

# Calculating the main and leakage inductance matrix of the 3-column 15-winding autotransformer

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**Abstract:** The aim of this paper is presentation and comparison of calculation methods of the inductance matrix of a 3-column multi-winding autotransformer. Main and leakage autotransformer inductance was obtained using finite elements method. Static calculations were made at the current supply for 2D and 3D models, and mono-harmonic calculations were made at the voltage supply. In the mono-harmonic calculations the eddy current losses were taken into account, this made it possible to study relationship between the autotransformer parameters and the frequency. Calculations were made using Ansys and the authors' own programs in Matlab.

**Key words:** autotransformer, leakage inductance, eddy current losses, proximity effect

## 1. Introduction

Uncontrolled AC/DC converters with autotransformer have broad practical applications, for example, in electroplating technique, in electrolysis, in frequency converters with voltage inverters used in electric drives, etc. They have very simple construction and low production costs because of low costs of electronic circuits (no control circuit) and low cost of autotransformer (in comparison with transformer). However, the relatively big content of higher harmonics in mains currents is the drawback of such converters. These higher harmonics are the reason for bigger electric energy losses in the grid and for higher harmonics in supply voltage, and they have negative influence on sensitive electronic devices. The problem of lowering of the higher harmonics content in the mains currents supplying uncontrolled converters is important also because they cause additional eddy current losses in electric machines and they increase the current in capacitive electrical loads.

The good measure of higher harmonics content in the mains currents is the THD coefficient. Current distortion coefficient  $THD_I$  is defined as follows ( $I_n$  is the  $n$ -th current harmonic):

$$THD_I = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \tag{1}$$

In order to limit higher harmonics content in the mains currents of the converters and thereby to lower mains currents deformation coefficient, passive and active filters are used as well as various modulation methods, also converters with higher number of pulses are build.

