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Corona current concept in lightning return-stroke models of engineering type

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Abstract: A role of radial corona current in a lightning discharge is discussed in the paper. It is shown that the corona current concept previously introduced by Cooray for lightning return stroke models of distributed-current-source (DCS) type, and later, by Maslowski and Rakov for lumped-current-source (LCS) type models enables to show duality between these two types of models. Further, it is demonstrated that the corona current is useful during consideration of dynamics of the lightning-channel corona sheath. As an example of application of presented approach a relaxation model of charge motion in the corona sheath is analysed together with plots which show the rate of expansion and shrinkage of the lightning corona sheath on both microsecond and millisecond time scales.

Key words: lightning, corona sheath, return stroke model, continuity equation

1. Introduction

In lightning protection, the most important event in a lightning flash is the return stroke stage because it causes most of the destruction and disturbance in transmission lines and in sensitive electronic and computer systems. Therefore, it is very important to examine all aspects of return-stroke models which represent physical phenomenon of lightning discharges. For example, three specific models are necessary to evaluate lightning-induced voltages on transmission lines and cables [1]. At the beginning it should consider a model for the current propagation along the channel and channel-base current [2]. The next problem is calculation of the lightning return-stroke electromagnetic field propagated over a real soil with finite conductivity. The final stage is coupling this field with different structures as transmission lines and cables [3].

In principle, lightning return stroke models can be separated into four classes: (1) the gas dynamic models, also known as the "physical" models or the electro-thermodynamic models, (2) the distributed-circuit models (3) the electromagnetic models based on antenna theory, and

(4) so called the "engineering" models. One can find a detailed analysis of different return-stroke models in [2].

In the "engineering" return stroke models, a spatial and temporal distribution of the current and charge density along the channel are specified based on measured channel-base current, the speed of the upward-propagating front, and the channel luminosity profile. It enables to achieve an agreement between the model-predicted electromagnetic fields and those observed during natural and rocket-and-wire triggered lightning. Further, this class of models is divided into two main types. The first type represents so called the Lumped Current Source (LCS) type models and the second contains the Distributed Current Sources (DCS) type models. In the LCS type models a current pulse is originating at ground level by the lumped current source and propagating from ground to cloud along the transmission line created by the leader. In DCS type models the leader channel is treated as a charged transmission line, and the longitudinal return stroke current is formed when the corona sheath progressively collapses into the highly conducting return stroke channel. It means that the return stroke current in DCS type models is caused by potential wave that travels along the channel from ground to cloud. Both type of models are very effective in engineering practice during calculations of induced effects. In this paper, a new insights into the "engineering" return stroke models are presented with special attention of corona current concept adopted previously in DCS type models by Cooray [4], and recently in LCS type models by Maslowski and Rakov [5, 6].

2. Role of radial corona current in a lightning process

Based on experimental data obtained through measurements of natural lightning discharges, mainly at towers equipped with current measuring devices, or rocket triggering lightning result that the deposited charge on a thin lightning channel core (away from the leader tip) create a radial electric field which exceeds the breakdown value and pushes the charge away from the core. As a result, the leader channel consists of a thin core surrounded by a radially formed corona sheath [7]. The corona sheath expands outward from the channel core until the radial electric field is less than the breakdown value, assumed to be about 2 MV/m by Baum and Baker [8] and 1 MV/m by Kodali et al. [9]. It is generally thought (e.g., [8] and [2]) that the bulk of the leader charge is stored in the corona sheath whose radius is of the order of meters, while the highly conductive channel core (probably less than 0.5 cm in radius) carries essentially all the longitudinal current. The return-stroke current wave traverses the leader-channel core and serves to bring it to ground potential. As a result, the leader charge stored in the corona sheath collapses into the channel core and is transferred to ground. More information on the role of corona envelopes in lightning processes can be found in [10].

