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MULTIPHASE CAGE INDUCTION MOTORS FOR CONTROLLED DRIVES

Abstract: General control possibilities of the drive with a multiphase cage induction motors having number of phases greater than 3 are subject of the paper. The motors have additional properties for speed control distinguishing them from the standard 3 phase motors. They are connected with operation at various sequences of supplying voltages due to the inverter control and possible operation with open-circuited phases. This allows for different no load speeds at the same frequency. Applying vector or scalar control this feature extends applications for miscellaneous drive demands including sensor-less control. The main features of the motors depend mainly on type of the stator winding for a given number of phases. Example of operation was presented for a 9-phase motor, though general approach was discussed. The motor was supplied from a voltage source inverter and controlled with field orientation under forced currents. The mathematical model of the motor was reduced to the minimum form incorporating all most important physical features and appropriate for the control law formulation.

1. Introduction

The multiphase cage induction motors, with number of phases $M > 3$, have features distinguishing them from the standard 3 phase motors and allowing for specific control methods.

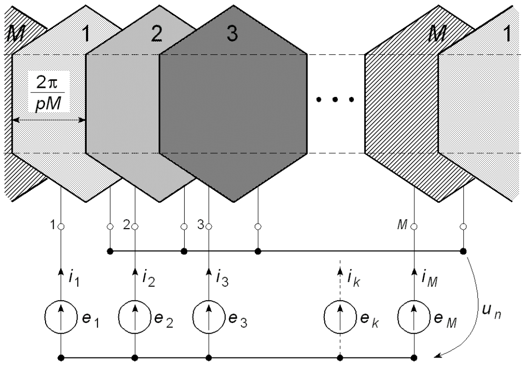

Fig. 1. M-phase winding system for pole pair number p = 1 *and M-phase supplying voltages – the phase groups of short or full pitched coils.*

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_k \\ \vdots \\ e_M \end{bmatrix} = E_s \begin{bmatrix} \sin(\phi_s) \\ \sin(\phi_s - m\frac{2\pi}{M}) \\ \sin(\phi_s - 2m\frac{2\pi}{M}) \\ \vdots \\ \sin[\phi_s - (k-1)m\frac{2\pi}{M}] \\ \vdots \\ \sin(\phi_s - (M-1)m\frac{2\pi}{M}) \end{bmatrix}_{\substack{k=1,2,\dots,M \\ m=0,1,2,\dots,M-1 \\ \phi_s = 2\pi\int_0^t f_s(\tau)d\tau}} \tag{1}$$

Greater number of phases gives the possibility for supply with different sequences of supplying voltages. For their sinusoidal form described by (1) the sequence is determined by number $m = 0, 1, 2, 3, \dots, M-1$.

Mechanical characteristics of the motor depend on the stator winding design – in practice two types of families of the characteristics can be obtained for two types of winding denoted by $S = 1$ and $S = 2$. Both of them differs each other with orders of spatial harmonics produced by resultant magneto motive force (MMF)

$$\nu = 1,\ (2-S)2,\ 3,\ (2-S)4,\ 5, \dots \tag{2}$$

The winding of type $S = 1$ produces MMF containing odd and even harmonics, whereas the winding of type $S = 2$ only odd harmonics. This is valid for the distributed and concentrated phase windings.

For each sequence number m and the same frequency f_s different no load speeds are achieved. The number of these speeds for one direction is

$$\left.\begin{aligned} m_M &= \tfrac{M-1}{2} \ \text{for odd } M \\ m_M &= \tfrac{M-2}{2} \ \text{for even } M \end{aligned}\right\} \tag{3}$$

For $m = m_{(+)} = 1, 2, \dots, m_M$ (forward sequences) the collusively positive no load speed is

$$\Omega_{0(+)} = \frac{2\pi f_s}{p \cdot m_{(+)}} \tag{4}$$

For backward sequences $m = m_{(-)} = M - m = M - 1,\ M - 2,\ ...,\ M - m_M$ the negative no load speed is

$$\Omega_{0(-)} = \frac{2\pi f_s}{p \cdot (m_{(-)} - M)} \qquad (5)$$

Hence, the change of supply sequence changes the number of poles produced by the resultant magnetic field. The multiphase motors should have in practice number of pole pair $p = 1$. When M is divisible by 3, then for $m = \frac{M}{3}$ the multiphase motor becomes the 3-phase one with phase coils connected in parallel (Fig. 2). One can notice (Fig. 2b) that the 6 phase motor is in fact the 3-phase one with switched number of poles for $m = 1$ and $m = 2$.

