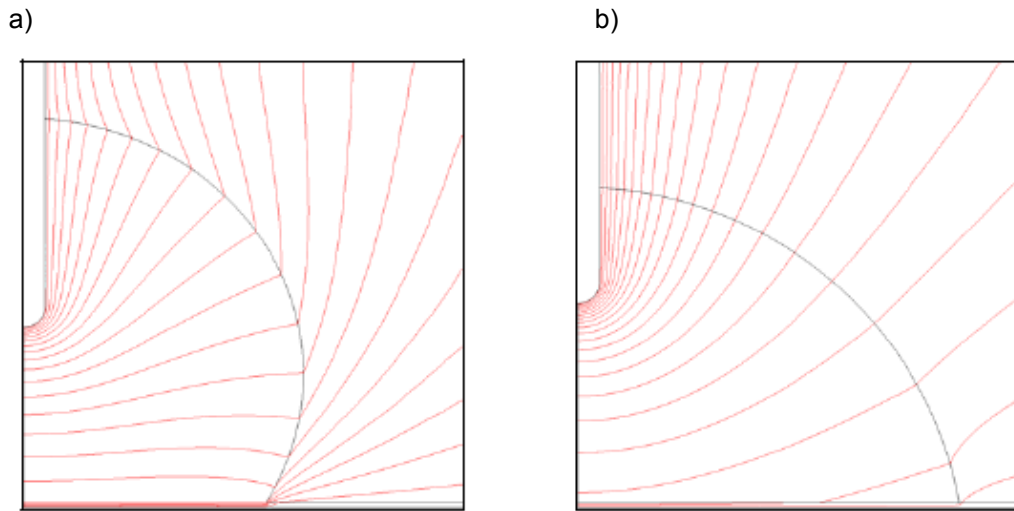
**Fig. 4. Flowchart to find surface control point by the hydrostatical equilibrium condition**

The free surface profile of the droplet can be obtained through numerical iteration, whose algorithm is based on the condition that the pressures on the free surface are the same. Figure 4 presents the overall flowchart of the procedure.

## 4. NUMERICAL RESULT

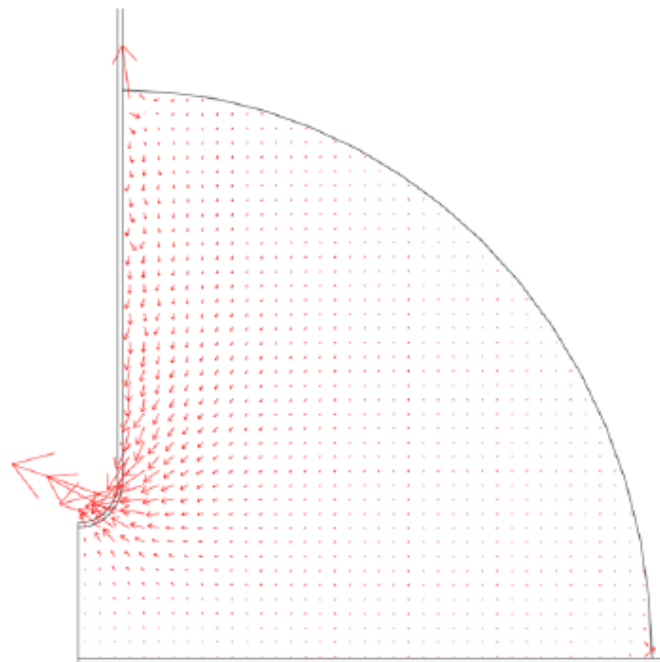
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Figure 5 shows equi-potential lines passing through the surface of initial and final shape when 200 V is applied.



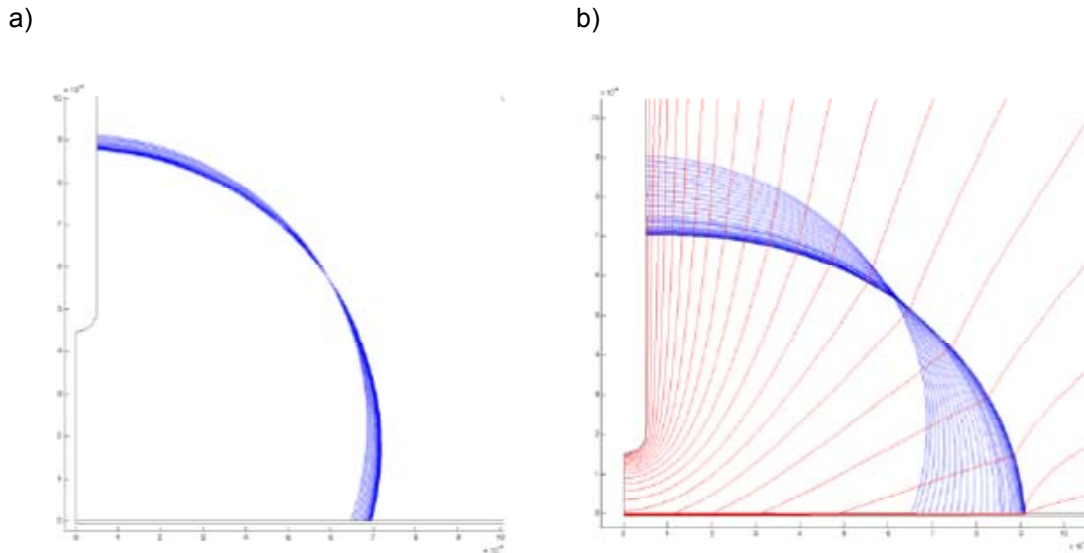
**Fig. 5. Body Surfaces of 1  $\mu\text{l}$  droplet and flux distributions of (a) initial and (b) final shape**

