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Shock safety modeling method for low-voltage electric devices

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Abstract: The article describes a shock safety modeling method for low-voltage electric devices, based on using a Bayesian network. This method allows for taking into account all possible combinations of the reliability and unreliability states for the shock protection elements under concern. The developed method allows for investigating electric shock incidents, analysing and assessing shock risks, as well as for determining criteria of dimensioning shock protection means, also with respect to reliability of the particular shock protection elements. Dependencies for determining and analysing the probability of appearance of reliability states of protection as well as an electric shock risk are presented in the article.

Key words: safety management process, expert system, Bayesian network, total probability, protection reliability states, appearance of touch and electric shock states, risk analysis and assessment

1. Introduction

In the shock safety management process, the following tasks are performed, among others, [1-3]: investigating electric shock incidents, analysing and assessing shock risks, as well as determining criteria of dimensioning shock protection equipment. The constructed expert system, supporting performance of these tasks [4], should have a knowledge base equipped with an appropriate shock safety model.

A shock safety model for low-voltage electric devices should make it possible to determine probabilities and intensity of appearance of different unreliability and shock states [1, 5].

The probability of experiencing unreliability states depends on the following probabilities: the occurrence of touch voltage on the particular elements of an electric device, and on touching these elements by a human.

The probability of touch voltage occurring on the particular elements of an electric device is dependent on the probability of experiencing the state in which the particular elements of the device are damaged, as well as on the reliability structure of the applied shock protection mean.

The probability of a human touching these parts of an electric device, on which touch voltage might occur, depends on the frequency and duration of such a contact.

The probability of appearance of shock states depends on the probability of experiencing the state of a touch, and on the probability of pathophysiologic touch effects occurring in this state.

The probability of appearance of pathophysiologic touch effects taken into consideration in dimensioning shock protection of low-voltage electric devices (sensation of a current flowing, an inability to free oneself, burn, and ventricular fibrillation) is dependent on a number of circumstances and factors such as [6]: the values of the touch voltage and the touch circuit impedance, which affect the touch current values, the touch duration, and on the type of current – in the case of the direct current on its flow direction, and in the case of the alternating current – on the frequency, the body weight, the path the current flows along through the body, and other factors.

The analytic method of shock safety modeling for low-voltage electric devices, applied in the [2, 3, 7] works, is characterised by rigid, aggregated reliability structures of a limited number of combinations of damages and of a lack of damages in protection elements.

The method of modeling shock safety for low-voltage electric devices, described in the present article, based on using a Bayesian network, does not require constructing rigid reliability structures for shock protection means. This method allows for taking into account all possible combinations of the reliability and unreliability states for the shock protection elements under concern.

2. Concept of modeling method

The suggested shock safety modeling method consists in:

- mapping the total probability of appearance of combinations of the considered damages and a lack of damages of the particular elements of shock protection means in electric devices, with the use of a Bayesian network;
- computer identification of the occurring touch scenarios, i.e. of touch voltage appearing in the considered areas of contacting parts of electric devices for each of the considered combinations of damages and of a lack of damages in technical means of protection;
- determining the probabilities of appearance of touch effects on the basis of computer-calculated values of touch voltage and its duration as well as the pre-set other circumstances and factors affecting the shock appearance for each of the identified touch scenarios.

The structure of a Bayesian network mapping a generalized shock safety model is presented in Fig. 1. It contains non-conditional (D_b, D_p contacts and F_p, F_w, F_d protection damages) and conditional nodes (D contacts, F protection damages, Z_r, R, Z_b protection reliability states, and S touch effects), as well as the so-called value nodes ($\lambda_d, \lambda_f, \lambda_{zr}$, $\lambda_r, \lambda_{zb}, \lambda_s$ itensity of appearance of particular events or states, $\bar{t}_d, \bar{t}_f, \bar{t}_{zr}, \bar{t}_r, \bar{t}_{zb}$ – their average duration times, and $P(s \times k, t)$ shock safety).

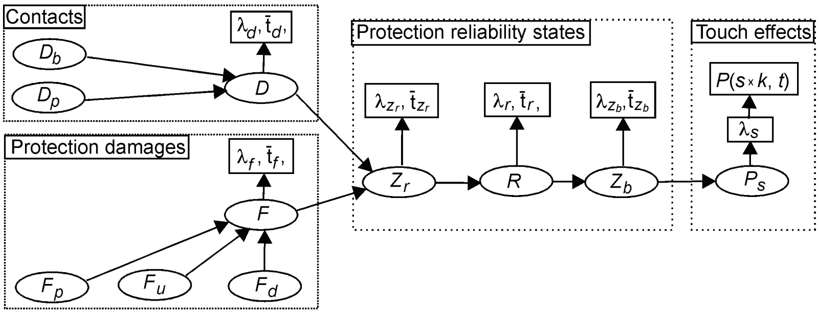

Fig. 1. Bayesian network structure mapping a generalized shock safety model.
The symbols are explained in the text

The D_b and D_p contacts, as well as the F_p, F_u and F_d protection damages, which constitute the non-conditional nodes of the above Bayesian network (Fig. 1), are two-state random variables determining the occurrence or non-occurrence of the following:

- $D_b(d_b, \bar{d}_b)$ direct and $D_p(d_p, \bar{d}_p)$ indirect contacts;
- $F_p(f_p, \bar{f}_p)$ damages of basic protection elements, $F_u(f_u, \bar{f}_u)$ damages of fault protection elements, and $F_d(f_d, \bar{f}_d)$ damages of additional protection elements.