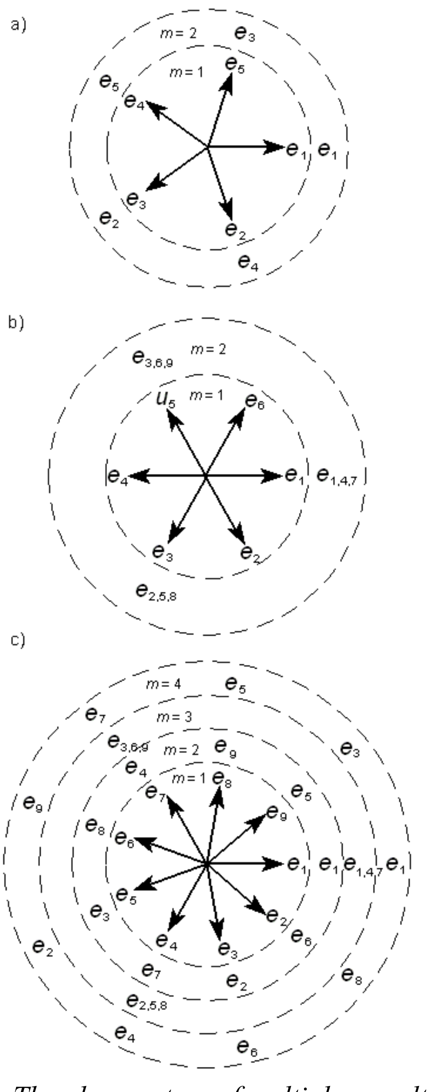

Fig. 2. The phasor stars of multiphase voltages for numbers of supply sequences m and number of phases: a) $M = 5$, b) $M = 6$, c) $M = 9$.

For sinusoidally distributed multiphase winding the motor can operate only for $m = 1$ and $m = M - 1$.

The families of mechanical characteristics typical for both the types of stator winding are shown in Fig. 3. For the first type winding (Fig. 3a) the characteristics for different sequence *m* are for the same voltage and frequency. For the second type (Fig. 3b) the voltage must be regulated according to the sequence *m*.

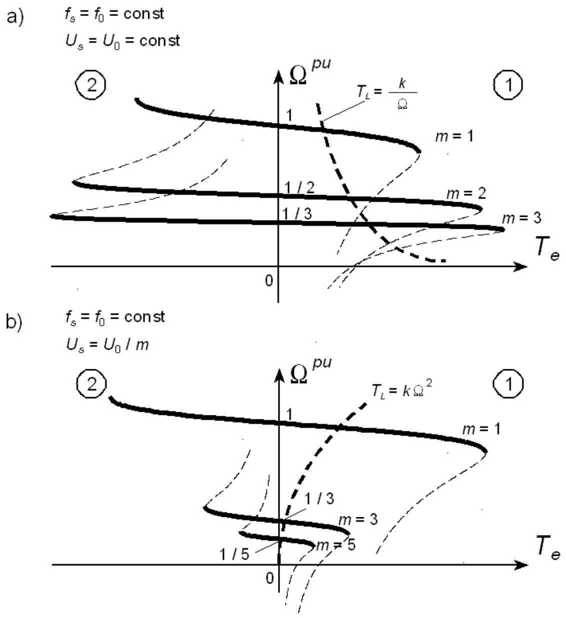

Fig. 3. Types of mechanical characteristics: a) $S = 1$, b) $S = 2$.

Bounding the work at variable sequences with variable frequency gives new possibilities for vector, scalar and direct torque control. Particularly low speeds can be obtained at the same frequency and voltage as for high speeds. This is important for sensor-less control with speed estimation or applying observers of state variables, since even at low speeds the measured voltages (for a given frequency) are relatively high [15]. Practically, the motor can be fed from a multi-phase inverter working as a voltage or a current commutator (VSI, CSI).