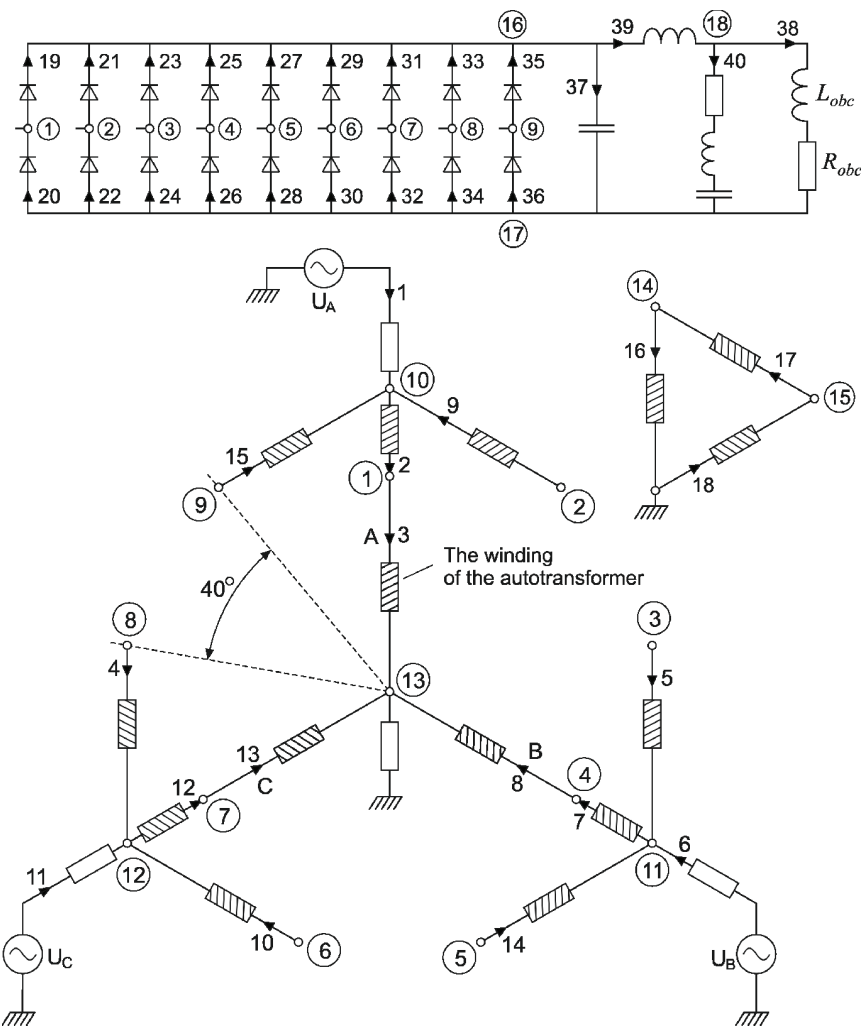


Fig. 1. Analyzed circuit of the 18-pulse AC/DC converter supplied by the autotransformer with 5 windings on each column

Figure 1 presents the 18-pulse rectifier circuit supplied by examined autotransformer. The leakage inductances of the autotransformer have great impact on the THD value of the con-

verter mains currents as they directly influence diode commutation processes. In the presented paper the main and leakage inductance matrix of the autotransformer was investigated.

Extensive theoretical and empirical research has taken place to establish the variation of winding losses with frequency. The methodology is now well understood and is often provided as an application example in commercial software packages of finite element analysis. The development of the finite element analysis software as the integral part of a procedure to calculate the high frequency behaviour of magnetic components has been developed extensively by Asensi et al [1] and Prieto [2]. This procedure has the function of providing models for practical electric circuit simulation of power applications, and this has also been addressed by Okyere & Heinemann [3] and Evans & Heffernan [4]. Unfortunately the procedure requires exhaustive finite element analyses of the transformer models, which become more finely meshed as the frequency increases thereby either increasing the simulation time required, or reducing the accuracy of the result. The resulting model obtained may also be cumbersome and exhibit convergence difficulties. Lopera et al. [5] and Pernia et al. [6] have developed a onedimensional approximation of the behaviour of the magnetic component windings including frequency variations. In this paper a model structure is proposed which models the leakage inductance variation with frequency, and can be characterized with a relatively small number of measurements or finite element analyses.

## 2. Methods for autotransformer matrix inductance calculation at DC supply

In energetic method we use the equation for magnetic energy contained in magnetic field of the autotransformer.

$$E = \frac{1}{2} \int_V \vec{B} \cdot d\vec{H} \, dV = \frac{1}{2} [I]^t \cdot [L] \cdot [I] \Rightarrow [L] = ? \quad (2)$$

where:  $[I]$  is the vector of winding current having size  $n$ .

Demerdash's method uses the linearity of the magnetic field relative to supplying currents and thus superposition method [7]. Component  $(i, j)$  of the autotransformer inductance matrix  $[L]$  is then:

$$L_{ij} = \int_V \vec{B}_i \cdot \vec{H}_j \, dV, \quad (3)$$

where  $B$  is magnetic inductance of  $i$ -th winding supplied by unit current, and  $H$  is magnetic field strength of  $j$ -th winding supplied by unit current.

### 2.1. 2D single window model of the autotransformer

Figures 2 and 3 present simplified geometry of 2D single window of the examined autotransformer. Inductance matrix  $L(10 \times 10)$  was obtained as if there were 5 windings associated with the left and other 5 with the right column ( $n = 5$ ).

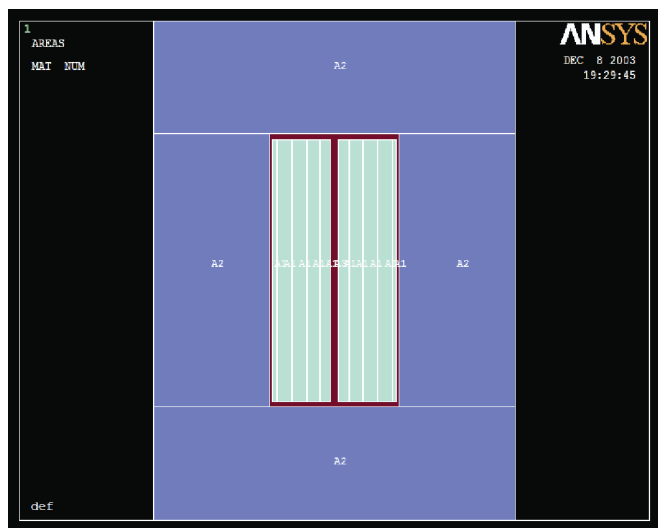


Fig. 2. Geometry of the 2D model of single window of the autotransformer cross-section