Wagner and Hileman [11] were the first to suggest that the return stroke current is generated by the neutralization of the corona sheath in range of microseconds. Rao and Bhattacharaya [12] suggested another scenario. They assumed that the charge deposited inside the corona sheath is neutralized in range of milliseconds, and the long continuing current is generated by this process. Lin et al. [13] assumed that only part of the return stroke current is generated by the neutralization of the corona sheath. Heckman and Williams [10] concluded based on the fundamental electrostatics and laboratory results on coronas that the longitudinal current due to radial charge motion is, in every phase of the lightning discharges, considerably smaller than observed lightning currents. On the other hand assuming longitudinal extension of the channel one can calculate currents which are the same order of magnitude as observed currents. Finally, they stated that longitudinal currents during all stages of a lightning flash are governed by longitudinal extension of the channel.

Arima et al. [14] and Cebrera and Cooray [15] performed laboratory experiments similar to what happens during leader to return stroke transition during natural lightning discharges. They investigated the mechanism of the neutralization of space charge clouds created in the laboratory when the potential of the high-voltage electrode was suddenly brought to ground potential. The experiments showed that the neutralization of the space charge cloud takes place through the action of streamer discharges moving out from the high-voltage electrode with speeds of the order of 10^5 m/s. Later, Cooray [16-18] suggested that the return stroke current is generated by the neutralization of the corona sheath by positive streamers from the core of the lightning return stroke. He assumed that these streamers propagate into the corona sheath with speeds of the order of 10^5 m/s.

Expansion of the luminous region of the lightning return stroke channel has been investigated by Takagi et al. [19] using High-Speed Line-Scanning Camera with frame rates exceeding 7800 scans per second. It means that time interval between each frame was 128 μs. The average expansion velocity of the natural lightning corona sheath was about 10^5 m/s, that is, it was the same order as measured by Cebrera and Cooray during laboratory experiments. Therefore, one can assume the discharges causing the luminous expansion in negative lightning are positive streamer discharges, and this process can be responsible for draining the charge which is deposited in the corona sheath by the preceding leader.

Maslowski and Rakov [5], based on electric field measurements [20] and their consideration of the lightning corona sheath dynamics, inferred the existence of two zones around the lightning channel core during the return stroke stage. The inner zone (Zone 1) has net positive charge, and the outer zone (Zone 2) contains negative charge, with the net charge inside the entire corona sheath being equal to zero after the return stroke stage. Similar electrical structure of the return-stroke corona sheath has been considered by Gorin [21], who studied corona processes during the return stroke stage of long laboratory sparks. Recently, two improved models for prediction of charge motion in the corona sheath are proposed and the radial expansion of Zone 1 and its shrinking was examined [22, 23]. Both improved models include the motion of negative leader charge from the outer to the inner zone, towards the core and can be viewed as generalizations of the model proposed by Maslowski and Rakov [5]. They enable to calculate the corona sheath radius and velocity of the expansion and shrinkage of the corona sheath which was of the order of 10^6 m/s at the beginning of the return stroke stage, and it was of the order of 10^4 m/s at the end of the return stroke process. Note that the average speed of the corona sheath expansion calculated theoretically by Maslowski and Rakov was of the order of 10^5 m/s.

3. Lightning return-stroke models of engineering type

A lightning return-stroke model with specified longitudinal current distribution (an engineering return-stroke model) is usually defined as an equation relating the longitudinal channel current $i(z', t)$ at any height z' and any time t to the current $i(0, t)$ at the channel origin, $z' = 0$ (e.g. [7]), that is

$$i(z',t) = u(t - z'/v_f)P(z')\,i(0,t - z'/v)\,, \tag{1}$$

where: u is the Heaviside function equal to unity for $t \geq z'/v_f$ and zero otherwise, $P(z')$ is the height-dependent current attenuation factor introduced by Rakov and Dulzon [24], v_f is the upward-propagating front speed, and v is the current-wave propagation speed. An equivalent formulation in terms of the line charge density $\rho(z', t)$ expressed as the sum of so called transferred and deposited charge density components, was proposed by Thottappillil et al. [25].