Applications of the multiphase induction motors described in the literature were limited to the 5 and 7 phase motors in most [1, 7, 9, 12, 13, 14, 16]. Additionally the dual 3-phase induction motors with two sets of three phase stator windings spatially shifted by 30°(el.) are also well known [6]. In the cited papers only operation for $m = 1$ voltage sequence was considered and for the stator winding of the type $S = 2$.

The approach presented in this paper was previously developed in monograph [11] and based on earlier papers [3, 4, 6, 8, 10]. So, the main task of this paper is to show additional possibilities of the motors for controlled drives that have not been practically considered in the publications. The multiphase motors have found application in locomotive traction, electric ship propulsion and the aircraft equipment. However, they can be more attractive for the traction drives and electrical cars [12, 17] utilising the switched supply sequence giving for the loading torque similar effect to the mechanical gear box with switched runs. One must be mentioned here that the switched sequence does not change the moment of inertia reduced to the motor shaft.

2. On mathematical modelling and the control law

2.1. Properties of the mathematical model

Set of equations describing the multiphase motor for natural variables has typical form for the network composed of resistances and inductances:

$$\begin{bmatrix} \mathbf{U}_{s1} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{s1} & \\ & \mathbf{R}_{r1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{s1} \\ \mathbf{I}_{r1} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \mathbf{L}_{s1} & \mathbf{M}_{sr1} \\ \mathbf{M}_{sr1}^T & \mathbf{L}_{r1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{s1} \\ \mathbf{I}_{r1} \end{bmatrix} \tag{6}$$

The simplest mechanical equation

$$J\frac{d\omega}{dt} = T_e - T_L \tag{7}$$

Electromagnetic torque

$$T_e = \mathbf{I}_{s1}^T \frac{\partial}{\partial \varphi} \mathbf{M}_{sr1} \mathbf{I}_{r1} \tag{8}$$

Vectors of stator phase voltages $\mathbf{U}_{s1}$ and the phase currents $\mathbf{I}_{s1}$ have the same dimension M and vector of rotor currents $\mathbf{I}_{r1}$ has dimension N equal to the number of rotor cage meshes (rotor phases). The matrix of stator resistances $\mathbf{R}_{s1}$ is usually diagonal and the matrix of rotor resistances $\mathbf{R}_{r1}$ incorporates self and mutual resistances of cage meshes. The matrices of leakage inductances are included into main stator and rotor inductances $\mathbf{L}_{s1}$ and $\mathbf{L}_{r1}$. The most important is matrix $\mathbf{M}_{sr1}$ of mutual inductances between stator and rotor circuits, since main features of the motor are coded in this matrix. Only this matrix depends on the rotor rotation angle φ.

Transformation to symmetrical components, separately for the stator and rotor vectors, gives two profits: natural variables become space vectors represented by appropriate symmetrical components, the set of equations assumes useful order allowing for control law formulation.

The transformation matrix for the stator and rotor variables is given below by (9). After transformation the vector of stator voltages assumes form given by (10). Similar form has vector of currents.

$$\mathbf{S}_X = \frac{1}{\sqrt{X}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \underline{a} & \underline{a}^2 & \cdots & \underline{a}^{X-1} \\ 1 & \underline{a}^2 & \underline{a}^4 & \cdots & \underline{a}^{2(X-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \underline{a}^{X-1} & \underline{a}^{2(X-1)} & \cdots & \underline{a}^{(X-1)^2} \end{bmatrix} \tag{9}$$

$\underline{a} = e^{j\frac{2\pi}{X}}$, $X = M$ for the stator and $X = N$ for the rotor.

$$\mathbf{U}_{s2} = \mathbf{S}_M \mathbf{U}_{s1} = \Big[u_s^{(0)} - \sqrt{M}u_n, \underline{u}_s^{(1)}, \underline{u}_s^{(2)}, \ldots, \underline{u}_s^{(w)}, \ldots, \underline{u}_s^{(M-w)}, \ldots, \underline{u}_s^{(M-2)}, \underline{u}_s^{(M-1)}\Big]^T \tag{10}$$

