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BEMLAB OPEN SOURCE LIBRARY AND PROBLEMS EXPRESSED BY COMPLEX NUMBERS

ABSTRACT *BEMLAB [7, 8] is the open source Boundary Element Method (BEM) library designed for solving potential problems. BEM requires Partial Differential Equations (PDEs) to be expressed in the boundary integral form. To simplify solution of some common time based problems, PDEs can be expressed using complex calculus. Boundary Integral Equations (BIEs) have to be adapted accordingly. Therefore it is essential to introduce complex numbers into BEMLAB library. The article announces mathematical backgrounds and introduces architecture areas, which are complex specific. It also presents an example of library usage.*

Keywords: *BEMLAB, BEM, boundary element method, objective library, numerical software, open source, GNU LGPL, C++*

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Nomenclature

Abbreviations

BEM – Boundary Element Method,
 FEM – Finite Element Method,
 BIE – Boundary Integral Equation,
 PDE – Partial Differential Equation,
 UML – Unified Modelling Language.

Symbols

Ω – domain,
 Γ – boundary of the domain,
 φ, V, u – potential,
 q – potential normal derivative $\frac{\partial u}{\partial n}$,
 G – Green function,
 \mathbf{r} – position vector,
 k – wave number,
 f – domain function, source,
 ω – pulsation.

1. INTRODUCTION

In 1970s the term Boundary Element Method (BEM) has been proposed by A. Brebbia [1]. It was accepted by the scientists and describes the method which this article concerns. Since 1970s it is being developed rapidly. Commonly it is being compared with Finite Element Method (FEM), which is sometimes stated as the main „rival” of BEM. BEM, as the name states, is the boundary method, in comparison to FEM which is the domain one. When the problem is solved using BEM, the discretized boundary plays the main role in calculations. When problem is homogeneous within the whole area or its particular regions, no discretization of the domain is necessary. Moreover, solution is characterised by the steady error, which comes from all boundary elements, within the whole domain. Unlike BEM, FEM needs the whole domain to be discretized, which increases the size of problem by one. The advantages of BEM make it a better choice for many problems, especially when the knowledge of the solution inside the whole domain is not required. BEM is less effective, when the solved problem has badly conditioned shape (number of finite elements in FEM is close to the elements in BEM) or the domain has to be densely discretized. Moreover set of equations created by BEM is characterised by the dense, non-symmetric matrix (though the size of matrix is lower than in FEM) and all elements of boundary mesh have to be directed unambiguously.

BEM can be successfully used in a range of problems, however the market lacks of software implementing BEM [7, 3]. Probable reasons can be found in a complicated singular integration or problems with providing universal multi-regional algorithms. Therefore BEMLAB project was started, the unique architecture was created, new algorithms were implemented. All copyrighted by the open source licenses. The demand for complex calculations was one of the library specification requirements. The following sections will bring closer specificity of complex calculations in the BEMLAB library.

2. BEM COMPLEX PROBLEMS

The most common PDEs, which are solved using numerical methods, are shown in (1) and (2). They are used in many disciplines, inter alia: electromagnetics, thermodynamics, acoustics, optics, seismology.

- Poisson:

$$\nabla^2 u(x) = -f(x) \quad (1)$$

- Helmholtz:

$$\nabla^2 u(x) - k^2 u(x) = -f(x) \quad (2)$$

When $-f(x) \neq 0$, both, Poisson and Helmholtz, need the domain to be discretized. When $-f(x) = 0$, no domain discretization is needed while using BEM. When $-f(x) = 0$, (1) is called the Laplace equation.

Unlike FEM which uses PDEs, BEM solves BIEs shown in the analytical form:

$$c_i(r_i)u_i(r_i) + \int_{\Gamma} u(r_i) \frac{\partial G(r_i, r)}{\partial n} d\Gamma(r) = \int_{\Gamma} q(r_i)G(r_i, r) d\Gamma(r) + \int_{\Omega} f(r_{\Omega})G(r_i, r_{\Omega}) d\Omega(r_{\Omega}) \quad (3)$$

Sometimes, problems expressed by equations using real numbers are complicated and calculations are difficult to proceed. However there is a range of examples where calculations using complex numbers simplify equations and consequently the solution.