Table 1. Lightning return-stroke current change along the channel defined for LCS and DCS type models ($t \geq z'/v_f$)

LCS type models		**DCS type models**	
Model TL (Transmission Line) [26]	$i(z', t) = i(0, t - z'/v)$	**Model BG** (Bruce-Golde) [29]	$i(z', t) = i(0, t)$
Model MTLL (Modified Transmission Line with Linear Attenuation) [27]	$i(z',t) =$ $= \left(1 - \frac{z'}{H}\right) i(0, t - z'/v)$	**Model TCS** (Traveling Current Source) [30]	$i(z', t) = i(0, t + z'/c)$
Model MTLE (Modified Transmission Line with Exponential Attenuation) [28]	$i(z',t) =$ $= e^{-\frac{z'}{\lambda}}\, i(0, t - z'/v)$	**Model DU** (Diendorfer-Uman) [31]	$i(z',t) = i(0, t + z'/c) +$ $+ i(0, z'/v^*) e^{-(t - z'/v_f)/\tau_D}$
H – total length of the return-stroke channel, v – velocity of current-wave propagation ($v = v_f$), λ – const		v_f – velocity of the upward-propagating front, c – velocity of current-wave propagation (speed of light), $v^* = v_f / (1 + v_f/c)$, τ_D = const	

Arbitrary defined lightning return-stroke current change is depicted in Table 1 for different LCS and DCS type models respectively. Note that, for LCS type models, current-wave is propagated upward with speed $v = v_f$, while, for DCS type models, current-wave is propagated downward with speed c, and at the same time the return-stroke front is propagated upward with speed v_f (see also Fig. 1).

4. Continuity equations for DCS and LCS type models in terms of the corona current

Maslowski and Rakov [5] first introduced corona current concept in lightning return-stroke models of LCS type. The equivalent corona current generated by lateral positive streamers occurring during the return-stroke stage can be represented by sinks which are progressively activated by the upward moving return-stroke front. It means that longitudinal current is governed for LCS type models by the lumped current source situated at the lightning channel base.

On the other hand, any engineering model of DCS type can be formulated in terms of sources directed into the channel core and distributed along the channel which are progresssively activated by the upward moving (with speed of v_f) return stroke front. Distributed sources represent corona currents which are generated by the neutralization of the corona sheath. The cumulative effects of these corona currents generate the longitudinal return stroke current. Note that direction of corona current is not consistent for DCS type models with actual corona current. Note also that the longitudinal current wave propagates downward with speed of light c

Schematic representations of engineering return-stroke models that employ distributed current sources along the channel and a lumped current source at the lightning channel base are shown in Figure 1.

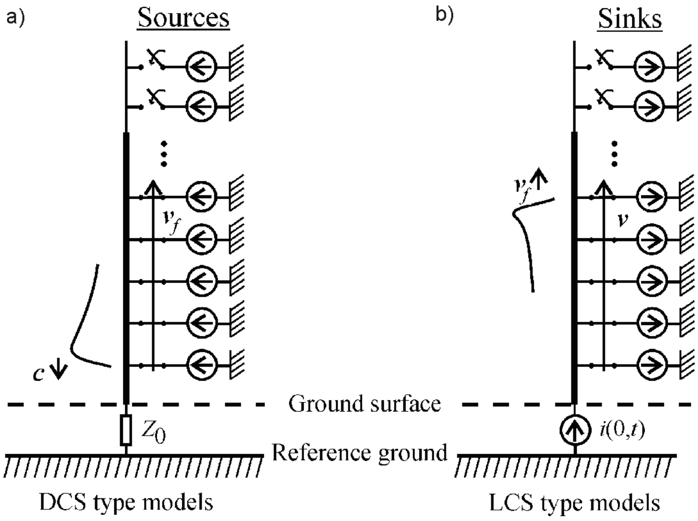


Fig. 1. Schematic representation of the “engineering” return-stroke models that employ a) distributed current sources along the channel (DCS type models), and b) a lumped current source at the lightning channel base (LCS type models); Z_0 is the characteristic impedance of the lightning channel introduced for DCS type models in order to avoid reflections from ground surface. Note that for LCS type models $v_f = v$

