Jan Rusek
AGH University of Science and Technology, Chair of Electrical Machines

INDUCTION MACHINE DIAGNOSTICS BASED ON *FFT* ANALYSIS OF CURRENT SPACE VECTOR

Abstract: *FFT* analysis of one of the stator currents has become a standard procedure for diagnosing the state of an induction machine. In this procedure the harmonics are all as if positive. However, if the stator currents are aggregated to a space vector, a direction of rotation of complex harmonics is determinable. The sign of a harmonic conveys additional information, important for diagnostic purposses.
The paper presents analyses of calculated currents, based on an *FFT* algorithm for two currents. As, typically, a machine is fed through three cables, two currents are sufficient to establish a space vector of stator currents. The space vector is a complex quantity which can be put into an *FFT* analysis. A developed software is fitted with graphical interface. Unlike traditional display, separate amplitudes of harmonics are displayed above and below a horizontal frequency axis. The harmonics rotating in positive direction are displayed above, and the others below the frequency axis. The software automatically indicates zones where the slot harmonics are expected. The algorithm for this is based on a specific harmonic balance model. Based on the frequency of a slot harmonic, the rotor speed is determined. Preliminary analyses are uncluded.

1. Introduction

The so called Current Signature Analysis [1], based on analysis of one current, has now become a standard in motor diagnostics. However, the current harmonics are deprived of one of a very important attribute: they are sign agnostic, i.e. they are always as if positive. But if all stator currents are considered, and if they are aggregated to a space vector, being a complex quantity, the direction of rotation of a complex harmonic can be determined. That could be of some importance for proper diagnosis in more complex cases, met in real industrial diagnoses. The paper presents a software accomplishing an *FFT* algorithm for two currents. As, typically, a machine is fed through three cables, without neutral, two currents allow establishing a space vector of stator currents. This complex quantity undergoes *FFT* analysis, always carried out in a complex domain. The software was developed in one of the contemporary programming environments, in a C# language. The developed software *Sp2ph* is fitted with a graphical interface. Unlike traditional display, separate amplitudes of the harmonics are displayed above and below the horizontal frequency axis. The harmonics rotating in positive direction are displayed above, and the others below the *x* axis. The software automatically indicates zones where slot harmonics are to be looked for. This was accomplished via making use of a specific form of the harmonic balance model. The paper contains preliminary examples of applications of the developed software.

2. Space vector of stator currents

If instant currents, flowing through stator phases, are i_1, i_2, i_3, then the space vector $\boldsymbol{i_S}$ is

$$\boldsymbol{i}_S = \frac{2}{3}\left(i_1 + a i_2 + a^2 i_{3)}\right) \tag{1}$$

where $a = \exp(j2\pi/3)$.

For a typical three-phase supply, without neutral conductor, the sum of all three currents vanish. Hence

$$i_3 = -(i_1 + i_2) \tag{2}$$

Recognizing (2) in (1) the real and imaginary parts of a space vector are:

$$\boldsymbol{i}_{S,Re} = i_1 \tag{3}$$

$$\boldsymbol{i}_{S,Im} = (i_1 + 2i_2)/\sqrt{3} \tag{4}$$

As input currents i_1 and i_2 are sequences of samples, taken at the same time instants, also the real and imaginary parts (3) and (4) are sequences of, say, N samples. In accordance with Shanon's theorem, the sampling apparatus must include an anti-aliasing filter rejecting harmonics of orders higher or equal to $N/2$. The *FFT* algorithm requires that the number of samples N is a power of two. It can be taken e.g. from [2]. As, generally, N samples do not cover exactly an integer number of periods of currents, current samples must be multiplied by e.g. Hann window function. The *FFT* algorithm transforms N samples (3) and (4) into N complex numbers. In the case of classical Current Signature Analysis, based on a single current, the components resulting from the *FFT* algorithm, the order of which is higher than $N/2$, are complex conjugate to appropriate harmonics of orders lower than $N/2$. Hence, they do not convey any new information. This, however, is no longer the case if the input signal is complex, as is a space vector $\boldsymbol{i_S}$ in (1). In fact a harmonic of the order of N-μ (after *FFT* analysis) rotates with exactly the same speed as does the harmonic of the order μ, except that it rotates in opposite direction. If the sequence of input currents i_1 and i_2 is such that i_1 leads i_2, then the fundamental harmonic rotates forward. If it is not the case, current i_2 should be considered as i_1 and vice versa. Such obvious change is included in the developed software, used in the analyses further down.

