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APPLICATIONS OF REGRESSION METHODS TO HUMIDITY SENSORS CALIBRATION

ABSTRACT *The classical and inverse linear calibration methods based on the regression of y on x and the regression of x on y, were applied to several humidity sensors. The predicted values for given values of the output value of sensors were calculated, and conclusions concerning the accuracy of prediction were drawn.*

Keywords: *humidity sensor calibration, classical calibration method, inverse prediction method.*

1. INTRODUCTION

Accurate calibration of sensors plays significant role in modern building conditioning systems and environment monitoring. Relative humidity (RH) sensors are of special interest for precise humidity control (e.g. HVAC sensors) [12]. However, the progress in RH sensor calibration accuracy is about one order worse than for other related quantities (temperature or pressure). Because of this, a need for better calibration methods arouses.

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The classical method of sensor calibrating – including the RH sensors – is relatively fast, simple and inexpensive, but not highly accurate [8, 12]. During the first stage of the calibration experiment, a set of observed pairs of values of x and y is collected. Then a statistical model of function, called „calibration curve equation" is fitted to the data. In many cases, the simplest, linear fit is quite sufficient, and the equation of classical linear calibration: $y = A + Bx$ is obtained as the result of regression of y on x. The last stage – the most useful one – is the prediction of the unknown value x_1 applying the calibration equation to the measured value y_1 (e.g. the value of a measured capacitance C of a capacitive RH sensor).

Theoretical research has revealed that better prediction results can be achieved if the regression of x on y is considered, since predictions based on inverse calibration have lower mean squared error within the calibration range [1, 2, 5, 6, 11]. It seems worthwhile to check the usefulness of applying the inverse calibration to humidity sensors.

2. THE BASIC PRINCIPLES OF CLASSICAL AND INVERSE CALIBRATION METHOD

For a given set of data points, the coefficient of linear correlation R should be calculated; if the value of R is grater than 0.995, the calibration curve is considered as linear; however, sometimes supplementary tests are necessary [13]. The approximation line coefficients: A (intercept) and B (slope) are established by the classical regression method. Then, from the equation $y = A + Bx$, the predicted value x_1 (for a given value y_1) can be obtained by simple inverting of the classical calibration equation: $x_{1r} = (y - A)/B$. One of the assumptions made for obtaining a valid approximation line is that the measurement errors of x's are negligible and the main source of errors is the uncertainty of y's. The whole procedure can be realized in inverse way if this assumption is reversed. Generally, the concept of inverse calibration is contested by some theoreticians [7, 9], but practicians are less severe [3, 4, 10].

As a rule, the modern RH sensor's output y is the voltage (or resistance, or capacitance); i.e. the electrical quantities which can be measured by DMM with high accuracy. On the contrary, the accuracy of calibrating RH standards is at least one order worse than the accuracy of DMM [7]. The inverse equation, obtained using the regression of x ($\equiv Y$) on y ($\equiv X$), can be written as: $Y = a + bX$. The predicted value x_1 ($\equiv Y_1$) [for a given value y_1 ($\equiv X_1$)] can be obtained as:

$x_{1t} = Y_1 = a + bX_1$. The predictive formulae more useful for statistical computations are elaborated; for classical calibration:

$$x_c = \bar{x} + \frac{\sum_1^n (x_i - \bar{x})^2}{\sum_1^n (x_i - \bar{x})(y_i - \bar{y})}(y_1 - \bar{y}) \, , \tag{1}$$

and for inverse calibration:

$$x_\upsilon = \bar{x} + \frac{\sum_1^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_1^n (y_i - \bar{y})^2}(y_1 - \bar{y}) \, . \tag{2}$$

3. CLASSICAL AND INVERSE CALIBRATION METHOD APPLIED TO HUMIDITY SENSORS

For several humidity sensors, the calibration point sets (published by the manufacturers in sensor data sheets) were subjected to both classical and inverse linear method; the calibration equation based on the regression of x on y, and the regression of y on x were obtained, and approximation errors were estimated. The calibration coefficients and maximum approximation errors as well as correlation coefficients are shown in Table 1.