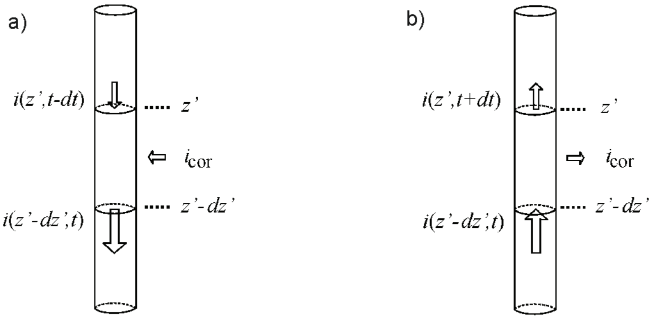
Cooray [4], considering a differential channel segment (see Fig. 2a), derived the following relation for DCS type models,

$$i'_{cor}(z',t) - \frac{1}{c}\frac{\partial i(z',t)}{\partial t} = -\frac{\partial i(z',t)}{\partial z'}, \tag{2}$$

where: $i(z', t)$ is the longitudinal channel current at height z', and time t (see Tab. 1), and $i'_{\mathrm{cor}}(z', t)$ is the radial corona current per unit length at height z' and time t injected into the channel.

Maslowski and Rakov [6], using the channel segment shown in Figure 2b, derived a similar equation for LCS type models, as follows:

$$i'_{\mathrm{cor}}(z',t) + \frac{1}{v}\frac{\partial i(z',t)}{\partial t} = -\frac{\partial i(z',t)}{\partial z'}. \tag{3}$$



DCS type models LCS type models

Fig. 2. Differential channel segments used in deriving continuity equations for a) DCS type models and b) LCS type models, i and i_{cor} are the longitudinal and corona currents, respectively

Expressions (2) and (3) represent continuity equations for DCS and LCS type models in terms of longitudinal and corona currents, respectively. In fact, the total charge per unit length for DCS type models can be expressed, for $t \geq z'/v_f$, as [6]:

$$\rho(z',t) = -\frac{i(z',t)}{c} + \int_{z'/v_f}^{t} i'_{\mathrm{cor}}(z',t)dt. \tag{4}$$

Taking the time derivative on both sides of (4), one can get,

$$\frac{\partial \rho(z',t)}{\partial t} = i'_{\mathrm{cor}}(z',t) - \frac{1}{c}\frac{\partial i(z',t)}{\partial t}. \tag{5}$$

Substituting (5) into (2) one can obtain the general form of the continuity equation,

$$\frac{\partial \rho(z',t)}{\partial t} = -\frac{\partial i(z',t)}{\partial z'}. \tag{6}$$

Similarly, the total charge per unit length for LCS type models can be expressed, for $t \geq z'/v$, as

$$\rho(z',t) = \frac{i(z',t)}{v} - \frac{dP(z')}{dz'} \int\limits_{z'/v}^{t} i(0, t - z'/v)dt, \tag{7}$$

where the first term of (7) is equal to the transferred charge density $\rho_{\mathrm{tran}}(z', t)$ and the second term is equal to the deposited charge density $\rho_{\mathrm{dep}}(z', t)$. These two kinds of charges together with equation (7) have been introduced by Thottappillil et al. [25]. The time derivative of (7) is

$$\frac{\partial\rho(z',t)}{\partial t} = \frac{1}{v}\frac{\partial i(z',t)}{\partial t} - \frac{dP(z')}{dz'}\, i\,(0, t - z'/v), \tag{8}$$

where the second term on the right hand side represent the unipolar corona current per unit channel length i'_{cor} for LCS type models [5]

$$i'_{\mathrm{cor}} = -\frac{dP(z')}{dz'}\, i\,(0, t - z'/v). \tag{9}$$

One can see from (9) that the corona current $i'_{\mathrm{cor}}(z', t)$ per unit channel length for LCS type models, is equal to the time derivative of the deposited charge density $\rho_{\mathrm{dep}}(z', t)$. Hence, as the current injected at the ground surface traverses the lightning channel, the radial corona current in general will cause a reduction of this current and serve to neutralize the usually negative charge deposited along the channel by preceding leader. Finally, after substitution (9) into (8) and then (8) into (3), one can get for LCS type models, as expected, the same general form of the continuity equation (6) as for DCS type models.

Cooray [4] showed, using the continuity equation (2), that any engineering model implying a lumped current source at the lightning channel base (any LCS type model) can be formulated in terms of sources distributed along the channel and progressively activated by the upward moving return stroke front. This has been previously demonstrated for one model (modified transmission line model with exponential current decay with height) by Rachidi and Nucci [32]. Recently, the approach suggested by Cooray [4] was used by Rachidi et al. [33] to generalize five engineering models in order to take into account a tall strike object.

