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PORÓWNANIE METOD NUMERYCZNEGO WYZNACZANIA PRĄDU TRANSFORMATORÓW NIEOBCIĄŻONYCH

COMPARITION OF NUMERICAL METHODS FOR CURRENT DETERMINATION UNDER NO-LOAD TRANSFORMERS

Abstract: In this paper, the new Modified Time Stepping Finite Element Method (MTSF) and Field- Circuit Method (FCM) for the inrusch current calculations of transformers have been presented. The computation time for MTSF and classical Time Stepping Finite Element Method (TSF) in 2D magnetic field analysis have been compared. The MTSF is about two times faster than the TSF method. We analyzed two constructions of the 1-phase transformers. The first one is amorphous modular core transformer (T1) and the second one is the conventional construction with laminated carbon steel core (T2). The comparision of the calculation and measurement results gives a good agreement.

1. Introduction

In many industrial power systems, the transformers are switched on and off many times during their using. In the switching on moments a non-sinusoidal transient inrush current arises. In some cases, the magnitude of the inrush current is several times higher than the operational load current. Its value depends mostly on the voltage magnitude of the supplying source and the residual flux in the transformer core, as well as the dynamic inductance.

The benefits of the computer simulations and the current determination are well recognized. Knowledge of their values is also important for correct determination of the shelters parameters [2]. A accurate approximation of the inrush current requires detailed information regarding the transformer parameters [1]. If its physical model is approachable, the equivalent circuit parameters can be simply obtained from its measurements. However, during the transformer designing they can be obtained from computer simulations, e.g. from magnetic field calculations [9, 10].

The transients of transformers were analyzed in many works e.g. [8, 9]. Not always the residual (remanent) flux is considered for the transformer soft magnetic material core. In some cases, the hysteresis effects were also taken into account [9]. In this work we carried out the calculations using the equivalent circuit parameters, which have been obtained from numerical analysis. We included different values of the residual flux. We have simulated the single-phase transformer operation and its transient states.

Using magnetic field analysis, the non-linear characteristic of the dynamic inductance, as a function of magnetizing current, has been determined. Also, the leakage inductances have been computed. In the computations the material characteristics have been included and the magnetic residual flux has been indirectly taken into account for the initial value of the magnetizing current fixing.

In this work have been compared commercial programs with program created by authors as part of grant of Polish Ministry of Science and Higher Education. In this algorithm is used modified Time Stepping Method. This modification consists on using calculated magnetic flux values in every step to determine inductance.

2. Analyzed objects and mathematical models

2.1. Analyzed transformers

For investigations, the new construction of the 1-phase transformer with amorphous modular core (T1) and a conventional 1-phase transformer (T2) have been studied [8] (Fig. 1). Each of the amorphous transformer columns consists of two hollow cylinders (toroids). Yokes of the T1 transformer have rectangular shape with two rounded thinner sides (Fig. 1a). Contrary to the T1, the T2 transformer has a core package made from thin sheets of high-grade steel. In Fig. 1, the main dimensions of the transformers and the assumed Cartesian coordination systems are shown.

For the T1 transformer, the turn number of the windings is N=232, while for the conventional one (T2), the winding is wounded with N=182 turns.

a) *b)*

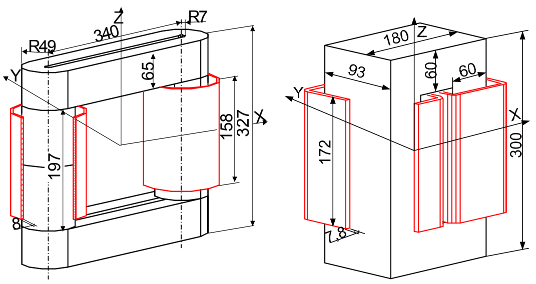


Fig. 1. Outline of the analyzed transformers with: a) amorphous core - T1, b) traditional core – T2.