For example alternating sinusoidal field equation (left side of (4)) can be expressed in the symbolic form (right side of (4)) as Helmholtz equation in complex domain. For detailed transformations see [2, pp. 235-236].

$$\nabla^2 V - \mu\gamma \frac{\partial V}{\partial t} - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \Rightarrow \nabla^2 \underline{V} - \Gamma^2 \underline{V} = -\frac{\underline{\rho}}{\varepsilon} \quad (4)$$

$$\Gamma^2 = -\omega^2 \mu\varepsilon + j\omega\mu\gamma$$

where

- μ – permeability;
- ε – permittivity;
- γ – conductivity;
- ρ – density.

Another examples of complex numbers applications are diffusion equations in frequency domain [5, pp. 58-60] or diffusion models of light transport for: distributed sources ([5, pp. 65-69], located on the boundary ([5, pp. 70-74]) or located inside the region ([5, pp. 75-79]). They are defined by the governing equation (5).

$$\nabla^2 \Phi(r, \omega) - k^2 \Phi(r, \omega) = Q \quad (5)$$

where Q – source function (here, when $Q \neq Q_0$: magnitude).

3. BEMLAB COMPLEX NUMBERS IMPLEMENTATION

The open source BEMLAB software is protected and freely distributed under GNU LGPL (Lesser General Public License) terms [9]. BEMLAB is designed and implemented objectively. The reference command-line application uses common API (Application Programming Interface) delivered by the library. Complex numbers had to be introduced in number of classes. The main condition was to use the same code as for real numbers. No over-coding was a noble purpose. This was provided by using templates (C++) or generics (Java). Only few methods had to be overloaded.

The bird's eye view of the BEMLAB library can be found in [7, 6]. The following figures show UML class diagrams of main template classes supporting complex numbers. Classes marked with template can be instantiated against real or complex number type – for double precision it will be `double` or `complex<double>` type.

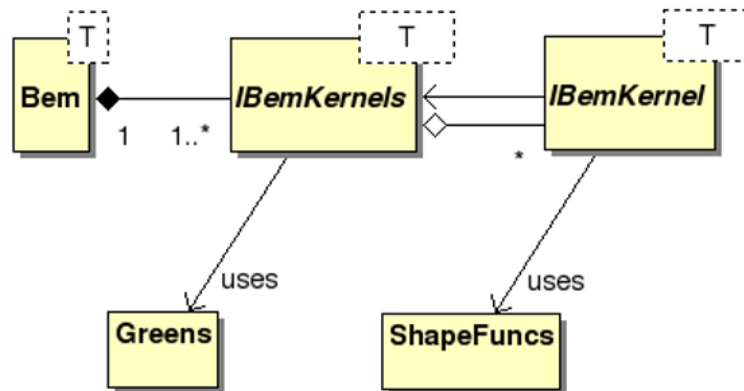


Fig. 1. Class diagram showing leading philosophy of bem package – selected classes only