3. Harmonic balance model

Application of a harmonic balance model [3] for direct calculation of amplitudes of separate harmonics, though possible [4], is not practical, mainly due to the fact that strict constancy of rotor speed leads to overestimation of the amplitudes of calculated harmonics. It, however, can constitute a starting point for deriving a specific harmonic balance model [1,5]. The assumptions for the latter one are:

- the three-phase stator winding is symmetrical, without parallel branches,
- the air-gap is uniform: there is neither static nor dynamic eccentricity,
- the slot dents are of no meaning,
- the cage is symmetric,
- the iron is of infinitely high permeance,
- the shaft flux vanish.

Despite that none of the machines fulfills these rather strong assumptions, the model proved to be very fruitful in diagnosing even big power machines run in the industry [6]. It allows to foresee the value of the so called main slot harmonic index *mShi*, also referred to as a parameter h, indicating a zone where the main slot harmonic is to be looked for. The specific harmonic balance model also allows establishing of the so called sequence index *Si*, also referred to as a *category*, indicating the sequence of symmetrical components of higher harmonics. Both, the *mShi* and *Si* indexes result from one, rather simple, equation (5), derived in [1] and [5], with p meaning the number of pole-pairs and N_r the number of rotor slots.

$$\begin{Bmatrix}1\\0\\2\end{Bmatrix}+3k=\frac{hN_r}{2p} \tag{5}$$

Equation (5) is to be understood in a following manner. One looks for the smallest natural number h fulfilling (5) for either 1, or 0, or 2 from the curly brackets. This value of h coincides with the value of the *mShi* index. If the above fulfillment takes place for 1, from the curly brackets, the sequence index *Si = 012*, if for 0 the *Si = 111*, and if for 2 the *Si = 210*.

For *Si = 012* the sequence of symmetrical components of stator current higher harmonics, in equations of the specific harmonic balance model, is:

$$\begin{bmatrix}I^{s0}_{1,hN_r}\\I^{s1}_{1,0}\\I^{s2}_{1,-hN_r}\end{bmatrix} \tag{6}$$

For *Si= 111* this sequence is:

$$\begin{bmatrix}I^{s1}_{1,hN_r}\\I^{s1}_{1,0}\\I^{s1}_{1,-hN_r}\end{bmatrix} \tag{7}$$

For *Si= 210* this sequence is:

$$\begin{bmatrix}I^{s2}_{1,hN_r}\\I^{s1}_{1,0}\\I^{s0}_{1,-hN_r}\end{bmatrix} \tag{8}$$

In (6) to (8), out of an infinite number of symmetrical components of higher harmonics, only the three central ones are explicitly exposed. The superscripts are indexes of symmetrical components. The subscripts determine the speed of rotation of separate symmetrical components, of higher harmonics. For example, the subscript $1,hN_r$ in (6) determines the angular speed ω_h of rotation of a complex higher harmonic:

$$\omega_h=1\cdot\omega_e+hN_r\cdot\Omega_m \tag{9}$$

where

$\omega_e = 2\pi f_L$, $f_L = 50$ Hz, is a power supply frequency,

Ω_m – is a mechanical speed, in rad/s.

Formulas (6) to (8) determine the layout of a specific harmonic balance model. However, for further analysis, of importance are the following notes:

- The top and the bottom harmonics in (6) to (8) constitute pairs of main slot harmonics. They occur in slot zones determined by $mShi = h$.
- If a top or a bottom harmonic contributes to a zero or homo-polar symmetrical component, it must vanish, if there is no neutral conductor. Hence, in cases (6) and (8) only one main slot harmonic remains.
- The distance between absolute values of speeds of both main slot harmonics is $2f_L$, if both exist.
- If, as a result of *FFT* analysis, the angular frequency ω_h, of a given harmonic, is known, equation (9) allows determining a mechanical speed Ω_m.

Given f_L and Ω_m, the frequencies of the two diagnostic components, accompanying cage asymmetry, are [7]:

$$f_{l,r} = f_L(1 \mp 2s) \qquad (10)$$

where the slip s is:

$$s = (\omega_e - p\Omega_m)/\omega_e \qquad (11)$$

Formula (10) is used in the developed software to generate a hint for potential location of the diagnostic components, characteristic for cage asymmetry. However, cage asymmetry is not a subject of present contribution.