Figure 6 shows the electromagnetic body force density distribution near the tip of the electrode when the voltage is applied. The force density near the tip of the electrode is larger than at points far from the electrode.



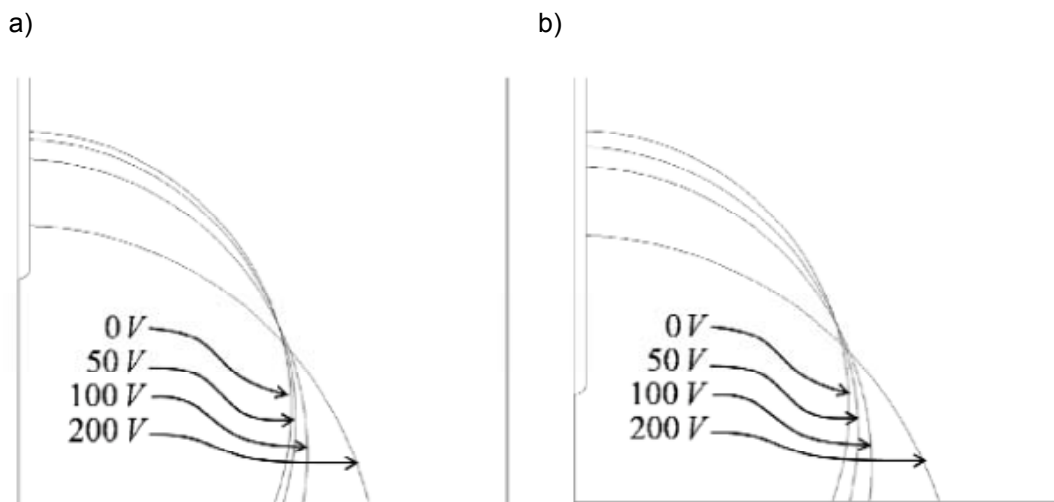
**Fig. 6. Body force density distribution. The arrows to visualize the body force density are drawn in each of the elements**

Figure 7 shows the trajectory of the droplet shape detected with the pressure equilibrium condition at control points.



**Fig. 7. Numerical analysis results. Trajectories of the 1  $\mu\text{l}$  droplet surface profile. Applied voltage: 50 V (a) and 200 V (b), The potential line for the last shape**

The final shapes of droplets according to the various applied voltages are shown in Figure 8. The volume constraint condition is adopted to determine the shape of droplets. The larger the voltage, the flatter the droplet shape. The result shows that the electrowetting phenomenon was successfully modeled and analyzed using the proposed approach.

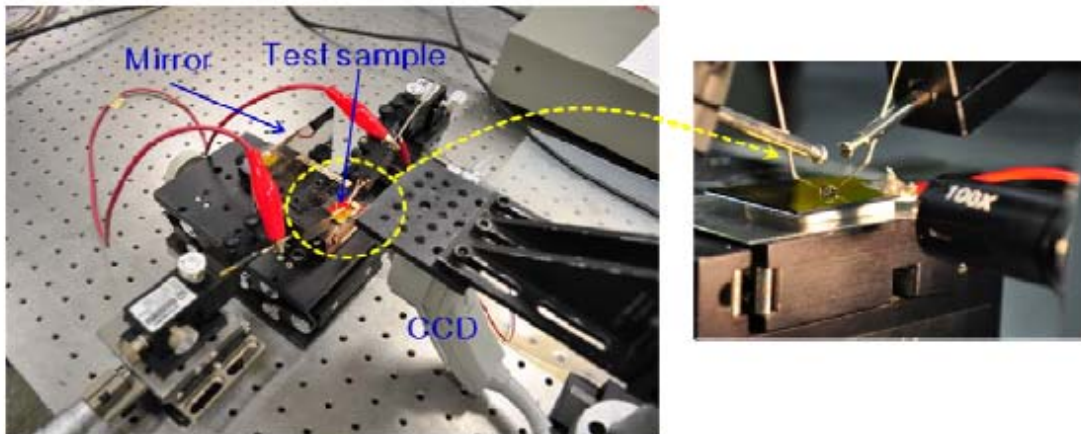


**Fig. 8. Comparison of droplet surfaces according to the applied voltage:**  
a) distance between tip and bottom – 1 mm; b) distance between tip and bottom – 0.5 mm