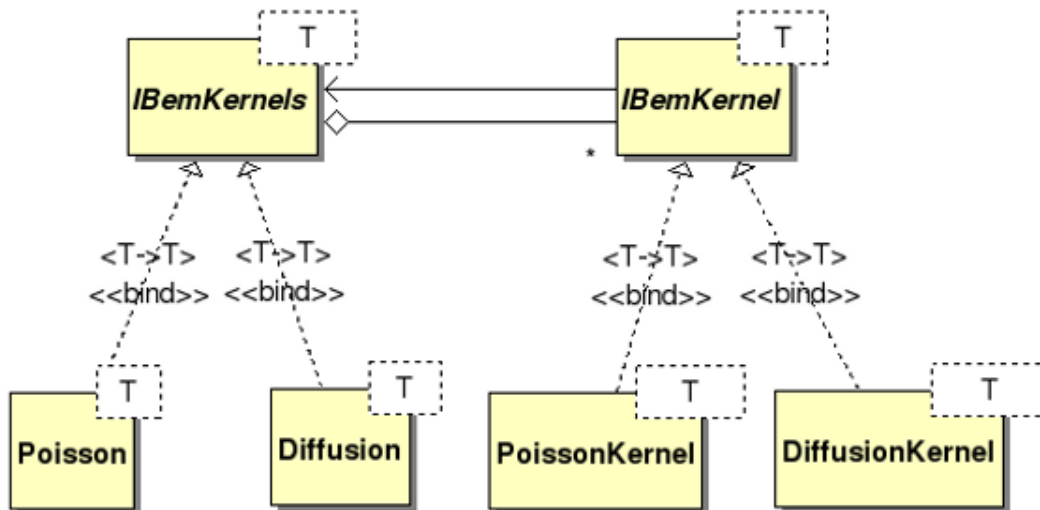


Fig. 2. Selected bem classes implementing two algorithms of BEM kernels

Bem template class (Fig. 1) includes universal BEM algorithms, not connected with particular PDEs like e.g. Laplace or diffusion. Bem can be instantiated for both real or complex types. Greens class, from bem package, contains common Green function definitions as static methods. Functions which allow using complex variables are defined as template ones. ShapeFuncs class is responsible for geometry only, therefore does not contain any template defined methods. IBemKernels and IBemKernel are interfaces, which are implemented by PDE specific classes. Figure 2 shows Diffusion and Poisson specific classes.

The next two figures show UML class diagrams of other BEMLAB packages. Figure 3 shows base package, which includes data containers and algorithms for data input/output. They are template defined. The integration package (Fig. 4) contains integration numerical algorithms which are used in kernels calculations from equation (3). Classes from auxiliary package (Fig. 4) are not template defined but particular methods, in order to persist the rules:

- no over-coding and
- reuse code.

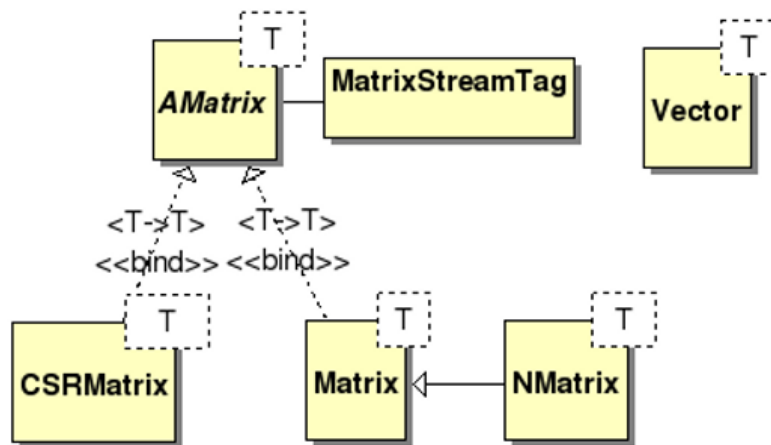


Fig. 3. Structure of base package – selected classes only

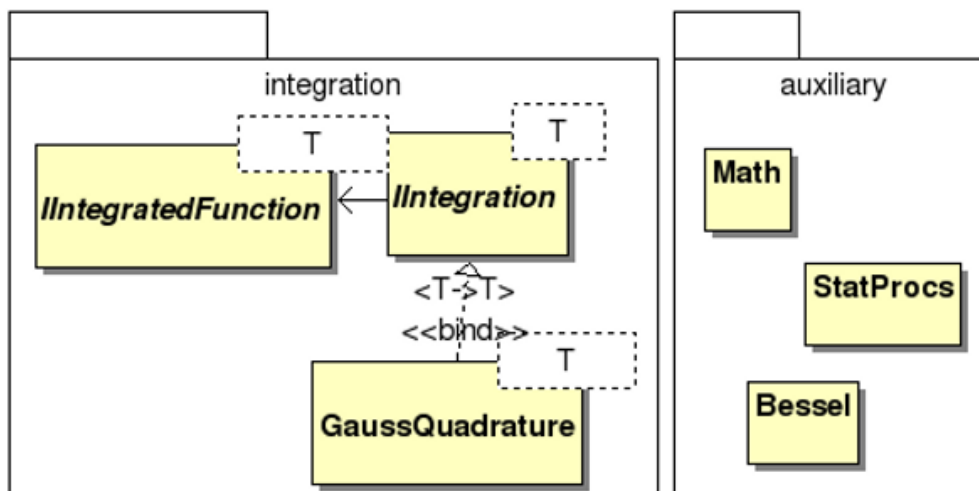


Fig. 4. integration and auxiliary packages in a class diagram – selected classes only