4. Centric suspension of rotor

Fig. 1 shows a spectrum of stator a currents space vector. It, as well as all other figures, refers to a 132 kW cage induction motor. The number of stator slots was $N_s = 72$ and that of the rotor was $N_r = 56$.

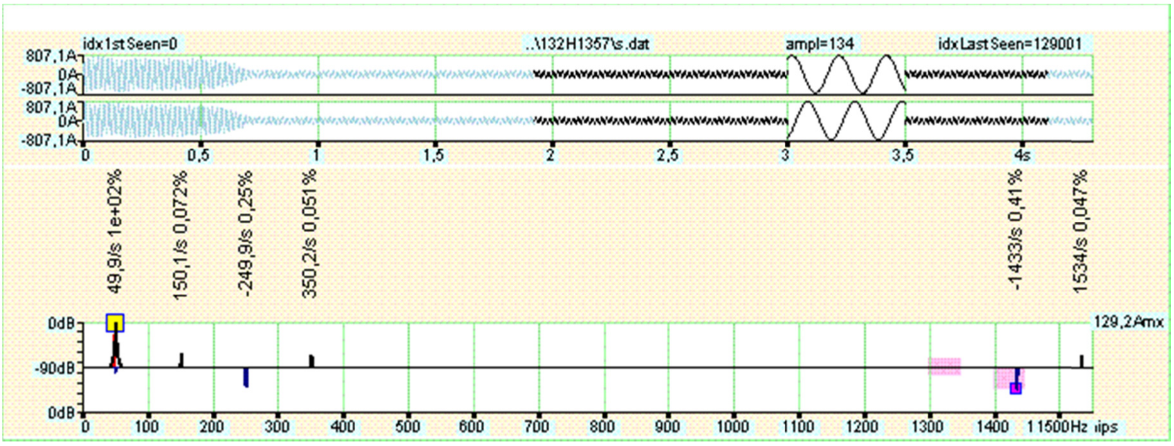


Fig. 1. Stator currents and spectrum of a current space vector for steady-state interval. Centric suspension of the rotor.

The stator winding had two parallel branches in each phase. The calculations were accomplished with account for slotting and parallel branches. In next paragraphs also eccentricities are taken into account. Saturation was accounted for only through fictitious magnification of the air-gap length (thickness), depending on the magnitude of the supply currents, related to nominal currents. The supply voltage of 1000 volts was assumed to consist of a fundamental 50 Hz harmonic plus third, fifth and seventh higher harmonics. To depict typical power supply conditions the fifth harmonic was assumed to constitute a negative sequence. In the spectrum, the fundamental 50 Hz harmonic is marked with bigger square and the main slot harmonic with smaller square. The calculations encompass both the start up and the steady state. The start up interval is here meaningless. The long enough steady state interval guarantee extinguishing of all transient components. The time interval of currents, really *FFT* analyzed, is plotted in bold. Frequency of the main slot harmonic is −1433 Hz. Its location is indicated by a downward directed broad bar. Location of this bar results from a specific harmonic balance model. The negative rotation of the main slot harmonic, foreseen by a specific harmonic balance model, is confirmed by the *FFT* analysis of a current space vector. Thanks to this model, the slot harmonic was not mixed with the one of the frequency of 1534 Hz, also present in Fig. 1.

5. Static eccentricity

Fig. 2 shows a spectrum of a current space vector for a motor the rotor of which is afflicted by a static eccentricity of 90% of the geometrical air-gap length (thickness). In the spectrum, the harmonics already present in Fig. 1, are flushed to the top rim of the spectral field. A new harmonic, of the frequency of 1333 Hz, is shown conspicuously lower. It is located in the left subzone of the main slot harmonic zone, encompassing left and right slot subzones, as it was discussed in the third paragraph. Its location, for static eccentricity, was already foreseen in [6]. Its amplitude is rather small due to the parallel branches allowing for equalizing currents.