Maslowski and Rakov [6] showed that any engineering return stroke model can be expressed, using Cooray's continuity equation (2) and their new continuity equation (3), in terms of either lumped or distributed current sources, with the resultant longitudinal current distribution along the channel being the same. This property can be viewed as duality of engineering models. In fact, it was shown in [6] that the conversion alters the actual corona current of the model. For LCS type models the actual corona current is unipolar and directed radially out of the channel core (see Fig. 2b), while for DCS type models it is unipolar and directed into the channel core (see Fig. 2a). For LCS type models converted to DCS type models and for the Diendorfer-Uman model converted to the equivalent LCS type model the corona current is the sum of the negated actual corona current (if any) and a fictitious corona current, the latter being bipolar. It means that the fictitious corona current component is flowing in both directions, that is, outward from the channel core, and then, it is flowing into the core. For the TL model (no longitudinal current attenuation with height) expressed in terms of distributed sources, there is only fictitious bipolar corona current component. Conversion of the traveling current source and Bruce-Golde models to equivalent LCS type models involves replacement of the actual, unipolar corona current with a fictitious one, the latter current being bipolar near the channel base and unipolar at higher altitudes [6].

5. Dynamics of the corona sheath

One can determine dynamics of the corona sheath using corona current concept and defined in [25] the charge transferred and deposited along the lightning channel. It is generally known that at the beginning of the return-stroke stage the corona sheath expands outward from the channel core inside the leader corona sheath, and then, it shrinks up to a nearly zero radius (see Figure 3). In order to estimate the rate of expansion and shrinking of the corona sheath it is considered a closed cylindrical surface (Gaussian cylinder) *dS* that is coaxial with and surrounding a segment of channel core whose length is *dz*. According to Gauss' law,

$$\varepsilon_0 \oint_{dS} \mathbf{E}\cdot d\mathbf{S} = dQ, \tag{10}$$

where **E** is the electric field on closed surface *d***S** and *dQ* is the total charge inside this surface.

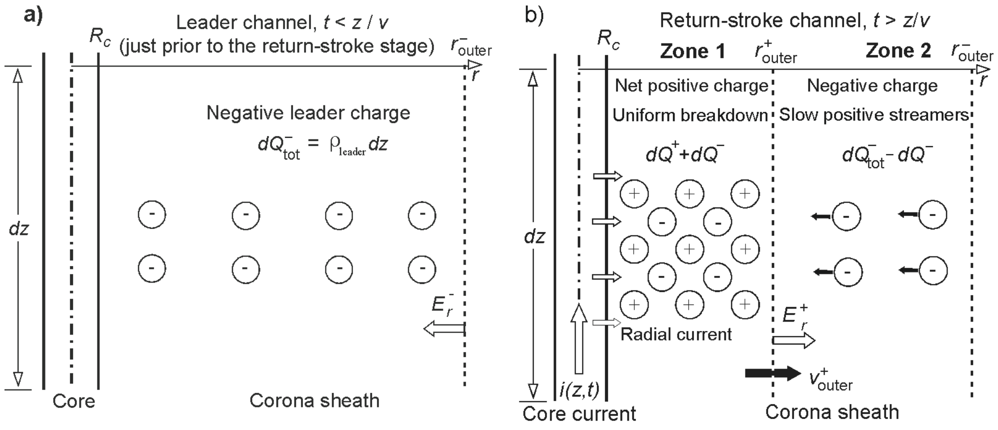


Fig. 3. a) Leader channel composed of a narrow core of radius R_c and negatively charged corona sheath and b) two zones of the corona sheath during the return-stroke stage. Negative charges drift from Zone 2 to Zone 1 so that charge in Zone 2 decays exponentially, while $r^-_{\text{outer}} = \text{const}$

Maslowski and Rakov [5] showed that Equation (10) can be rewritten in the equivalent form, as follows

$$2\pi r^+_{\text{out}}\varepsilon_0 E^+_r dz' = dQ^+ + dQ^-, \tag{11}$$

where r^+_{outer} is the outer radius of the inner zone of corona sheath (Zone 1) containing positive charge deposited by the radial conduction current flowing during the return stroke stage, E^+_r is the constant radial electric field on the lateral surface of radius r^+_{outer}. The total charge *dQ* enclosed by *dS* consists of the negative charge dQ^- deposited by the preceding leader and the positive charge dQ^+ associated with the return stroke stage. In order to estimate r^+_{outer}, the radial electric field E_r^+ must be chosen.