The symmetrical components $\underline{u}_s^{(w)}$ and $\underline{u}_s^{(M-w)}$ are conjugated, whereas u_n is the neutral voltage between star points of the wye-connected supply voltages and stator windings (Fig. 1). If the supplying voltages are sinusoidal (1), then

$$\mathbf{U}_{s2} = \frac{\sqrt{M}}{2j} E_s \Big\{ \big[0,0,\ldots,c^{(w)},\ldots,0,\ldots,0\big]^T e^{j\phi_s} - \big[0,0,\ldots,0,\ldots,c^{(w)},\ldots,0\big]^T e^{-j\phi_s} \Big\} \tag{11}$$

$$\left. \begin{aligned} c^{(w)} &= 1 \text{ for } w = m \\ c^{(w)} &= 0 \text{ for } w \neq m \end{aligned} \right\} \tag{12}$$

2.2. The control law

The properties of (11) and (12) were utilised to formulate simplified mathematical model of the motor constituting the background for the field oriented vector control. For this simplified model the spectrum of spatial harmonics (2) must be limited to maximum number $\nu_{max} = p(M - 1)$. This gives the possibility that equations describing the motor can be separated for every symmetrical component (magnetic and electric coupling disappears). Every two equations for $\underline{u}_s^{(w)}$ and $\underline{u}_s^{(M-w)}$ are conjugated carrying the same information. Thus, similarly as for the 3-phase motor, only equation for $\underline{u}_s^{(w)}$ is sufficient where $w = 1,\ 2,...,\ m_M$ (3). However, for sinusoidal supply at the sequence m, only $\underline{u}_s^{(m)} \neq 0$ and the remaining voltages represent short circuit. Hence, the reduced mathematical model for sinusoidal multiphase supply has the same form as for the mono harmonic 3-phase motor in the stationary reference frame (α-β).

$$\begin{bmatrix} \underline{u}_s^{(m)} \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & \\ & R_r^{(m)} \end{bmatrix} \begin{bmatrix} \underline{i}_s^{(m)} \\ \underline{i}_r^{(m)} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \underline{\psi}_s^{(m)} \\ \underline{\psi}_r^{(m)} \end{bmatrix} - \\ - j\, m\, p\, \omega \begin{bmatrix} \underline{\psi}_s^{(m)} \\ \underline{\psi}_r^{(m)} \end{bmatrix} \tag{13}$$

Flux vectors

$$\begin{bmatrix} \underline{\psi}_s^{(m)} \\ \underline{\psi}_r^{(m)} \end{bmatrix} = \begin{bmatrix} L_s^{(m)} & L_\mu^{(m)} \\ L_\mu^{(m)} & L_r^{(m)} \end{bmatrix} \begin{bmatrix} \underline{i}_s^{(m)} \\ \underline{i}_r^{(m)} \end{bmatrix} \tag{14}$$

$$L_s^{(m)} = L_{\sigma s} + L_\mu^{(m)}, L_r^{(m)} = L_{\sigma r}^{(m)} + L_\mu^{(m)} \tag{15}$$

Electromagnetic torque

$$T_e = 2\, m\, p \frac{L_\mu^{(m)}}{L_{\sigma r}^{(m)} + L_\mu^{(m)}} \mathrm{Im}\left\{ \underline{i}_s^{(m)}\, \underline{\psi}_r^{(m)*} \right\} \tag{16}$$

Structure of the above equations is the same as for 3-phase motors and they are described in the stationary reference frame. So, all known control methods can be applied here for operation at a given sequence number m. Some parameters (main inductance and rotor parameters) must change their values with respect to m.

Transformation of the variables to the reference frame (d_m-q_m), attached to the rotor flux vector $\underline{\psi}_r^{(m)}$, means that for every m individual transformation must be performed. For this reference frame the electromagnetic torque takes the following form

$$T_e = 2\, p\, m \frac{L_\mu^{(m)}}{L_{\sigma r}^{(m)} + L_\mu^{(m)}} \psi_r^{(m)}\, i_{sq}^{(m)} \tag{17}$$

This expression indicates the control law of the drive with the multiphase motor. It means that for every m the rotor flux vector and the position $\vartheta^{(m)}$ of the (d_m-q_m) reference frame must be determined separately. Parameters of flux controller must be adapted to m. Keeping the product $\psi_r^{(m)} i_{sq}^{(m)}$ constant independently of m allows for operation with greater T_e for increasing m and a lower speed – the motor works with constant power at wide range of speed (Fig. 3a).