In order to determine the probability of the D_b, D_p, F_p, F_u and F_d non-conditional nodes, dependencies applying to a stationary random process with exponential distributions of the times of experiencing the particular states can be applied.

For example, the non-conditional probability of appearance of the $P(d_b)$ direct contact and the $P(\bar{d}_b)$ lack of direct contact for the D_b node, is determined from the dependence

$$P(d_b) = \frac{\lambda_{d_b} \cdot \bar{t}_b}{1 + \lambda_{d_b} \cdot \bar{t}_b}, \quad P(\bar{d}_b) = \frac{1}{1 + \lambda_{d_b} \cdot \bar{t}_b}, \tag{1}$$

where: λ_{db} − intensity of appearance of direct contacts, $\bar{t}_b$ − average duration time of direct contact.

The non-conditional probability of the F_p, F_u and F_d nodes has to be determined taking into consideration the regular inspection of the equipment. In order to do this, approximate dependencies for the probability of occurrence of unreliability and reliability states in the periods between inspections [8], can be applied.

For example, the non-conditional probability for the F_p node, of the $P(f_p)$ appearance or the $P(\bar{f}_p)$ lack of appearance of damage in an element of the basic protection, is determined from the following dependence

$$P(f_p) = \frac{\lambda_{fp} \cdot \bar{t}_{fp}}{1 + \lambda_{fp} \cdot \bar{t}_{fp}}, \quad P(\bar{f}_p) = \frac{1}{1 + \lambda_{fp} \cdot \bar{t}_{fp}}, \tag{2}$$

where the average duration time of the damage is determined from the [6] formula

$$\bar{t}_{fp} = \frac{T_k}{1 - e^{-\lambda_{fp} T_k}} - \frac{1}{\lambda_{fp}}, \tag{3}$$

where: λ_{fp} − intensity of damages to an element of basic protection, $\bar{t}_{fp}$ − average duration time of the damage ('waiting' for inspection), T_k − time between inspections.

The conditional nodes of the created Bayesian network (Fig. 1) are:

- $D(d, \bar{d})$ cases of contacts with a device part;
- $F(f, \bar{f})$ damages to shock protection elements;
- reliability states of: $Z_r(\bar{z}_r, z_r)$ touch danger, $R(\bar{r}, r)$ touch, and $Z_b(\bar{z}_b, z_b)$ safety unreliability;
- $S(\bar{s}, s)$ electric shock states.

The conditional probability of the D node for the particular combinations of the D_b and D_p contacts are given in Tab. 1, whereas that of the F node for the particular combinations of the F_p, F_u and F_d damage combinations are listed in Tab. 2.

Tabele 1. D node conditional probability for particular combinations of D_b and D_p contacts

No.	Combination of contacts		$P(D/D_b, D_p)$	
	D_b	D_p	$P(d/D_b, D_p)$	$P(\bar{d} / D_b, D_p)$
1	$\bar{d}_b$	$\bar{d}_p$	0	1
2	$\bar{d}_b$	d_p	1	0
3	d_b	$\bar{d}_p$	1	0
4	d_b	d_p	1	0

The F node conditional probabilities (Tab. 2) take into account two cases: a lack of a quick switching off of a given combination of damages to the protection elements, and a quick switching off of a damage, performed by one of the two protections which is still functioning (the fault protection damage, or the additional one). If there is no quick switching off, then $P(f / F_p, F_u, F_d) = 1$, and thus. $P(\bar{f} / F_p, F_u, F_d) = 0$ In the case where a quick switching off of a damage to the protection elements takes place, the role of the F node conditional probabilities is to replace the $[P(\bar{f}_p), P(f_p)]$ and $[P(\bar{f}_u), P(f_u)]$ non-conditional probabilities determined from the (2) formula for the damage duration times with 'waiting' for inspection, with the $[P(\bar{f}_{pwyl}), P(f_{pwyl})]$ and $[P(\bar{f}_{uwyl}), P(f_{uwyl})]$ probabilities determined for the times of switch-offs by overload devices. It was assumed that the average switch-off time amounted to $\bar{t}_{wyl} = 0{,}63 \cdot t_{wyl}$, where t_{wyl} is the switch-off time required by the regulations in force.