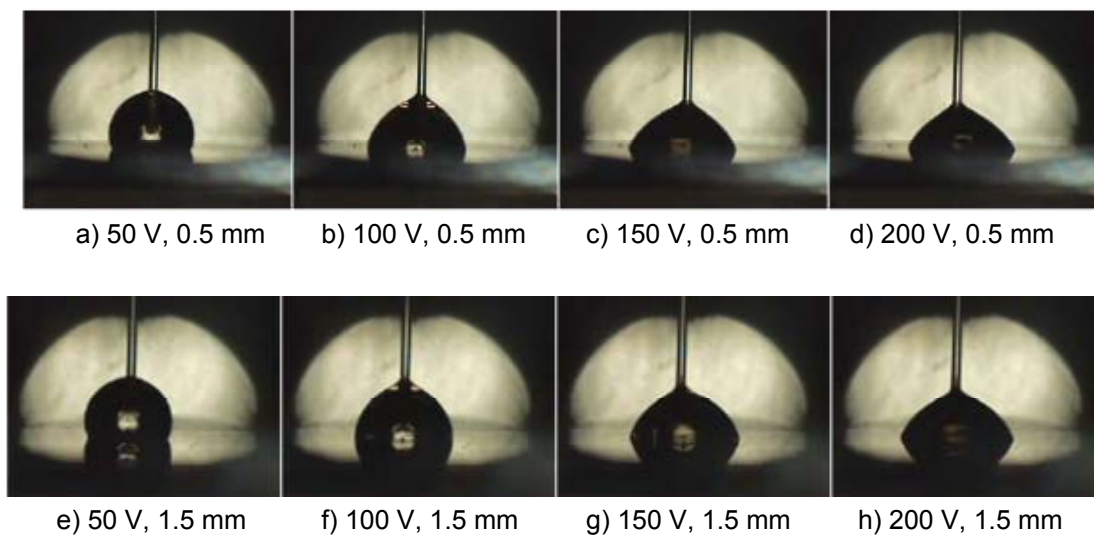
## 5. EXPERIMENT

In Figure 9 presents photos of the experiment.



**Fig. 9** Experimental equipments used to observe electrowetting phenomena

A detailed description of the measurement procedure follows. First, a 5  $\mu\text{l}$  water droplet is placed on the grounded metal plate coated with Teflon. Second, voltage is applied to the electrode in the droplet. The distance between the tip of the electrode and coated metal plate was 1.5 mm and 0.5 mm . A picture of the droplet shape is taken using a CCD camera.



**Fig. 10.** Photographs of shape deformation with applied voltage and depth of the electrode

## 6. CONCLUSION

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In this paper, we present a numerical approach to represent the effect of electric field in the electrowetting phenomenon. The electrowetting phenomenon can be regarded as a result of electromagnetic body force and surface tension. The proposed approach was successfully modeled and analyzed qualitatively for the electrowetting phenomenon. We have obtained a significant result although the experimental part permits only to analyze qualitatively the electrowetting phenomenon. We can get many advantages to analyze the electrowetting phenomenon from electrical viewpoint. Variations of voltage, the depth of electrode, the sort and thickness of the dielectric coating are easily treated from the electromagnetic viewpoint. The reason why the contact angle is saturated, fingering effect near contact line, contact line instability radiation of light can be explained reliably, if electrowetting is continuously studied from the electrical aspect.

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## NUMERYCZNA SYMULACJA ZWILŻANIA ELEKTRYCZNEGO PRZY UŻYCIU ELEKTRYCZNEJ SIŁY JEDNOSTKOWEJ KORPUSU I NAPRĘŻENIA POWIERZCHNIOWEGO

Tan II SUNG

**STRESZCZENIE** *Nawilżanie elektryczne polega na tym, że kształt kropli jest odkształcany poprzez zewnętrzne pole elektryczne. Większość prac dotyczących nawilżania elektrycznego została dokonana przez badaczy z mechanicznego punktu widzenia.*

*Artykuł przedstawia numeryczną metodę obliczania kształtu kropli z uwzględnieniem wpływu zewnętrznego pola elektrycznego, naprężenia powierzchniowego i grawitacji. Numeryczna analiza obliczania kształtu wykorzystuje warunki równowagi ciśnienia hydrostatycznego w sprzężonym układzie zewnętrznego pola elektrycznego i naprężenia powierzchniowego w obecności grawitacji. Model jest wdrożony numerycznie i sprawdzony przy użyciu standardowej procedury elementów skończonych.*

*Proponowana metoda jest numerycznie testowana i sprawdzana poprzez problem kropli wody umieszczonej ponad przewodnikiem pokrytym dielektrykiem w zewnętrznym polu magnetycznym. Zjawisko nawilżania elektrycznego jest z powodzeniem modelowane i analizowane przy użyciu proponowanego podejścia do zagadnienia.*