The control law is based on the simplified model but is applied to the real motor supplied usually with the inverter. Hence, the supplying voltages are not sinusoidal causing electrical coupling between equations for every symmetrical component. More important is normally existing magnetic coupling between circuits for all stator and rotor symmetrical components expressed by the stator-rotor matrix of mutual inductances $\mathbf{M}_{sr2}$. For example this matrix, for the motor with $M = 9$, $N = 28$, $S = 1$, $p = 1$ has 252 non-zero elements. The simplified model utilises only 16 elements from this matrix.

3. Control system and waveforms

Similarly as for the 3-phase motors the field oriented control of the multiphase motor can be performed using current or voltage control. The structure of the control system results from equations (13) – (17). So, the difference appears only with adaptation to the sequence control.

For the control with forced phase currents the structural scheme has been shown in Fig. 4. The main reference signals are speed ω^{ref} and rotor flux ψ^{ref}. The additional signal is number m of the supply sequence. The system has two main controllers: $R\omega$ – speed controller (usually PI or P type with saturation), $R\psi$ – flux controller (usually PI type). The output signals: $i_{sq}^{(m)\mathrm{ref}}$ – armature command current, $i_{sd}^{(m)\mathrm{ref}}$ – excitation command current, are transformed in $T1$ and $T2$ to the reference signals of phase currents $i_{1,2,...,M}^{\mathrm{ref}}$. These signals are compared with the phase current signals and due too on-off controllers Ri for all phases control transistors of voltage source inverter F. The output signals Q_1, Q_2,..., Q_k,..., Q_M assume values +1 or –1. The positive signal switches on the positive transistor of a half-bridge for a given phase (the negative one is blocked) and vice versa. Block Ψ is the flux estimator and block ϑ determines position $\vartheta^{(m)}$ of the rotor flux vector. Block S selects m-th symmetrical component (voltage and current) eliminating the remaining and the

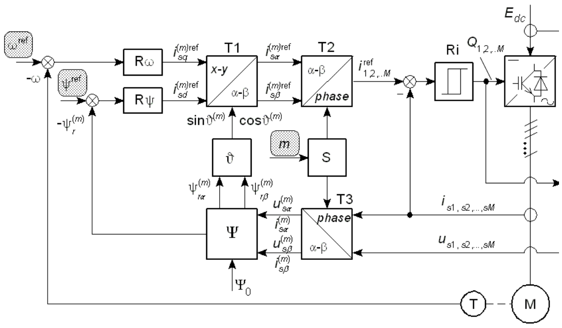

Fig. 4. Control scheme for the multiphase induction motor drive operating with forced currents.

effect of coupling between the components. Stator phase voltages need not be measured and can be calculated (block U)

$$u_{sk} = \frac{E_{dc}}{2}\left(Q_k - \frac{1}{M}\sum_{l=1}^{M} Q_l\right)\Bigg|_{k=1,2,\ldots,M} \tag{18}$$

Operation of the drive was illustrated for the 9-phase motor with mentioned above parameters. This was starting and reverse to the desired speed ω^{ref} = 120/m rad/s for m = 1, 2, 3, 4 at the same reference value for the rotor flux ψ^{ref} = 0,45 Wb. Results of operation are shown in Fig. 5.

The drive operates stable for every m. Speed controller $R\omega$ assures work with a maximum value $i_{sq}^{(m)\text{ref}}$ = 20 A during speed transients for every m. Maximum electromagnetic torque is for m > 1 m-times greater than for m = 1 for the reference speeds m times lower – work with constant power. The controller $R\psi$ forces rotor flux to the required value $L_\mu^{(m)} i_{sd}^{(m)\text{ref}}$ in about 0.25 s.

The command signal $i_{sd}^{(m)\text{ref}}$ increases for increasing m and significantly for m = 4, since the pole pairs of the air-gap field is 4 and the magnetising current for the same flux per pole pair must be greater. The speed control behaves similarly as other drives and similarly reacts on loading torque changes – the quality depends mainly on type of $R\omega$.