Table 2. F node conditional probability for particular combinations of F_p, F_u and F_d damage combinations

No.	Damage combination			$P(F / F_p, F_u, F_d)$	
	F_p	F_u	F_d	$P(f / F_p, F_u, F_d)$	$P(\bar{f} / F_p, F_u, F_d)$
1	$\bar{f}_p$	$\bar{f}_u$	$\bar{f}_d$	0	1
2	$\bar{f}_p$	f_u	$\bar{f}_d$	$P(f / \bar{f}_p, f_u, \bar{f}_d) = 1$ or $[P(\bar{f}_{pwył}) / P(\bar{f}_p)] \cdot$ $\cdot [P(f_{uwył}) / P(f_u)]$ for $\bar{t}_{uwył} = \bar{t}_{dwył}$ 2.	$P(\bar{f} / \bar{f}_p, f_u, \bar{f}_d) = 0$ or $1 - \{[P(\bar{f}_{pwył}) / P(\bar{f}_p)] \cdot$ $\cdot [P(f_{uwył}) / P(f_u)]\}$ for $\bar{t}_{uwył} = \bar{t}_{dwył}$ 3.
3	f_p	$\bar{f}_u$	$\bar{f}_d$	$P(f / f_p, \bar{f}_u, \bar{f}_d) = 1$ or $P(f_{pwył}) / P(f_p)$ for $\bar{t}_{pwył} = \min\{\bar{t}_{uwył}, \bar{t}_{dwył}\}$	$P(\bar{f} / f_p, \bar{f}_u, \bar{f}_d) = 0$ or $1 - [P(f_{pwył}) / P(f_p)$ for $\bar{t}_{pwył} = \min\{\bar{t}_{uwył}, \bar{t}_{dwył}\}$
4	f_p	f_u	$\bar{f}_d$	$P(f / f_p, f_u, \bar{f}_d) = 1$ or $[P(f_{pwył}) / P(f_p)] \cdot$ $\cdot [P(f_{uwył}) / P(f_u)]$ for $\bar{t}_{pwył} = \bar{t}_{uwył} = \bar{t}_{dwył}$ 4.	$P(\bar{f} / f_p, f_u, \bar{f}_d) = 0$ or $1 - \{[P(f_{pwył}) / P(f_p)] \cdot$ $\cdot [P(f_{uwył}) / P(f_u)]\}$ for $\bar{t}_{pwył} = \bar{t}_{uwył} = \bar{t}_{dwył}$
5	$\bar{f}_p$	$\bar{f}_u$	f_d	$P(f / \bar{f}_p, \bar{f}_u, f_d) = 1$ or $[P(\bar{f}_{pwył}) / P(\bar{f}_p)] \cdot$ $\cdot [P(\bar{f}_{uwył}) / P(\bar{f}_u)]$	$P(\bar{f} / \bar{f}_p, \bar{f}_u, f_d) = 0$ or $1 - \{[P(\bar{f}_{pwył}) / P(\bar{f}_p)] \cdot$ $\cdot [P(\bar{f}_{uwył}) / P(\bar{f}_u)]\}$
6	$\bar{f}_p$	f_u	f_d	$P(f / \bar{f}_p, f_u, f_d) = 1$ or $[P(\bar{f}_{pwył}) / P(\bar{f}_p)]$	$P(\bar{f} / \bar{f}_p, f_u, f_d) = 0$ or $1 - \{[P(\bar{f}_{pwył}) / P(\bar{f}_p)]\}$
7	f_p	$\bar{f}_u$	f_d	$P(f / f_p, \bar{f}_u, f_d) = 1$ or $[P(f_{pwył}) / P(f_p)] \cdot$ $\cdot [P(\bar{f}_{uwył}) / P(\bar{f}_u)]$ for $\bar{t}_{pwył} = \bar{t}_{uwył}$	$P(\bar{f} / f_p, \bar{f}_u, f_d) = 0$ or $1 - \{[P(f_{pwył}) / P(f_p)] \cdot$ $\cdot [P(\bar{f}_{uwył}) / P(\bar{f}_u)]\}$ for $\bar{t}_{pwył} = \bar{t}_{uwył}$
8	f_p	f_u	f_d	1	0

3. Determining the probability of appearance of the protection reliability states

The Z_r, R and Z_b conditional nodes of the shock safety model (Fig. 1), concerning the shock protection reliability states, were classified as follows:

1. The $\bar{z}_r$ states of a lack of a touch danger take place if no elements of the shock protection are contacted or damaged, or if only one of these cases occurs.
2. The z_r states of a touch danger occur if there is at least one contact with a device, and at least one element of the shock protection is damaged at the same time.
3. The $\bar{r}$ states of a lack of a touch take place in such states of a touch danger, in which there was no touch voltage in the contact area under concern.
4. The r states of a touch occur in such states of a touch danger, in which there was touch voltage in the contact area under concern.
5. The $\bar{z}_b$ safety reliability states can only appear in such touch states in which there are single damages, pursuant to the $f_p \veebar f_u \veebar f_d$ condition, to an (f_p) basic protection element, an (f_u) fault protection element, or to an (f_d) additional protection element, alternatively.
6. The z_b safety unreliability states can only take place in such touch states in which there are damages to at least two types of protection, pursuant to the $(f_p \wedge f_u) \vee (f_p \wedge f_d) \vee$ $\vee (f_u \wedge f_d)$ condition, to (f_p) basic protection elements, (f_u) fault protection elements, or to (f_d) additional protection elements.

The conditional probabilities, used for classifying the particular combinations of the $(f, \bar{f})$ damages and a lack of damages of shock protection elements, and the $(d, \bar{d})$ for contacts or a lack of contacts as Z_r, R and Z_b reliability states, are listed in Tab. 3.