2.2. Field-Circuit Method (FCM)

Generally, the inrush current of the transformer was analyzed by using Field-Circuit Method (FCM). In this model the calculations of the transients are based on the transformer equivalent circuit (Fig. 2), which is described by the following system of the differential equations [3], [9]:

$$\begin{cases} L_s \dfrac{di}{dt} = u - R \cdot i + R_{Fe} \cdot (i_\mu - i) \\ L_d(i_\mu) \dfrac{di_\mu}{dt} = -R_{Fe} \cdot (i_\mu - i) \end{cases} \quad (1)$$

In the expressions (1), the leakage inductance value is L_s=732 μH for T1 and L_s=177 μH for T2, the RMS value of the excitation voltage U=220 V, the coil resistance value - R=0.24 Ω for T1 and R=0.136 Ω for T2. The core losses resistances of R_{Fe}=1913 Ω (for T1) and R_{Fe}=2186 Ω (for T2) have been determined. The currents i and i_μ are unknown functions. The nonlinear dynamic inductance $L_d(i_\mu)$ should be determined by the finite element (FEM) calculations.

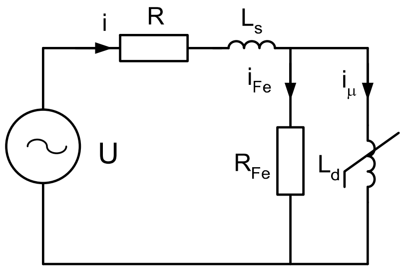


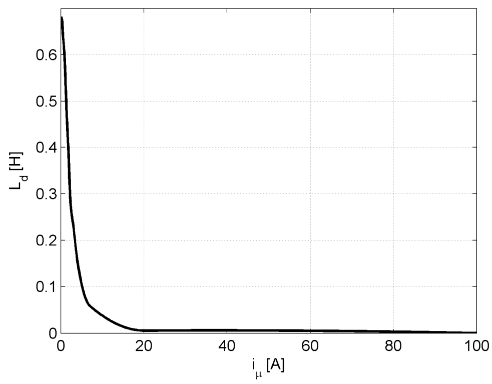
Fig. 2. Equivalent circuit of the transformer in the no load state.

As was mentioned in the section 1, the residual flux value B_r and the flux magnetic path l_μ, influence on the current value i_μ

$$i_\mu(0) = \frac{B_r \cdot l_\mu}{\mu \cdot N} \quad (2)$$

The leakage inductance L_s value of the transformer winding has been determined by using the field analysis under short-circuit state. Resistance R_{Fe} has been determined from measurements under no-load state, whereas the dynamic inductance L_d curve has been created using the field models. The assumed excitation current values in the models has changed from 0.2 to 100 A. In Fig. 3 the dynamic inductance verso excitation current is presented.

a)



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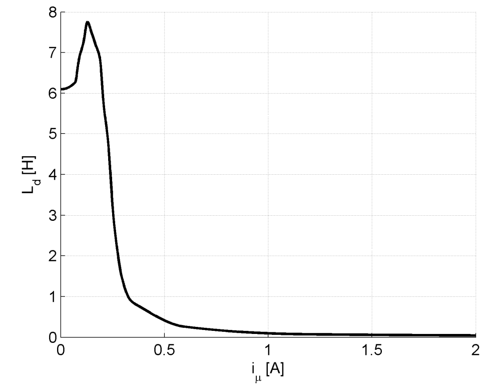


Fig. 3. Dynamic inductance Ld vs. coil excitation current i_μ: a) T1 transformer, b) T2 construction.

2.3. Modified Time Stepping FEM (MTSF)

In the second mathematical model (called MTSF) the governing expressions for the magnetic field is represented by Maxwell's equation with the magnetic vector potential $\underline{\vec{A}}$. If the eddy currents in the iron core are neglected, the magnetic field can be expressed by the partial differential equation (PDE)

$$\nabla \times \left(\frac{1}{\mu(\underline{\vec{B}})} \nabla \times \underline{\vec{A}}\right) = 0 \tag{3}$$

$\mu(\underline{\vec{B}})$ is the nonlinear permeability of the material.
In the area of the windings, the magnetic field can be governed by the equation

$$\nabla \times \left(\frac{1}{\mu(\underline{\vec{B}})} \nabla \times \underline{\vec{A}}\right) = \underline{\vec{J}} \tag{4}$$

where $\underline{\vec{J}}$ is the total current density.