4. Conclusions

The drives with multiphase cage induction motors offer additional properties comparing with 3-phase motor drives:

- low speeds at relatively high frequency and voltage can be obtained for the supply sequence m > 1; a wide range of speed can be realised at voltage depth of modulation not lower than 0.45,
- with respect to the loading torque the supply sequence switching gives similar effect to the

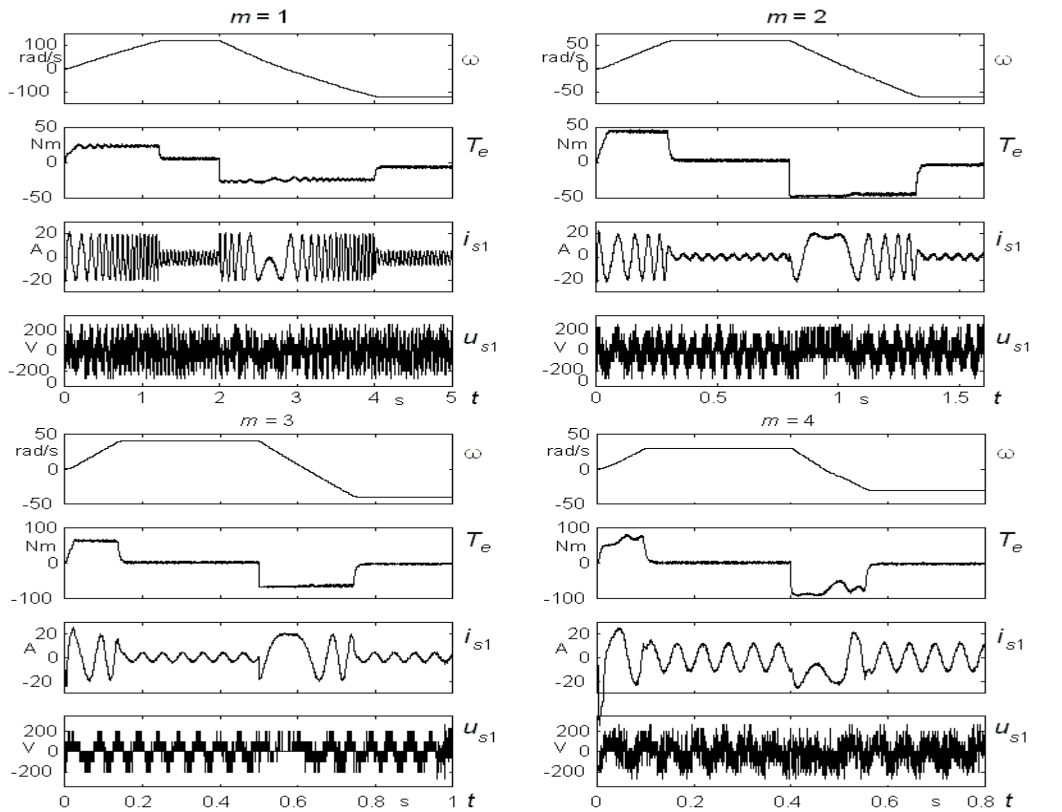

Fig. 5. Waveforms illustrating work of the drive for various m and low reactive loading torque.

mechanical gear box (this is not valid for reducing of the mechanical inertia according to the transmission ratio); the inverter can be designed for lower currents, since greater torque at lower speeds can be obtained due to greater *m* (17),

- theoretically the motor can work with no more than ($M - 3$) breaks in phase current flow,
- system of vector control with forced voltages is more complicated than the presented in Fig. 4, though more universal – can be utilised for open loop control (e.g. "scalar control").

The control law was based on the simplified two-phase motor model for every supply sequence *m* (13-16). The same structure model is usually used for the 3-phase motors. Hence, all sensor-less control methods invented and studied for the standard motors can be trained as applications for the multiphase motors. For example the MRAS speed estimator could be tried in its standard form or the improved one presented in [15]. However must be noticed that parasitic interaction between the phases is stronger than in the 3-phase motors. So, the motors must be appropriately designed to diminish those unnecessary interactions.

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