Table 3. Conditional probabilities of Z_r, R, Z_b nodes for particular F damages and D combinations of contacts

Reliability state	Conditional probability of appearance of the state	Damage and contact combination for which the state appears
$Z_r(z_r, \bar{z}_r)$	lack of touch danger $P(\bar{z}_r / D, F) = 1$	$\bar{f} \vee \bar{d}$
	touch danger $P(z_r / D, F) = 1$	$f \wedge d$
$R(r, \bar{r})$	lack of touch $P(\bar{r} / z_r) = 1$	$z_r \wedge \bar{u}_d$
	touch $P(r / z_r) = 1$	$z_r \wedge u_d$
$Z_b(z_b, \bar{z}_b)$	safety reliability $P(\bar{z}_b / r) = 1$	$r \wedge z_a$ whereby $z_a = f_p \veebar f_u \veebar f_d$
	safety unreliability $P(z_b / r) = 1$	$r \wedge z_k$ whereby $z_k = [(f_p \wedge f_u) \vee (f_p \wedge f_d) \vee (f_u \wedge f_d)]$

Note: The conditional probability of appearance of a given state equals zero if the required combination of damages and contacts does not occur

The total probabilities in the D, F, Z_r, R, Z_b and S conditional nodes are determined according to the principles of cause and effect inference, applied in Bayesian networks.

The $P(D)$ total probabilities, made up of the $P(d)$ probabilities of appearance of a contact and the $P(\bar{d})$ probabilities of appearance of a lack of contact, are determined on the basis of the D_b and D_p non-conditional probabilities and the D conditional node (Tab. 1). They are expressed with the following dependencies

$$P(D) = P(D_b) \cdot P(D_p) \cdot P(D / D_b, D_p) = P(d) + P(\bar{d}), \tag{4}$$

$$P(d) = P(D_b) \cdot P(D_p) \cdot P(d / D_b, D_p) = P(\bar{d}_b) \cdot P(d_p) + P(d_b) \cdot P(\bar{d}_p) + P(d_b) \cdot P(d_p), \tag{5}$$

$$P(\bar{d}) = P(D_b) \cdot P(D_p) \cdot P(\bar{d} / D_b, D_p) = P(\bar{d}_b) \cdot P(\bar{d}_p). \tag{6}$$

The $P(F)$ total probabilities, made up of the $P(f)$ probabilities of occurrence of damages and the $P(\bar{f})$ probabilities of a lack of damages, are determined on the basis of the F_p, F_u and F_d non-conditional probabilities as well as the conditional probabilities of the F node (Tab. 2). They are expressed with the following dependencies

$$P(F) = P(F_p) \cdot P(F_u) \cdot P(F_d) \cdot P(F / F_p, F_u, F_d) = P(f) + P(\bar{f}), \tag{7}$$

$$\begin{aligned} P(f) &= P(F_p) \cdot P(F_u) \cdot P(F_d) \cdot P(f / F_p, F_u, F_d) = P(\bar{f}_{pwyl}) \cdot P(f_{uwyl}) \cdot P(\bar{f}_d) + \\ &+ P(f_{pwyl}) \cdot P(\bar{f}_u) \cdot P(\bar{f}_d) + P(f_{pwyl}) \cdot P(f_{uwyl}) \cdot P(\bar{f}_d) + P(\bar{f}_{pwyl}) \cdot P(\bar{f}_{uwyl}) \cdot P(f_d) + \\ &+ P(\bar{f}_{pwyl}) \cdot P(f_u) \cdot P(f_d) + P(f_{pwyl}) \cdot P(\bar{f}_{uwyl}) \cdot P(f_d) + P(f_p) \cdot P(f_u) \cdot P(f_d), \end{aligned} \tag{8}$$

$$P(\bar{f}) = P(F_p) \cdot P(F_u) \cdot P(F_d) \cdot P(\bar{f} / F_p, F_u, F_d) = P(\bar{f}_p) \cdot P(\bar{f}_u) \cdot P(\bar{f}_d). \tag{9}$$

The probabilities of appearance of the particular reliability states of the protection are determined from the following dependencies

$$P(\bar{z}_r) = P(D) \cdot P(F) \cdot P(\bar{z}_r / D, F) = \sum_k P(\bar{z}_{rk}), \tag{10}$$

$$P(z_r) = P(D) \cdot P(F) \cdot P(z_r / D, F) = \sum_l P(z_{rl}), \tag{11}$$

$$P(\bar{r}) = P(D) \cdot P(F) \cdot P(z_r / D, F) \cdot P(\bar{r} / z_r) = \sum_m P(\bar{r}_m), \tag{12}$$

$$P(r) = P(D) \cdot P(F) \cdot P(z_r / D, F) \cdot P(r / z_r) = \sum_n P(r_n), \tag{13}$$

$$P(\bar{z}_b) = P(D) \cdot P(F) \cdot P(z_r / D, F) \cdot P(r / z_r) \cdot P(\bar{z}_b / r) = \sum_p P(\bar{z}_{bp}), \tag{14}$$

$$P(z_b) = P(D) \cdot P(F) \cdot P(z_r / D, F) \cdot P(r / z_r) \cdot P(z_b / r) = \sum_r P(z_{br}), \tag{15}$$