The Galerkin's approach is the most popular method for matrix of elements formulation. In this proposed model, the weighting functions are the same as the shape functions for this particular weighted residual method. According to the Galerkin's method, the magnetic vector potential can be expressed as

$$A = \sum_{j=1}^{3} N_j A_j \tag{5}$$

where N_j are the element shape functions and the A_j are the approximations of the vector potential at the nodes of the elements. Thus, the formulation for the field, in the current currying regions, is expressed by:

$$\iint_s \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \frac{1}{\mu} \sum_{j=1}^{3} N_j A_j + \right. \\ \left. + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \frac{1}{\mu} \sum_{j=1}^{3} N_j A_j + N_i \left(\frac{i}{S} \right) \right] ds = 0 \tag{6}$$

For the other subregions of the transformer the functional is expressed

$$\iint_S \left[\frac{\partial N_i}{\partial x} \frac{\partial}{\partial x} \frac{1}{\mu} \sum_{j=1}^{3} N_j A_j + \frac{\partial N_i}{\partial y} \frac{\partial}{\partial y} \frac{1}{\mu} \sum_{j=1}^{3} N_j A_j \right] ds = 0 \tag{7}$$

Taking into account the average length path l of the flux in the coil and its cross-section S, the electric circuit can be described by the:

$$u = \frac{l}{S} \left[\iint_{S_1} N_i \frac{\partial A}{\partial t} ds - \iint_{S_2} N_i \frac{\partial A}{\partial t} ds \right] + \\ + Ri + L_s \frac{di}{dt} = 0 \tag{8}$$

The integrals in (8) refer the region s_1 with the positive direction of the excitation current and the region s_2 with the negative current direction. Contrary to the commercial computational methods of the time variable FEM [5], we have solved the equation (8) with our software. For the field calculation with the discretized functionals we used the FEMM software. In this modeling method the 2D field calculations have been done. Due to field distributions are only approximate especially for amorphous transformer. Its geometry is difficult to accurate 2D field calculate. Therefore depth of this object was fitted for magnetic flux value.

Our method characterizes simplicity and multitask system. Thus the computational platform doesn't need to execute so many iterations like the classic TSF algorithm. Contrary to the TSF method, the values of the dynamic inductance, which concern the field values in the step "*i*", are stored in the separate matrix, which is located in the RAM memory. Thus, we don't need to compute integrals within the eq. 8 in each step of the computational process. It is a great advantage of the MTSF, because only at several time steps the problem must be solved.

It should be emphasized, that in the MTSF method the dynamic inductance values are computed for the demanded current values. In FCM, the dynamic inductance curve has to be interpolated with using the fixed excitation current values.

3. Calculation results

In this paper the supplying voltage for the primary winding under the no load state of the transformer has been assumed. However, the magnetic flux density distribution has been calculated for many values of the current excitation. For example, in Fig. 4 the flux density is presented. The field analysis is devoted to the dynamic inductance determination.

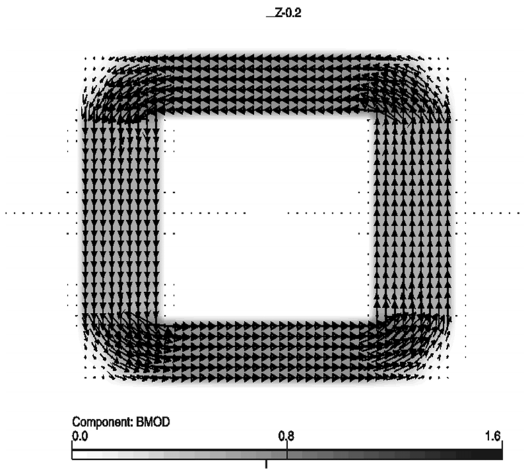
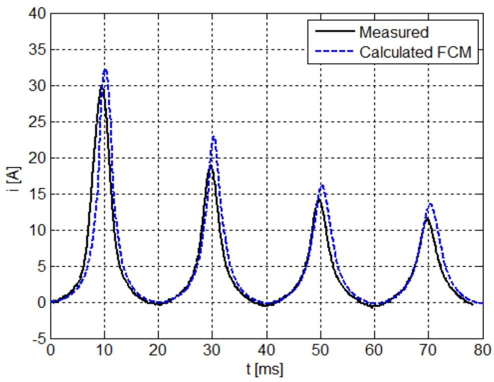


Fig. 4. Flux density distribution for I=1.5 A.