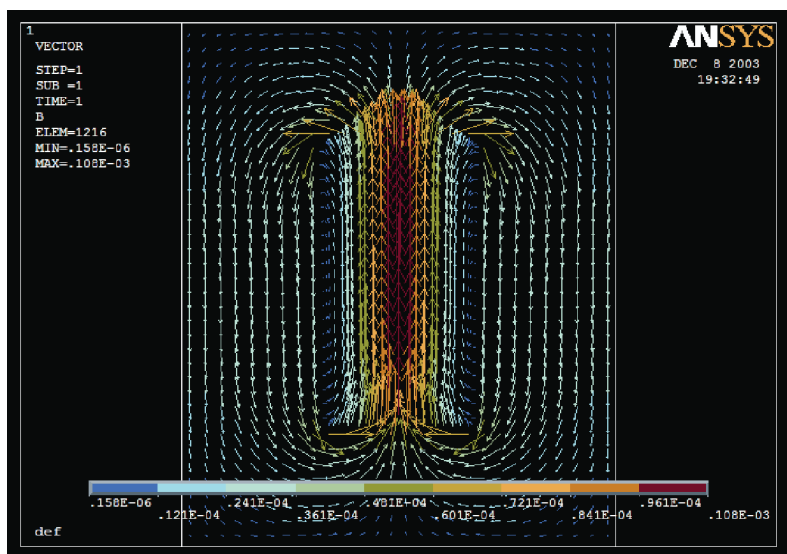


Fig. 3. Magnetic field distribution in Ansys visualization

Then it was mapped into the actual 3D geometry of the autotransformer by the formula:

$$M = Z^* \begin{bmatrix} \times 1 & \times 0.25 & \times 0 \\ \times 0.25 & \times 1 & \times 0.25 \\ \times 0 & \times 0.25 & \times 1 \end{bmatrix} * Z^* I_{sr} \quad (4)$$

where  $Z$  is the diagonal matrix containing numbers of winding turns along its diagonal,  $l_{sr}$  is the average length of turns of windings.

With this formula it is shown in  $(5 \times 5)$  sub-matrices, how the  $L$   $(10 \times 10)$  matrix is positioned within the  $M$   $(15 \times 15)$  matrix and what multipliers are used in order to map it into 3D structure.

## 2.2. 2D model of the 3-column autotransformer cross-section

Figures 4 and 5 present simplified geometry of 2D two windows of the examined autotransformer. The model in the picture forms a cross-section of the entire autotransformer surrounded by air. Zero Dirichlet's condition was assumed at the edge of this model. During calculations the inductance matrix  $L$   $(15 \times 15)$  was obtained.