4. DIFFUSE TOMOGRAPHY EXAMPLE

Diffuse tomography [5, 6] uses near infra-red light which wavelength is included in the range 760-830 nm. Diffuse tomography is one of the areas where Diffusion equation in frequency domain is used. The calculations can be simplified if complex numbers domain is used. An example has been calculated using BEMLAB software. The modelled diffusive object is the sphere which radius is 10. The yellow marker on figures 5 and 6 marks the source of the collimated near infra-red light. The source is coherent and its frequency is 100MHz in the following example. Robin boundary conditions were applied on the boundary [5]. Figures 5 and 6 presents the results of calculations. Figure 5 presents the amplitude of the photon density and Figure 6 presents the phase. The material of the presented sphere object is highly absorbing and scattering, therefore the amplitude and the phase were presented using the logarithmic scale. The amplitude value decreases quickly while going away from the source point throughout the whole object. The phase value which represents the delay of light in the diffuse media increases while going away from the source point. The correctness of results can be compared with [4, 5]. Diffuse optical tomography (DOT) problems can also be solved by finite element method (FEM), Monte Carlo or others. However DOT model complexity for boundary element method is smaller. The described example illustrates the possibility of solving diffuse optical tomography forward problems. Without complex numbers implemented such problems could not have been solved.

5. CONCLUSIONS

BEMLAB aim is to fill the gap in the BEM software and become the universal library with multiple applications. Complex number calculations are one of the functionalities which BEMLAB provides. The project and the architecture were created to support common objective languages like C++ and Java. Reference implementation in C++ is ready to use. BEMLAB supports input/output format compatible with Matlab [10], Octave [11] and Scilab [12], which makes it easy to continue further calculations in these popular numerical packages. BEMLAB is available at the projects web site: <http://bemlab.org/>.

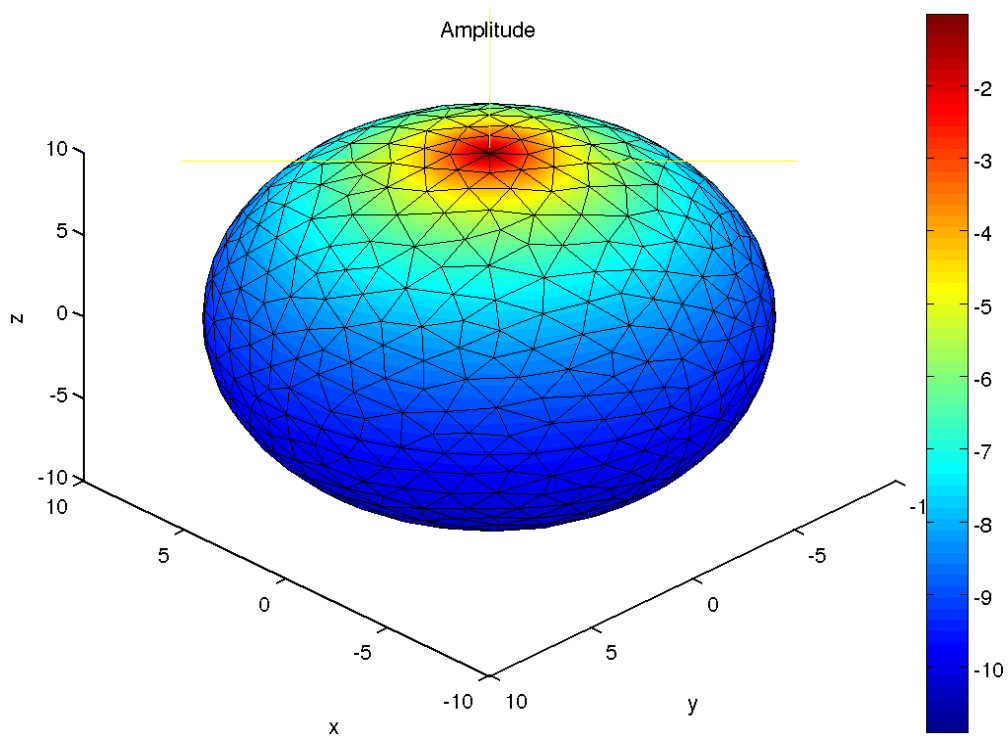


Fig. 5. Amplitude of photon density on the surface of the examined object presented in logarithmic scale