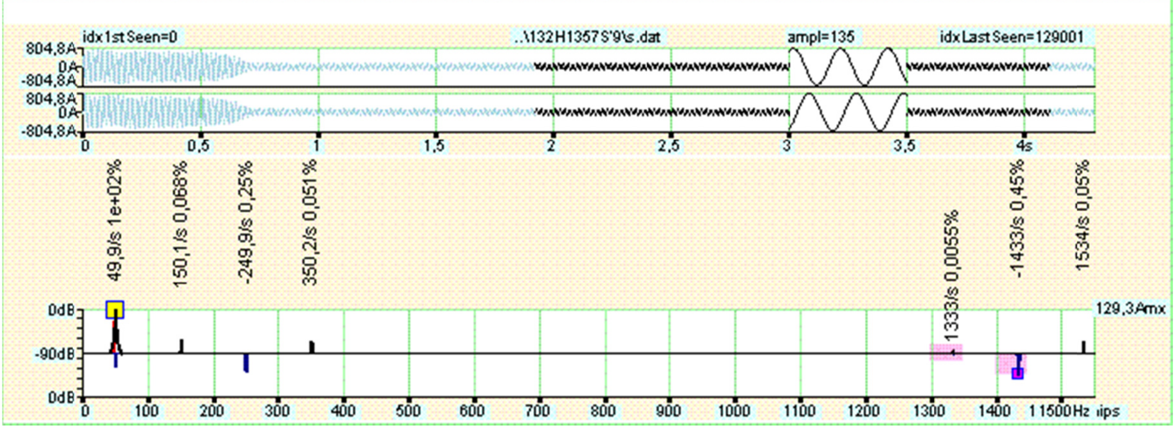


Fig. 2. Stator currents and spectrum of a current space vector for steady-state interval. Static eccentricity of 90% of air-gap.

6. Dynamic eccentricity

Fig. 3 shows a current space vector spectrum for a motor with dynamic eccentricity of 90% of the geometrical air-gap length. The harmonics characteristic for dynamic eccentricity are displayed lower than those already present by lack of eccentricity. Their frequencies are 99.33 Hz and 1383 Hz. The former value is only an approximation of −100 Hz, as a frequency resolution, due to the length of the *FFT* analyzed time interval, is about 0.5 Hz. The presence of the −100 Hz component was already reported in [8]. The here presented *FFT* analysis revealed that the 100 Hz component rotates in negative direction.

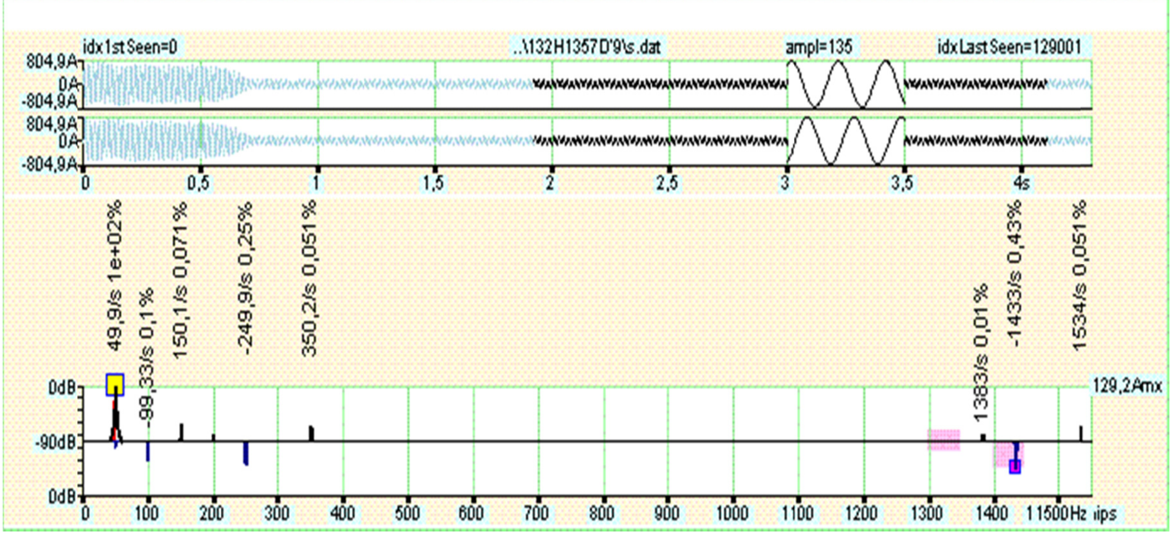


Fig. 3. Stator currents and spectrum of a current space vector for steady-state interval. Dynamic eccentricity of 90% of air-gap.