where: $P(D)\cdot P(F)$ − total probabilities concerning contacts and damages; $P(\bar{z}_r / D,F)$, $P(z_r / D,F)$, $P(\bar{r} / z_r)$, $P(r / z_r)$, $P(\bar{z}_b / r)$, $P(z_b / r)$ − conditional probabilities of appearance of the state of: a lack of a touch danger, touch danger, a lack of touch, touch, safety reliability and unreliability (in the given order); $P(\bar{z}_{rk})$, $P(z_{rl})$, $P(\bar{r}_m)$, $P(r_n)$, $P(\bar{z}_{bp})$, $P(z_{br})$ − probabilities of appearance of the state of: a lack of a touch danger, touch danger, a lack of touch, touch, safety reliability and unreliability (in the given order); *k*, *l, m*, *n*, *p* and *r* − the number of scenarios occurring at particular states; the meanings of the remaining symbols as in the text.

4. Determining the probability of appearance of electric shock states

The developed shock safety model allows for determining the probabilities of appearance of particular shock states. The probabilities are calculated from the following dependencies

$$P(\bar{s}\wedge\bar{z}_b) = P(D)\cdot P(F)\cdot P(z_r / D,F)\cdot P(r / z_r)\cdot P(\bar{z}_b / r)\cdot P(\bar{s} / \bar{z}_b) = \\ = \sum_p P(\bar{z}_{b_p})\ \cdot P(\bar{s} / \bar{z}_{b_p}) = \sum_s P(\bar{s}\wedge\bar{z}_{b_s}), \tag{16}$$

$$P(\bar{s}\wedge z_b) = P(D)\cdot P(F)\cdot P(z_r / D,F)\cdot P(r / z_r)\cdot P(z_b / r)\cdot P(\bar{s} / z_b) = \\ = \sum_r P(z_{b_r})\ \cdot P(\bar{s} / z_{b_r}) = \sum_t P(\bar{s}\wedge z_{b_t}), \tag{17}$$

$$P(s\wedge\bar{z}_b) = P(D)\cdot P(F)\cdot P(z_r / D,F)\cdot P(r / z_r)\cdot P(\bar{z}_b / r)\cdot P(s / \bar{z}_b) = \\ = \sum_p P(\bar{z}_{b_p})\ \cdot P(s / \bar{z}_{b_p}) = \sum_u P(s\wedge\bar{z}_{b_u}), \tag{18}$$

$$P(s\wedge z_b) = P(D)\cdot P(F)\cdot P(z_r / D,F)\cdot P(r / z_r)\cdot P(z_b / r)\cdot P(s / z_b) = \\ = \sum_r P(z_{b_r})\ \cdot P(s / z_{b_r}) = \sum_v P(\bar{s}\wedge z_{b_v}), \tag{19}$$

whereby

$$P(\bar{s}) = P(\bar{s}\wedge\bar{z}_b) + P(\bar{s}\wedge z_b), \quad P(s) = P(s\wedge\bar{z}_b) + P(s\wedge z_b), \tag{20}$$

where: $P(D)\cdot P(F)$ − total probabilities concerning contacts and damages; $P(\bar{z}_r / D,F)$, $P(z_r / D,F)$, $P(\bar{r} / z_r)$, $P(r / z_r)$, $P(\bar{z}_b / r)$, $P(z_b / r)$, $P(\bar{s} / \bar{z}_b)$, $P(\bar{s} / z_b)$, $P(s / \bar{z}_b)$, $P(s / z_b)$ − conditional probabilities of appearance of the state of: a lack of a touch danger, touch danger, a lack of touch, touch, as well as of a lack of appearance and of appearance of touch effect for two protection reliability states (in the given order); $P(\bar{s}\wedge\bar{z}_{bs})$, $P(\bar{s}\wedge z_{bt})$, $P(s\wedge\bar{z}_{bu})$, $P(s\wedge z_{bv})$ − shock scenarios without and with touch effects for two protection reliability states; p, r, s, t, u and v − the number of scenarios occurring at particular states; the meanings of the remaining symbols as in the text.

The $P(s/\bar{z}_{b_u})$ and $P(s/z_{b_v})$ probabilities of appearance of pathophysiologic reactions of a human body as a result of a touch are determined separately for each shock scenario. These scenarios come in one of the two reliability states, the first one with only one damaged protection (18), the other with at least two damaged protections (19). By definition, they are probabilities of exceeding the random value of the current which produces the effects under consideration by the touch current value [9].

In determining these probabilities, it is taken into account that the values of both the parameters are dependent on the electric parameters of the touch circuit and on other circumstances.

Random values of the human body impedance depend on [6, 10]: the u_d value and the f frequency of the touch voltage, the t touch duration time, the d touch path, the s epidermis state, the S replacement surface of the electrode contact, and the T temperature of the surroundings.

Values of random currents producing in humans such touch effects as the sensation of a current flowing, the inability of the person experiencing a touch to free themselves, and ventricular fibrillation, are dependent on [6, 11, 12]: the f frequency, the t touch duration time, the d touch path, and the m human body weight.