It is assumed that the corona sheath zone with uniform ionization extends outward from the channel core until the field becomes less than the positive breakdown electric field. Note that the radial electric field cannot be established instantaneously, but it will be used a constant value of breakdown field for simplicity. Note also that this constant value is assumed to be at the border of the corona sheath which initially expands during the first stage of the return stroke, and then, it shrinks. The radial electric field changes measured at a fixed point of space inside the corona sheath during rocket-and-wire triggered lightning experiments has been discussed in papers [20, 23, 34].

In this paper, it is assumed that the positive breakdown electric field is equal to $E_r^+ = 1.0$ MV/m [9]. The return-stroke charge dQ^+ can be specified using return-stroke models and represented as the sum of two components [25] as follows

$$dQ^+ = \rho_{\text{tran}} dz' + \rho_{\text{leak}} dz', \tag{12}$$

The first term of (12) is the charge transferred upward through the channel segment, and the second term represents the deposited charge that is spent to neutralize the leader charge previously deposited in the corona sheath of this segment.

Assuming a uniform radial distribution of the negative leader charge just before the return-stroke stage, one can show that dQ^- which is a portion of the total negative charge stored in the corona sheath located within the radial extent, r^+_{outer}, of positive charge dQ^+ can be expressed for $t \geq z'/v$, as

$$dQ^- = dQ_1^- + dQ_2^- = k\rho_L dz' + (\rho_L - k\rho_L)(1 - e^{-(t-z'/v)/\tau_{\text{CN}}})dz', \tag{13}$$

where $k = (r^+_{\text{outer}} / r^-_{\text{outer}})^2$, $dQ_1^- = k\rho_L dz'$ is the negative leader charge deposited within Zone 1 just before the return-stroke stage dQ_2^- is the negative charge that penetrates Zone 1 from Zone 2, and τ_{CN} is the decay time constant describing reduction of negative charge deposited within Zone 2, and, hence, the rate of motion of the negative charge from Zone 2 to Zone 1. In fact, Williams and Heckman [10] suggested that some slow breakdown processes (positive streamers) can develop outwards from the uniform breakdown region at fields in excess of as low as 0.2 MV/m.

According to Appendix B in [5], dQ_1^- can be expressed as

$$dQ_1^- = (2\pi\varepsilon_0 r^+_{\text{outer}} E_r^-)^2 dz' / \rho_L, \tag{14}$$

where E_r^- is the negative breakdown electric field which is assumed to be greater (in absolute value) than E_r^+ and equal to 1.5 MV/m [9], and ρ_L is the negative charge density per unit channel length prior to the return stroke stage. Note that r^+_{outer} is necessarily smaller than r^-_{outer}, the radial extent of the negative leader corona sheath. From (11), (12), (13), and (14) result

$$2\pi r^+_{\text{outer}} \varepsilon_0 E_r^+ = \rho_{\text{tran}} + \rho_{\text{dep}} + \rho_L (1 - e^{-(t-z'/v)/\tau_{\text{CN}}}) + \frac{(2\pi r^+_{\text{outer}} \varepsilon_0 E_r^-)^2}{\rho_L} e^{-(t-z'/v)/\tau_{\text{CN}}}. \tag{15}$$

The radius of Zone 1, r^{+}_{outer}, is a solution of quadratic equation (15) which reduces for enough long τ_{CN} to the model initially formulated by Maslowski and Rakov (Equation (20) in [5]), in which negative charge from Zone 2 does not penetrate Zone 1 during the return stroke stage.