In Figs. 5 and 6 the comparison of the calculated and measured inrush current waves for T1 transformer, under excitation phase φ=0 and φ=90° have been presented. The FCM method and the MTSF one give similar results. However, the first one is about 1.5 times faster. In the case of the φ=90°, both models give almost the same inrush current waves (Fig. 6).

a)



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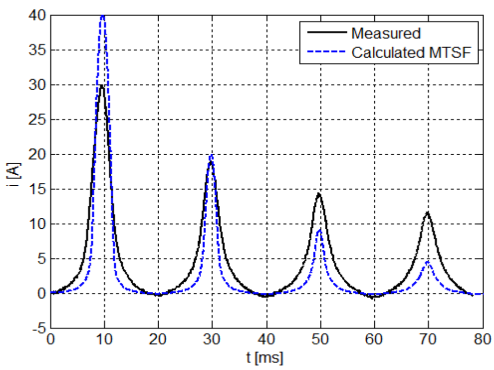
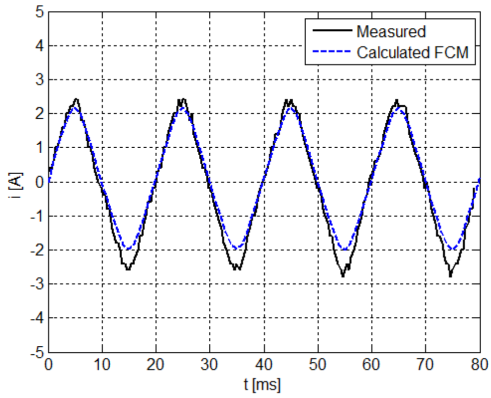


Fig. 5. Inrush current waves for T1 transformer (φ=0): a) FCM, b) MTSF.

a)



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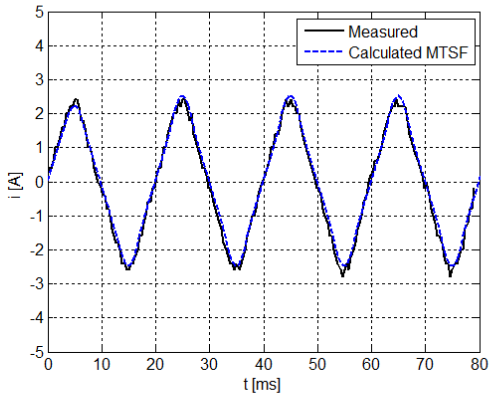


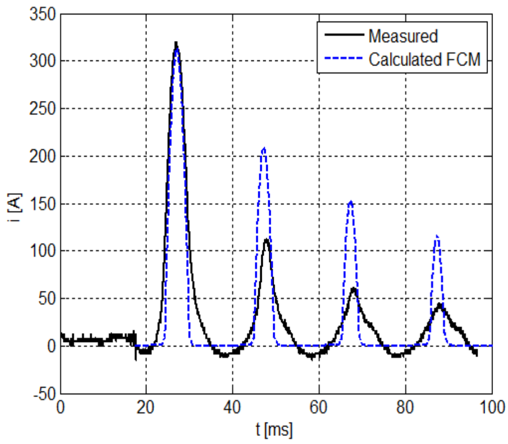
Fig. 6. Inrush current waves for T1 transformer (φ=90°): a) FCM, b) MTSF.

We also calculated the inrush currents for the conventional construction of the transformer T2. The calculations have been executed for three different values of the core residual flux density. The assumed switching on phase φ=0 has been chosen. The comparison of the calculation results for two computing methods shows, that the MTSF method is more adequate for transient calculations, (Figs. 7 and 8).

It can be observed a fine attenuation of the current waves in the case B_r=1.2 T. The MTSF calculation method gives more accurate results comparing to the simplified field-circuit method (Fig. 7).