5. Analysing the probability of appearance of touch and electric shock states

A shock safety model in the form of a Bayesian network (Fig. 1) allows for:

- rating probabilities of touch (13-15) and shock (18-20) scenarios;
- performing diagnostic inferencing pertaining to the probability of appearance of a selected combination of contacts with and damages of protection elements on condition that a touch or touch effects occur;
- determining the dependence of unreliability of the particular protection elements on the probability of appearance of touch and shock states.

The dependence of unreliability of the F_p, F_u, F_d particular protection elements and of the D_b, D_p probabilities of contacts, as well as of the $P(s/\bar{z}_{bp})$ and $P(s/z_{bp})$ conditional probabilities on the $P(s)$ probability of appearance of an electric shock state may be analysed by means of indices of importance as well as by means of absolute and relative contributions.

The importance of the i-th protection element in the reliability-safety shock protection structure is determined from the derivative of the $P(s)$ probability of occurrence of an electric shock with respect to the $P(f_i)$ unreliability of this protection element. For example, for a basic protection element, this importance is determined from the formula

$$\frac{\partial P(s)}{\partial P(f_p)} = P\left(s \wedge f_p, P\left(f_p\right)=1\right) - P\left(s \wedge \bar{f}_p, P\left(\bar{f}_p\right)=1\right), \tag{21}$$

where: $P(s \wedge f_p, P(f_p) = 1)$ and $P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)$ − probabilities of appearance of an electric shock state, considering shock scenarios with a damaged and undamaged basic protection element, determined for the $P(f_p) = 1$ damage and the $P(\bar{f}_p) = 1$ non-damage probabilities.

The unreliability contribution of the *i*-th protection element to the shock danger maps the increase in the probability of appearance of an electric shock state after replacing a protection element of the $P(f_i) = 0$ absolute reliability with an element of the $P(f_p) > 0$ unreliability. For example, the W_{fp} contribution of the $P(f_p)$ unreliability of a basic protection element in the $P(s)$ probability, is determined from the formula

$$W_{f_p} = P(f_p) \cdot \frac{\partial P(s)}{\partial P(f_p)} = P(s \wedge f_p) - P(f_p) \cdot P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1), \tag{22}$$

where: the meanings of the symbols as in (21).

This contribution increases as the importance and unreliability of a shock protection element get bigger.

It is more convenient to use relative contributions of the importance and unreliability of protection elements as well as of the particular conditional probabilities. For example, the W_{fp} relative contribution of the unreliability of a basic protection element to the $P(s)$ probability is determined from the formula

$$w_{f_p} = \frac{W_{f_p}}{\sum_i W_{xi}} = \frac{W_{f_p}}{W_{d_b} + W_{d_p} + W_{f_p} + W_{f_u} + W_{f_d} + \sum_p W_{s/\bar{z}_{bp}} + \sum_r W_{s/z_{br}}}, \tag{23}$$

where $\Sigma_i W_{xi}$ − the sum of contributions to the probability of appearance of an electric shock state; the sum is dependent on the probabilities of: particular contacts, the unreliability of protection elements, and appearance of touch effects.

The $P(s)$ probability of appearance of an electric shock state can be simply made dependent on the probability of appearance of any one state, e.g. the $P(d)$ contact probability, the $P(f_i)$ probability of a protection element damage, or of one of the $P(s/\bar{z}_{bp})$ or $P(s/z_{br})$ conditional probabilities. For example, the dependence of $P(s)$ on the $P(f_p)$ unreliability of one of the basic protection elements takes on the form of

$$\begin{aligned} P(s) = P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1) + W_{f_p} &= P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1) + \\ + P(f_p) \cdot \left[P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1) \right]&, \end{aligned} \tag{24}$$

where: the meanings of the symbols as in the (21) and (22) formulae.

After transforming the (24) formula, the following dependence for reliability of a basic protection element is obtained

$$P(f_p) = \frac{P(s) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)}{P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)}, \tag{25}$$

where: the meanings of the symbols as in the (21) and (22) formulae.

The above methods of analysing the probability of appearance of an electric shock state can be successfully applied to determining the most probable touch and shock scenarios occurring during the touch or shock incident under concern. In such a case, real states in the non-conditional nodes need to be taken into consideration. This means that if a particular *i*-th contact and an *i*-th damage are found while investigating an accident, then $P(d_i)=1$ and $P(\bar{d}_i)=0$ as well as $P(f_i)=1$ and $P(\bar{f}_i)=0$. The remaining non-conditional probabilities should be determined taking into account the time which has passed since the last inspection took place.

6. Electric shock risk analysis and assessment

The measure of an electric shock risk is the λ_s shock intensity [1/year]. As a measure of shock safety, the $P(\bar{s},t)$ probability of non-appearance of an electric shock in a specified *t* operation time (usually *t* = a year), can be applied.

The λ_s intensity of occurrence of an electric shock is the so-called residual risk and it allows for assessing the electric shock risk through its comparison to the tolerated (allowed) risk.