7. Mixed eccentricity: static + dynamic

Fig. 4 shows a spectrum of a current space vector for a motor the rotor of which is afflicted by both the static and dynamic eccentricities, both of 45% of the geometrical air-gap length (thickness). The left subzone does not contain a harmonic characteristic for a static eccentricity. Actually, the amplitude of this harmonic falls below the accounted for limit of −90 dBs. Around a fundamental 50 Hz harmonic a pair of the so called rotational harmonics appeared. Similar pair of rotational harmonics appeared around a main slot harmonic. Their frequencies are 25.18 Hz and 74.62 Hz in the first case, and −1409 Hz and −1458 Hz in the second. The rotational harmonics are those whose frequencies differ by a rotational speed of a rotor (expressed in revolutions per second) from the frequency of the harmonic giving raise to the pair of rotational harmonics under consideration. Here, both in the fundamental harmonic range and in the slot harmonic range, only one pare of rotational harmonics is present. However, there are cases where also higher order rotational harmonics can occur [6]. To be noted is that the direction of rotation of these harmonics coincide with that of a harmonic giving raise to rotational harmonics. The −100 Hz harmonic in Fig. 4 diminished its value, as the dynamic eccentricity is now smaller than that in Fig. 3. However, the most characteristic thing referring to a mixed eccentricity is that its spectrum is not just a superposition of the spectra for solo static and solo dynamic eccentricities. The mixed eccentricity is accompanied by totally new harmonics, that is by the just discussed rotational harmonics. Hence, the behavior of the static and dynamic eccentricities is multiplicative, and not additive. If one of these eccentricities vanish, vanish also all rotational harmonics, if, of course, they are not generated by some other phenomenon, such as torque pulsation [6].

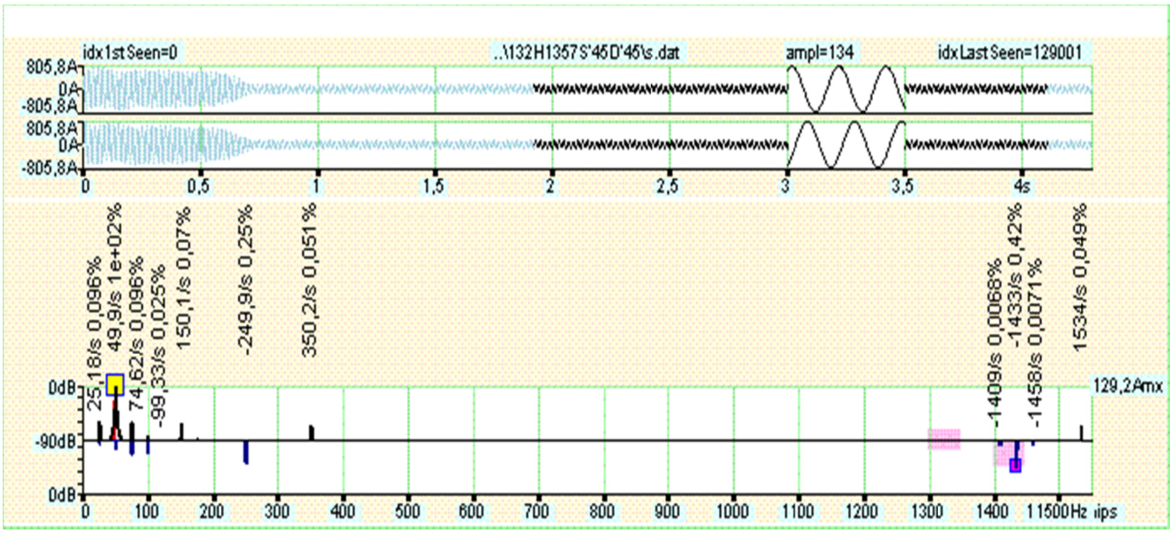
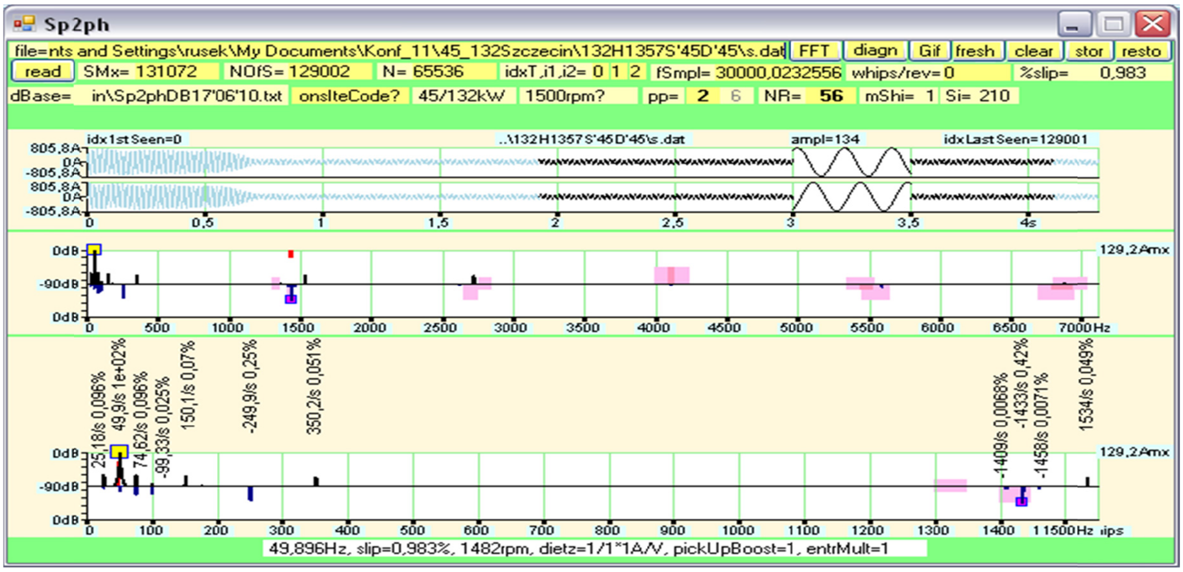