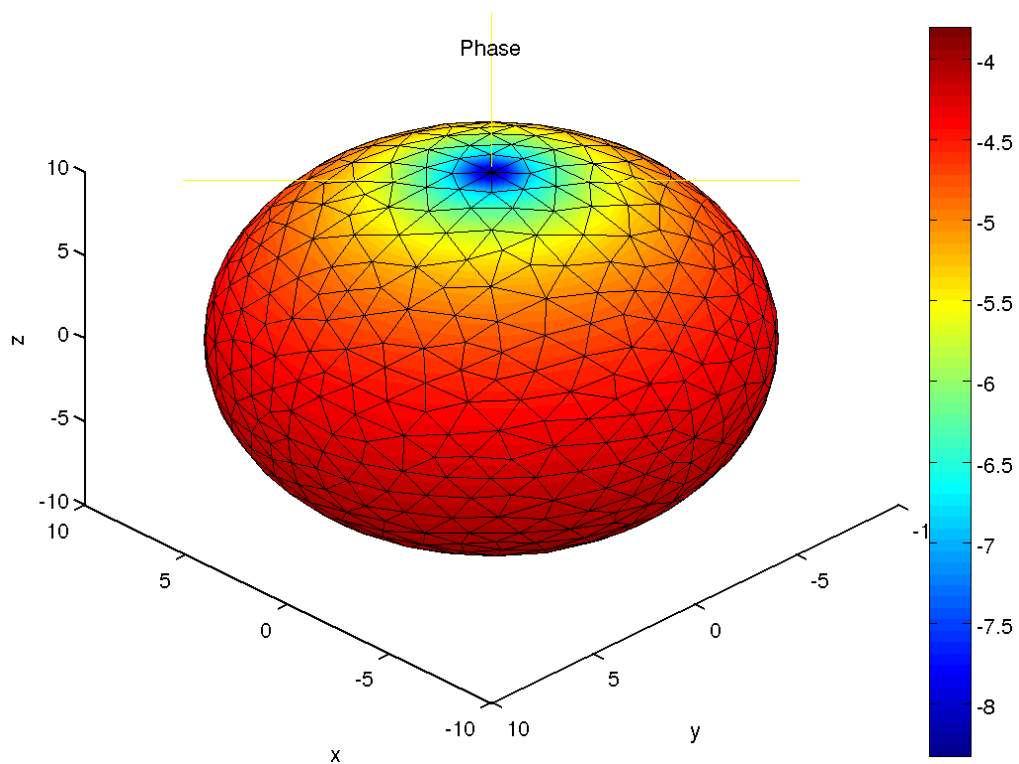


Fig. 6. Phase of photon density on the surface of the examined object presented in logarithmic scale

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BIBLIOTEKA OTWARTA BEMLAB
I PROBLEMY WYRAŻONE
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STRESZCZENIE *BEMLAB [7, 8] to biblioteka o otwartym kodzie źródłowym implementująca metodę elementów brzegowych (MEB). Umożliwia rozwiązywanie problemów potencjalnych opisanych równaniami różniczkowymi cząstkowymi, które muszą być przedstawione w postaci równania całkowo-brzegowego. Aby uprościć rozwiązywanie niektórych problemów zależnych od czasu, np. harmonicznym, równania różniczkowe cząstkowe można przedstawić używając liczb zespolonych. Równania całkowo-brzegowe muszą być również odpowiednio dostosowane. Z tego powodu wymagane było wprowadzenie obliczeń na liczbach zespolonych do omawianej biblioteki BEMLAB. Artykuł zawiera matematyczne podstawy oraz wprowadza w zagadnienia architektury odnoszące się do problemów opisanych w dziedzinie liczb zespolonych. Ponadto zawarto również przykład użycia biblioteki.*