Applying the principle of rarefaction and composition of the incidents, on the basis of the dependence for the probability of appearance of specific states, it is possible to determine the intensities of the occurrence of: the λ_d contacts, λ_f damages, particular reliability states of shock protection (λ_{zr} touch danger, λ_r touch, $\lambda_{\bar{z}b}$ safety reliability and λ_{zb} safety unreliability) and shock states in the two states as well as their sum ($\lambda_{s\bar{z}b}$ and λ_{szb} as well as λ_s). The knowledge of these intensities makes it possible to determine the average duration times and reliability states ($\bar{t}_d$, $\bar{t}_f$, $\bar{t}_{zr}$, $\bar{t}_r$, $\bar{t}_{zb}$).

Using this principle, the following expression for the intensity of a contact occurrence is obtained

$$\lambda_d = \lambda_{db} \cdot P(d \wedge d_b, P(d_b)=1) + \lambda_{dp} \cdot P(d \wedge d_p, P(d_p)=1) = \lambda_{db} + \lambda_{dp}, \tag{26}$$

and the average duration time of the contact is determined from the formula

$$\bar{t}_d = \frac{P(d)}{\lambda_d (1-P(d))}, \tag{27}$$

where: λ_{db} and λ_{dp} − intensities of occurrence of direct and indirect contacts, $P(d \wedge d_b, P(d_b) = 1$ and $P(d \wedge d_p, P(d_p) = 1$ − probabilities of contact appearance, considering direct and indirect contacts, determined for $P(d_b)=1$ probabilities of direct contact and $P(d_p) = 1$ probabilities of indirect contact; $P(d)$ − probability of contact appearance, determined from the (5) formula.

The expression for the intensity of damage occurrence, in turn, has the following form

$$\begin{aligned}\lambda_f = {} & \lambda_{fp} P(f \wedge f_p, P(f_p)=1) + \lambda_{fu} P(f \wedge f_u, P(f_u)=1) + \\ & + \lambda_{fd} P(f \wedge f_d, P(f_u)=1) = \lambda_{fp} + \lambda_{fu} + \lambda_{fd},\end{aligned} \tag{28}$$

and the average duration time of the damages is determined from the formula

$$\bar{t}_f = \frac{P(f)}{\lambda_f (1 - P(f))}, \tag{29}$$

where: λ_{f_p}, λ_{f_u} and λ_{f_d} − intensities of damage to basic, fault, and additional protection elements; $P(f \wedge f_p, P(f_p) = 1$, $P(f \wedge f_u, P(f_u) = 1$, and $P(f \wedge f_d, P(f_d) = 1)$ − probabilities of appearance of shock protection damage, determined for damaged protection elements: $P(f_p) = 1$ basic protection, $P(f_u) = 1$ fault protection, and $P(f_d) = 1$ additional protection, subsequently; $P(f)$ − probability of appearance of shock protection damage, determined from (8) formula.

Acting in an analogous way, the expressions for the intensity of occurrence of the shock protection reliability states:

− touch danger

$$\lambda_{z_r} = \lambda_{f_p} P(d_p) + \lambda_{f_u} P(d_p) + \lambda_{f_d} P(d_p) = \sum_l \lambda_{zr_l}, \tag{30}$$

− touch

$$\lambda_r = \lambda_{f_p} \cdot P(z_r \wedge f_p \wedge u_d) + \lambda_{f_u} \cdot P(z_r \wedge f_u \wedge u_d) + \lambda_{f_p} \cdot P(z_r \wedge f_p \wedge u_d) = \sum_u \lambda_{r_u}, \tag{31}$$

− safety reliability

$$\lambda_{\bar{z}b} = \sum_u \lambda_{r_u} \cdot P(\bar{z}_b / r_u) = \sum_w \lambda_{\bar{z}b_w}, \tag{32}$$

− safety unreliability

$$\lambda_{zb} = \sum_u \lambda_{r_u} \cdot P(z_b / r_u) = \sum_z \lambda_{zb_z}, \tag{33}$$

where: λ_{zrl}, λ_{ru}, λ_{zbw}, λ_{zbz} − intensities of occurrence of touch danger, touch, and safety reliability and unreliability; l, u, w, z − number of scenarios appearing in particular states; $P(z_r \wedge f_p \wedge u_d)$, $P(z_r \wedge f_u \wedge u_d)$, $P(z_r \wedge f_d \wedge u_d)$, $P(z_r \wedge d_b \wedge u_d)$, $P(z_r \wedge d_p \wedge u_d)$ − probabilities of appearance of touch danger scenarios for which touch voltage and specified damages to protection elements (basic, fault, and additional protection, subsequently) and contacts (direct and indirect one, subsequently) occurred; the meanings of the remaining symbols as in the 26 and 28 formulae.

The knowledge of these intensities allows for determining the mean duration times of these states, in a similar way as in the cases of contacts and damages.

The $\lambda_{s\bar{z}_b}$, λ_{sz_b} and λ_s intensities of an electric shock occurrence are determined from the following formulae

$$\lambda_{s\bar{z}_b} = \sum_p \lambda_{s\bar{z}_{bp}} \cdot P(s / \bar{z}_{bp}), \quad \lambda_{sz_b} = \sum_r \lambda_{sz_{br}} \cdot P(s/z_{br}), \tag{34}$$

whereby

$$\lambda_s = \lambda_{s\bar{z}_b} + \lambda_{sz_b}, \tag{35}$$

where: $\lambda_{s\bar{z}b}$ − intensity of electric shock occurrence in safety reliability states; λ_{szb} − intensity of electric shock occurrence in safety unreliability states; $P(s/\bar{z}_{bp})$, $P(s/z_{br})$ − conditional probabilities of appearance of human body pathophysiologic reactions for particular shock scenarios, with safety reliability and unreliability states, respectively; p and r − numbers of shock scenarios occurring in particular states.