Fig. 4. Stator currents and spectrum of a current space vector for steady-state interval. Mixed eccentricity: static eccentricity of 45%, plus dynamic eccentricity of 45%, of air-gap.

8. Interface of *Sp2ph*

Fig. 5 shows a screenshot with graphical interface of the developed software *Sp2ph*. The elements of the interface allow to recognize all the input parameters indispensable to carry out the two-current based *FFT* analysis. The main information displayed in the top strip is a qualified name of the file containing current samples. The second strip contains the value $N = 65536$ being the number samples selected for *FFT* analysis. A separate value of *0* is a zero-based index of a column with time samples. The separate values of *1* and *2* indicate that the column of index *1* contains samples of current i_1, leading current i_2. The third strip contains a qualified name of a database file, containing numbers of pole pairs and of rotor slots, of various machines, of which the one, the *45/132 kW* have been selected.



file=nts and Settings\rusek\My Documents\Konf_11\45_132Szczecin\132H1357S'45D'45\s.dat
SMx= 131072
NOfS= 129002
N= 65536
idxT,i1,i2= 0 1 2
fSmpl= 30000,0232556
whips/rev= 0
%slip= 0,983
dBase= in\Sp2phDB17'06'10.txt
onsIteCode?
45/132kW
1500rpm?
pp= 2 6
NR= 56
mShi= 1
Si= 210
idx1stSeen=0
..\132H1357S'45D'45\s.dat
ampl=134
idxLastSeen=129001
49,896Hz, slip=0,983%, 1482rpm, dietz=1/1*1A/V, pickUpBoost=1, entrMult=1

Fig. 5. Interface of the developed software Sp2ph for FFT based diagnostics of induction machines.

9. Conclusions

- The proposed *FFT* analysis of a stator currents space vector delivers information on the direction of rotation of separate complex, higher harmonics.
- Specific harmonic balance model removes ambiguity by decision making on which harmonic is the slot one.

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Author

Prof. Jan Rusek, AGH University of Science and Technology, Chair of Electrical Machines, Al. Mickiewicza 30, 30-059 Krakow, Poland, (48) 12 6341096,gerusek@cyf-kr.edu.pl.

Reviewer

Prof. dr hab. inż. Marian Pasko