Note that the corona-sheath model described by (15) enables one to predict both the radial expansion of Zone 1 and its shrinkage up to a nearly zero radius, once the decay time constant τ_{CN} is specified. It was estimated τ_{CN} based on the horizontal (radial) electric field component measured in the immediate vicinity of lightning channel during the 2000 triggered-lightning experiment at Camp Blanding, Florida [20, 23]. Electric field waveforms at horizontal distances from triggered lightning channel attachment point ranging from 0.1 to 1.6 m (most likely inside the corona sheath) have been measured with Pockels sensors. The horizontal electric field change measured during the leader stage was often overcompensated by the opposite-polarity electric field change during the return-stroke stage. Interestingly, the maximum horizontal electric field values during some return strokes were high enough (from 413 kV/m up to more than 520 kV/m) for starting positive breakdown. Time of field relaxation to practically zero level during the overcompensation stage is very similar for all measured waveshapes and is estimated to be approximately 2 ms [23, 34]. Hence, one can estimate the decay time constant to be about one-third of this value.

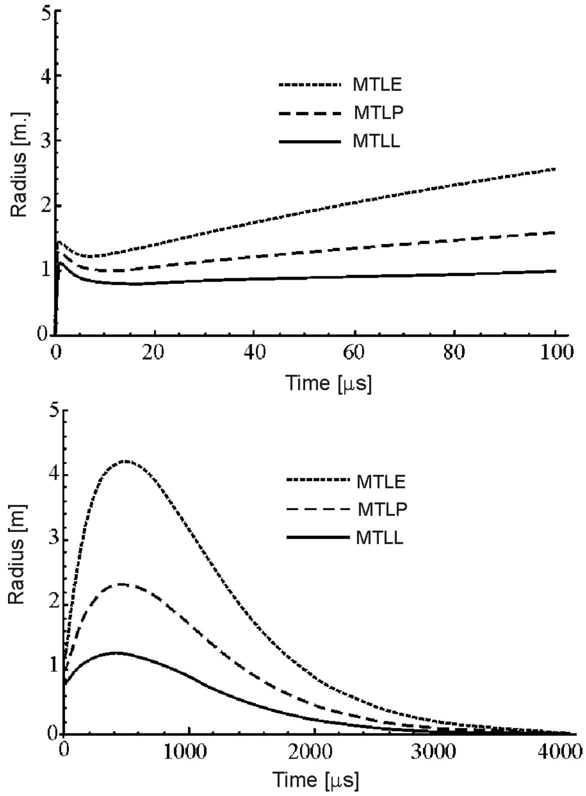


Fig. 4. Comparison of radius r^{+}_{outer} of Zone 1 versus time at a height of 10 m for three return stroke models, MTLL, MTLP, and MTLE ($H = 7500$ m, $\lambda = 2000$ m, $v = 130$ m/μs) shown on two timescales, 100 μs (left), and 4 ms (right), as predicted by (15). It was assumed in (15) that $\tau_{\text{CN}} = 650$ μs

Evolution of Zone 1 as a function of time at height z = 10 m for the three LCS return stroke models, MTLL (Modified Transmission Line model with Linear current decay with height), MTLP (Modified Transmission Line model with Parabolic current decay with height), and MTLE (Modified Transmission Line model with Exponential current decay with height), are shown in Fig. 4.

Channel base current used here is the same as that adopted in [5] and is characterized by a current peak of 12 kA and maximum current rate of rise of about 40 kA/μs. As seen in Fig. 4, Zone 1 expands approximately up to about 500 μs in the case of model described by Eq. 15. After this time Zone 1 shrinks. Note that for the model with exponential charge decay in Zone 2, one can adjust the rate of corona sheath expansion and its shrinking by changing the decay time constant.

6. Conclusion

A role of radial corona current in a lightning discharge was discussed in the paper. It was shown that the corona current concept enables to show duality between these two types of models. Further, corona current is useful during consideration of dynamics of the lightning-channel corona sheath. As an example of application a relaxation model of charge motion in the corona sheath is presented together with plots which show the rate of expansion and shrinking of the lightning corona sheath. Dynamics of the lightning-channel corona sheath was implicitly specified by engineering type lightning return-stroke models. Presented corona sheath radius described by LCS type models are comparable on both microsecond and millisecond time scales. Obtained results demonstrated a new insights into mechanism of lightning return stroke process and can be used in the near future to adjust longitudinal current distribution along the lightning channel more precisely.

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