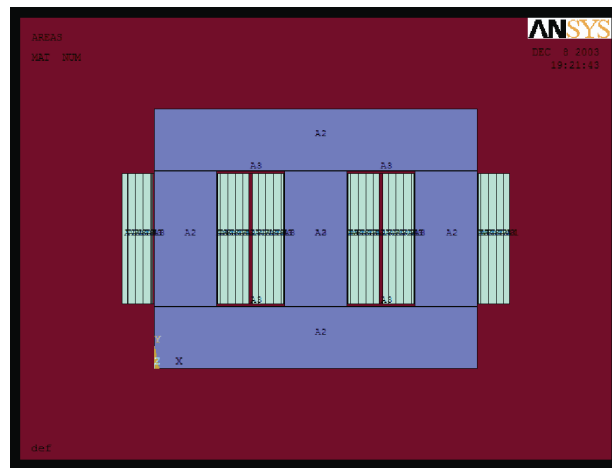


Fig. 4. Geometry of the 2D model of the autotransformer cross-section

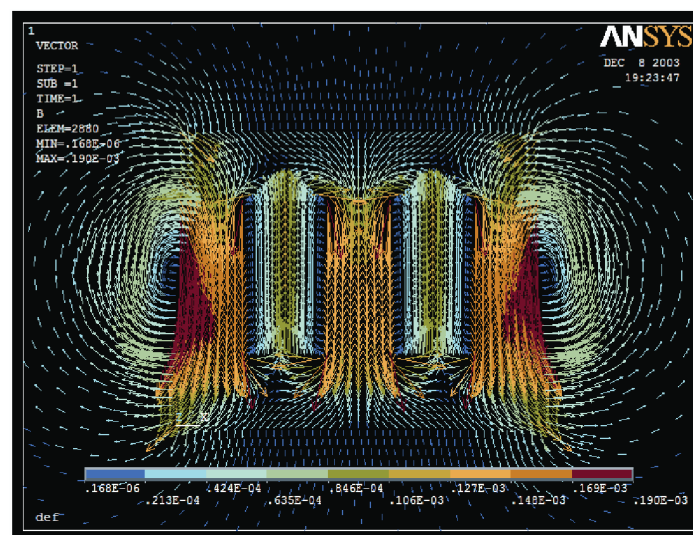


Fig. 5. Magnetic field distribution in Ansys visualization

### 2.3. 3D autotransformer model

To avoid any transfer of results of 2D analysis into actual 3D autotransformer, the best option would be to perform calculations directly for 3D model. Such model is shown in Figures 6 and 7.

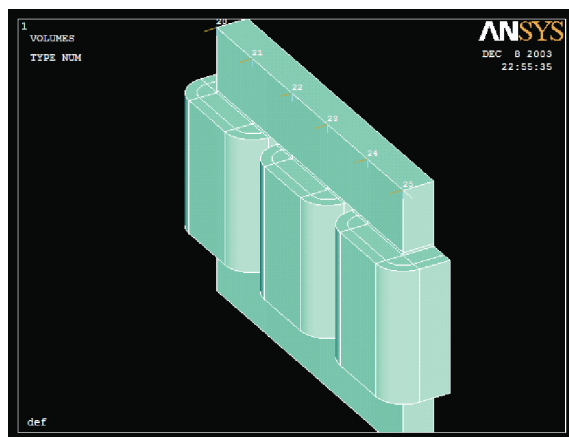


Fig. 6. 3D model of the autotransformer (in 1/2 symmetry)

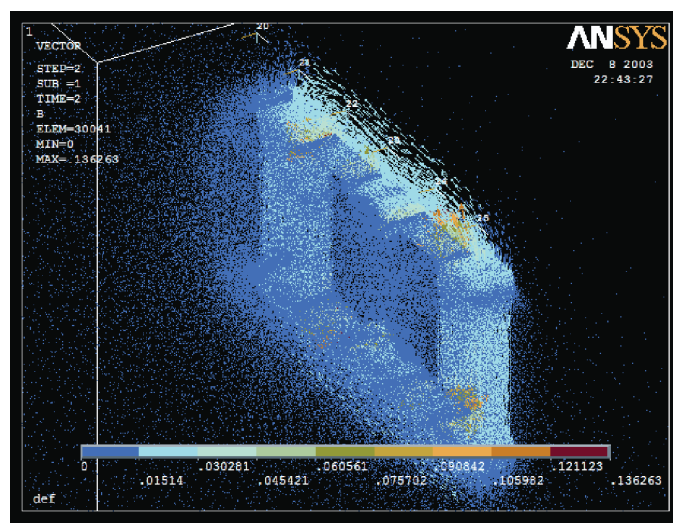


Fig. 7. The example of distribution of the magnetic field calculated in Ansys

### 3. Comparison of leakage inductance matrices obtained from different autotransformer models at current supply

The inductance matrix of the autotransformer  $[L]$  was decomposed into eigenvectors matrix  $[W]$  and eigenvalues diagonal matrix  $[D]$ :

$$[L] \cdot [W] = [W] \cdot [D] \Rightarrow [L] = [W] \cdot [D] \cdot [W]^t. \quad (5)$$