TABLE 1
The calibration coefficients and maximum errors of linear approximation for chosen RH sensors

Lp	Sensor symbol	Number of calibration points N	Calibration coefficients A	B	Maximum approximation error E	Coefficient of linear correlation R
		—	V	V/%[RH]	%[RH]	—
1	808H5V5	6	0.8210	0.0313	0.94	0.9992
2	HS-220	7	-0.0114	0.0331	0.43	0.9999
3	HS-230A	9	0.1961	0.0301	2.75	0.9982
4	HS-230B	9	0.3619	0.0326	2.87	0.9972
5	HS-1100	10	1.174	0.0237	0.21	0.9999
6	HS-1500-LF	18	1.078	0.0257	1.40	0.9998
7	HS-1101-LF	21	pF	pF/%[RH]	3.38	0.9991
			162.64	0.3087		

Then, for a given y_1 (close to the lower endpoint value of the output range) and y_2 (close to the higher endpoint value of the output range) the predicted values x_1 and x_2 were found using formulae (1) and (2) for both classical and inverse calibrations. The predicted values are presented in Table 2.

TABLE 2
The calibration ranges and predicted values at the range endpoints for chosen RH sensors

Lp	Sensor symbol	Input range		Output range		Response values		Predicted values			
								Classical method		Inverse method	
		x_{min}	x_{max}	y_{min}	y_{max}	y_1	y_2	x_{1c}	x_{2c}	$x_{1\upsilon}$	$x_{2\upsilon}$
		% [RH]		V		V		% [RH]			
1	808H5V5	30÷80		1.73÷3.30		1.80	3.25	31.29	77.64	31.33	77.61
2	HS-220	30÷90		99÷2.97		1.00	2.95	30.52	89.35	30.52	89.35
3	HS-230A	10÷90		0.58÷2.87		0.60	2.80	13.42	86.51	13.55	86.38
4	HS-230B	10÷90		0.70÷3.18		0.75	3.15	11.91	85.57	12.12	85.37
5	HS-1100	10÷100		1.41÷3.55		1.50	3.50	13.74	98.03	13.74	98.03
6	HS-1500-LF	10÷95		1.325÷3.555		1.35	3.45	10.24	92.63	10.92	91.98
7	HS-1101-LF	0÷100		pF		pF		1.16	98.35	1.25	98.26
				161.6÷193.1		163	193				

Finally, the uncertainties of the predicted values were calculated using both classical and inverse regression method. The uncertainties are given in Table 3.

TABLE 3
The uncertainties of predicted values at the range endpoints for chosen RH sensors

The method of regression		Uncertainties of predicted values at the endpoints of calibration range for chosen sensors						
		808 H5V5	HS -220	HS -230A	HS -230B	HS -1100	HS-1500-LF	HS-1101-LF
		% [RH]						
x_1	Classical	0.992	0.275	2.042	2.562	0.149	0.630	1.498
	Inverse	0.991	0.277	2.038	2.553	0.149	0.626	1.497
x_2	Classical	0.983	0.275	2.042	2.532	0.150	0.623	1.497
	Inverse	0.982	0.276	2.037	2.524	0.150	0.622	1.495

The manufacturers calibrated the sensors using the humidity generators, which are less cumbersome than the HFP solutions, but less accurate (1-2%) either; so the claims for acceptance of the inverse method as valid for RH sensors seem to be quite reasonable.

4. CONCLUSIONS

It can be seen that the predicted values calculated using both methods – – classical and inverse – are practically equal. The differences between certain predictive values are minute; much smaller than the maximum approximation error. That error seems to be dependent mainly on the correlation coefficient; if R is very close to 1, the number of calibration points has weak influence. The uncertainties of the predicted values obtained by both methods are also in good agreement – the results are identical within three decimal places. All calculations are favourable for the acceptance of the inverse calibration method as a useful tool for determining calibration curves of relative humidity sensors.

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ZASTOSOWANIE METOD REGRESJI W KALIBRACJI SENSORÓW WILGOTNOŚCI

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STRESZCZENIE *Zastosowano metodę klasyczną i metodę odwrotną kalibracji liniowej oparte, odpowiednio, na regresji* y *względem* x *oraz regresji* x *względem* y *do wyznaczenia równania prostej kalibracji dla kilku sensorów wilgotności oferowanych na rynku. Przedstawiono wyniki obliczeń wartości predykowanych wilgotności dla wybranych wartości sygnału wyjściowego sensorów oraz podano wnioski dotyczące dokładności predykcji.*

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