Shock protection is 'efficient' if the residual risk of an electric shock, represented by λ_s, is smaller than the tolerated $\lambda_s = \lambda_{s\bar{z}b} + \lambda_{szb} \le \lambda_{sdop}$. This condition is met if each $\lambda_{s\bar{z}bu}$ and λ_{szbv} partial residual risk of an electric current shock is appropriately limited.

Shock protection will be 'inefficient' if there is an untolerated risk of an electric current shock. It is then necessary to establish actions which would eliminate or decrease the residual risk of an electric shock. Reducing the λ_s electric shock risk is possible through decreasing unreliability of the shock protection elements (damage intensity and/or inspection frequency), and/or diminishing the $P(s/\bar{z}_{b_p})$ and $P(s/z_{b_r})$ conditional probabilities of touch effects appearance (e.g. through reducing the values of touch voltage and/or the values of duration times of operation of a shock protection means). In standards and regulations considering the shock protection construction and dimensioning, requirements related to the allowed electric shock risk and methods of ensuring this risk, should be included.

An analysis of the influence of protection elements unreliability, contacts, and conditional probabilities on this risk is useful in establishing actions which could be taken to reduce the risk of an electric shock. Such an analysis consists in determining the incident importance indices as well as the contributions of their elements unreliability to the intensity of an electric shock occurrence.

For example, the importance of a basic protection element in the aspect of an electric shock risk is expressed with the following formula

$$\frac{\partial \lambda_s}{\partial P(\lambda_{f_p})} = P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1), \tag{36}$$

where: $P(s \wedge f_p, P(f_p) = 1)$ and $P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)$ − probabilities of occurrence of electric shock states, determined for $P(f_p) = 1$ of a damaged and $P(\bar{f}_p) = 1$ of an undamaged basic protection element.

The contribution of the λ_{fp} damage intensity of a basic protection element to the electric shock risk is determined from the formula

$$W_{\lambda_{f_p}} = \lambda_{f_p} \cdot [P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)], \tag{37}$$

where: the meanings of the symbols as in the (21, 22), and (36) formulae.

The λ_s intensity of appearance of an electric shock can be made dependent on the intensity of occurrence of damages to each of the shock protection elements. For example, such a dependence, for the λ_{fp} intensity of one of the basic protection elements, takes on the form of

$$\lambda_s = \lambda(s \wedge \bar{f}_p, P(\bar{f}_p) = 1) + \lambda_{f_p} \cdot \left[P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1) \right], \tag{38}$$

where: $\lambda(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)$ − intensity of occurrence of shock states with a non-damaged basic protection element, determined from the (9) formula for $P(\bar{f}_p) = 1$, the remaining meanings of the symbols as in the (24) and (36) formulae.

After transforming the (38) formula, an expression for the dependence of the λ_{f_p} intensity of occurrence of damage to a basic protection element on the λ_s intensity of occurrence of electric shock states is obtained

$$\lambda_{f_p} = \frac{\lambda_s - \lambda(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)}{P(s \wedge f_p, P(f_p) = 1) - P(s \wedge \bar{f}_p, P(\bar{f}_p) = 1)}, \tag{39}$$

where: the meanings of the symbols as in the (24) and (36) formulae.

If a given basic protection element contributes to the electric shock risk to the greatest extent, then this element should be used to limit this risk in the first place. The λ_{f_p} allowed intensity of the occurrence of a damage to such a basic protection element may be determined (for the times between regular inspections, adopted in the calculations) from the (39) formula, substituting $\lambda_s = \lambda_{sdop}$ $\lambda_{sdop} \le 10^{-6}$ $10^{-8} 1/a$ is usually adopted as the tolerated level of the risk of a lethal electric shock. The determined λ_{f_p} allowed intensity makes it possible to calculate the $P(\bar{f}_p)$ required reliability of this element from the (25) formula.

The probability of a user of an electric device to undergo a touch k times within the t given time, can be determined from the Poisson distribution formula

$$P(s \times k, t) = \frac{(\lambda_s t)^k}{k!} e^{-\lambda_s t}. \tag{40}$$

If the calculation is made for $k = 0$ the formula for the $P(\bar{s}, t)$ probability of non-appearance of an electric shock within the t given time will be obtained.

7. Conclusions

The developed method of modeling shock safety in low-voltage electric devices, based on the use of a Bayesian network, allows for investigating electric shock incidents, analysing and assessing shock risks, as well as for determining criteria of dimensioning shock protection means, also with respect to reliability of the particular shock protection elements.

Applying a Bayesian network to shock safety modeling does not require constructing a 'reliability structure' of the shock protection mean, and at the same time it allows for determining probabilities, intensity, and duration times of experiencing different states of the protection functioning as well as for performing a comprehensive analysis of the influence of the particular protection elements on the electric shock risk.

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