The eigenvectors  $\bar{w}_1$ ,  $\bar{w}_2$ , related to the main magnetic flux and  $\bar{w}_3$  vector related to the zero component magnetic flux, appear also in the received leakage inductance matrix:

$$\begin{aligned} \bar{w}_1 &= [1, 1, \dots, 1 | -2, -2, \dots, -2 | 1, 1, \dots, 1]^t / \sqrt{6n} \\ \bar{w}_2 &= [1, 1, \dots, 1 | 0, 0, \dots, 0 | -1, -1, \dots, -1]^t / \sqrt{2n} \\ \bar{w}_3 &= [1, 1, \dots, 1 | 1, 1, \dots, 1 | 1, 1, \dots, 1]^t / \sqrt{3n}. \end{aligned} \quad (6)$$

	Eigenvalue related to vector:		
	$\bar{w}_1$	$\bar{w}_2$	$\bar{w}_3$
3D main inductance	0.539e-3	0.782e-3	$\approx 0$
<b>3D Model</b>	<b>0.805e-6</b>	<b>0.638e-6</b>	<b>1.713e-6</b>
<b>2 windows 2D</b>	<b>0.466e-6</b>	<b>0.438e-6</b>	<b>0.825e-6</b>
<b>1 window 2D</b>	<b>0.379e-6</b>	<b>0.361e-6</b>	<b>0.399e-6</b>

The presence of main magnetic components in leakage inductances is negligible. The first row presents their actual values which are 3 orders of magnitude bigger. However the zero component related eigenvalues can not be neglected. Zero component eigenvalue error is also the source of major differences at direct comparison of elements of leakage inductance matrices obtained with different methods.

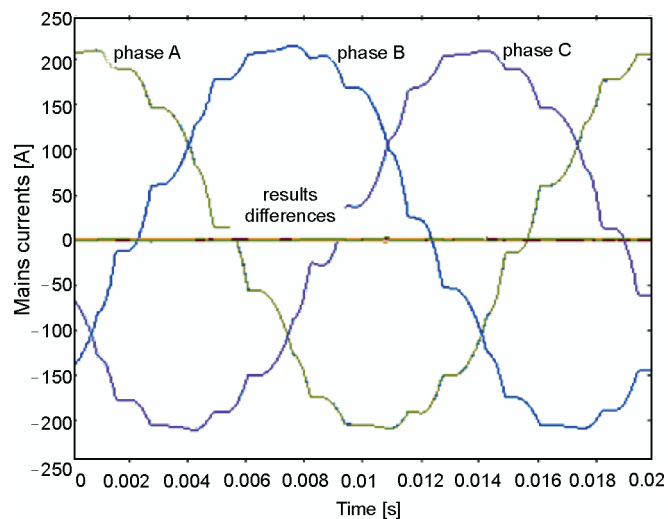


Fig. 8. Mains currents of the converter obtained using leakage inductance matrices calculated according to the three described methods

Assuming that results of this component obtained from 3D model are correct, inductance matrices  $L$  obtained from the other models can be corrected by  $\Delta L$  matrix:

$$\Delta L = \bar{w}_3 \cdot \bar{w}_3^t \cdot \Delta \lambda, \quad (7)$$

where  $\Delta \lambda$  is the zero component eigenvalue error.

In spite of big differences between elements of leakage inductance matrices obtained with presented methods, their leakage eigenvalues has very close value.

The leakage inductance matrices calculated with presented methods produce almost identical results when used in simulation of the converter circuit.

#### 4. Conclusion

The autotransformer leakage inductance calculation methods of different degree of complexity was presented and compared. The zero component related eigenvalue of the leakage inductance matrix is not calculated correctly in 2D methods. However after applying adequate correction, leakage inductances obtained with these methods can be used for converter simulations. The mono-harmonic calculations showed that the leakage inductances of the autotransformer remain constant for frequencies up to 4 kHz. This justifies the use of these inductances obtained in static calculations in the AC/DC converter simulations. The method of equivalent circuit creation of the autotransformer that uses LR ladder networks for modeling frequency dependencies of its parameters was presented.

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