Fig. 8 shows the calculated current waves for two values of the residual flux density: B_r=0 T and B_r=0.8 T. We can observe that the residual flux density strongly influenced on the inrush current. The current values in the first times period of the current wave are about one hundred times greater than those simulated without the residual magnetism.

a)



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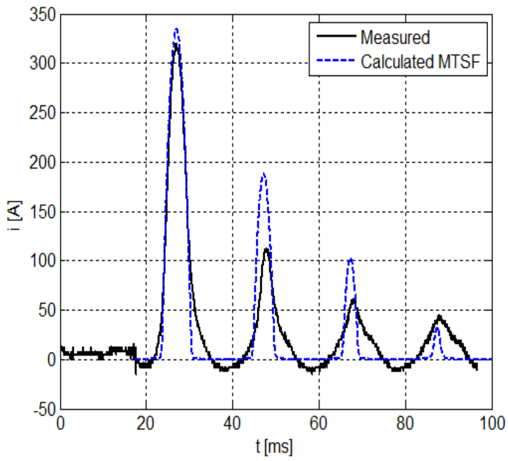
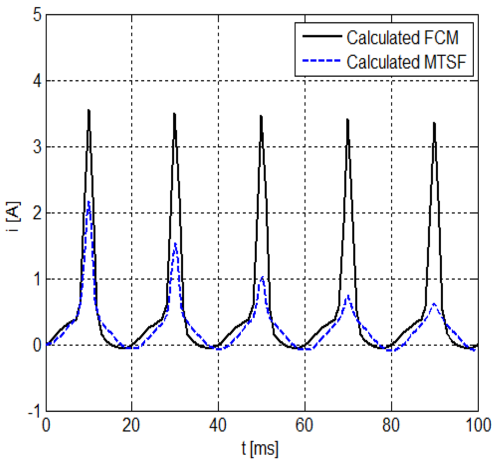


Fig. 7. Inrush current waves for T2 transformer for B_r=1.2 T: a) FCM, b) MTSF.

a)



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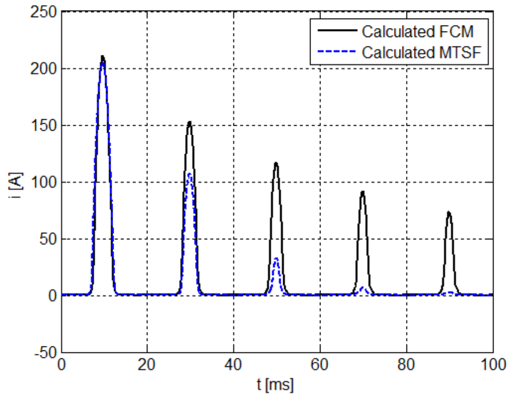

Fig. 8. Inrush currents for T2 transformer: a) B_r=0 T, b) B_r=0.8 T

The calculations of the currents have been executed with a TSF method, as well. The method is included in many commercial FEM applications. To compare the calculation times for all the methods, we also computed the problem using FCM model. The MTSF method was two times faster than the TSF one for the inrush current calculations (Tab. 1). The computations have been done with using the AMD64 3200+ processor and 3GB of RAM.

Tab. 1. Compared CPU times for analyzed T2 transformer

	FCM	TSF	MTSF
CPU time [s]	543	1782	846

4. Conclusions

The field-circuit method (FCM) and time stepping FEM (MTSF) for simulation of the inrush current waves in the two transformers have been studied. In the case of amorphous transformer T1 both methods give similar results. However, for T2 transformer analysis, the more accurate results arise from the MTSF method.

The MTSF has been compared with the commercial TSF, [4]. The CPU time is about two times shorter for the MTSF, which validates the algorithm.

The calculation method presented in this work has been validated by measurements of the single-phase transformers. We observed relatively good conformity between computed and measured current waves [Figs. 5, 6, 7]. The differences between calculation and measurement results arise from the simplifications in the field analysis and measurement errors. The main difficulty is precisely determine value of residual flux which has significant influence on inrush current. For example, the residual flux is difficult for testing and contributes to the errors of our method, as well. Whereas the second one is impossible has been take exactly air gap length. Moreover, we can see that the inrush current for amorphous modular transformer has maximal value several times lower compared to the conventional one.

5. Acknowledgements

This paper is partially supported by the Polish Ministry of Science and Higher Education under grant no. N N510 533739.

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