

## Real-time implementation of an interconnected observer design for $p$ -cells chopper

KAMEL LAIDI<sup>1,2</sup>, KHALIFA BENMANSOUR<sup>2</sup>, ABDELAZIZ FERDJOUNI<sup>1</sup>  
OUAHID BOUCHHIDA<sup>2</sup>

<sup>1</sup>*Laboratoire des Systèmes Electriques et Télécommunications,  
Université SAAD DAHLAB, Blida, Algérie*

<sup>2</sup>*Laboratoire de Recherche en Electrotechnique et en Automatique,  
Université YAHIA FARES, Médéa, Algérie*  
*e-mail: {kamlaidi/ k\_benmansour/ aferdjouni/ bouahid2000}@yahoo.fr*

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**Abstract:** This paper deals with an observer design for a  $p$ -cell chopper. The goal is to reduce drastically the number of sensors in such system by using an observer in order to estimate all the capacitor voltages. Furthermore, considering an instantaneous model of a  $p$ -cell chopper, an interconnected observer is designed in order to estimate the capacitor voltages. This is realized by using only the load current measurement. Simulation results are given in order to illustrate the performance of such observer. To demonstrate the validity of our approach, experimental results based on Digital Signal Processor (DSP) are presented.

**Key words:**  $p$ -cell chopper, observer design, interconnected observer, switching modes

### 1. Introduction

Important developments have been witnessed in the area of power electronics. This is due to the developments in power semiconductor and new conversion systems energy. Among these systems, Multi-Cell Choppers are based on the association in series of the elementary cells of commutation. This structure appeared at the beginning of the 90's [11, 13], and made it possible to share the constraints in tension and to improve the harmonic contents of the wave forms [3].

From a practical point of view, the series multi-cell converter designed by the LEEI (Toulouse-France), led to a safe series association of components working in switching mode [5, 6]. This new structure combines additional benefits: attenuation of the voltage jump and modularity of the topologies [7]. All these qualities made this new topology very attractive for various industrial applications. For instance, GEC/ACEC implemented this topology to realize the input chopper which supplies "T13" locomotives in power. Three-phase inverters called "symphony" are developed by Alstom to drive electric motors are also based on the same principle. To benefit

as much as possible from the great potential of multicellular structure, researchers investigated different directions.

The normal operation of the series  $p$ -cell converter is obtained when the voltages are  $v_{ci} = iE/p$ ,  $i = 1, \dots, p - 1$  (see Fig. 2). These voltages are generated when a suitable control of switches are applied in order to obtain a specific value. The control inserted for the switches allow to cancel the harmonics at the switching frequency  $F_{sw}$  and reduces the chopped voltage ripple. However, these properties are lost if the voltages of these capacitors drift. On the other hand, if a specific control is desired, it is advisable to measure these voltages in order to implement it. This task is not easy because extra sensors are necessary for measuring these voltages, which increases the implementation cost. For this reason, it should be avoided, and the estimation of the voltages becomes an attractive and economical option. Due to this reason, an original method is required to eliminate such sensors by using an observer. Theoretically, an observer is considered as a software sensor used to estimate the unmeasurable variables of a system.

Several approaches have been considered to develop new methods to the  $p$ -cell converter control and observation. Initially, models have been developed to describe their instantaneous behaviours [3], harmonics [4] or averaging [1]. These various models were used to develop control laws in open-loop [11] and in closed-loop [8, 14]. Until now, all these  $p$ -cell converters are driven successfully by means of a fix frequency modulator based on pulse width modulation (PWM). Current control algorithms do not take into account the fact that any power converter is a discrete and discontinuous plant or at least a hybrid one. Nevertheless, the profitable skills of PWM technique are to ensure a well-known steady state behaviour which is "optimal" for the electric load with respect to harmonic attenuation. Furthermore, some representations of the  $p$ -cell converter considered complex models and they need a discretisation in purpose to design and implement a discrete observer [2].

In all proposed methods a considerable number of feedback signals are required which is associated with an extra cost of sensing devices. To reduce the cost of sensors, a methodology is necessary to estimate the voltages in the capacitors. In this article, we develop an observer for  $p$ -cell chopper based on an instantaneous model describing the dynamical behaviour of the  $p$ -cell converter. This model is constructed in order to design an observer estimating each flying capacitor voltage. The proposed observer design is based on the class of nonlinear systems which can be written in a form of affine state systems, for which the problem of state observer design has been studied. This class of observer is based on the excitation condition in order to guarantee its convergence.

The objective of this work is the design of an observer for a  $p$ -cell chopper converter to estimate the unmeasurable voltages of the capacitors using the load current  $i$  and the voltage of the source  $E$ , and give an experimental results to validate the observer design. The block diagram describing the proposed observation scheme is illustrated in (Fig. 1).

The paper is organized as follows: In Section 2, the instantaneous model of  $p$ -cell chopper is introduced. In Section 3, the observability properties of the  $p$ -cell chopper model is given. The observer design based on new representation of the instantaneous model of the converter

is presented in the Section 4. In Section 5, a model of 5 cells chopper is used and simulation results are shown to illustrate the performance of the proposed observer.

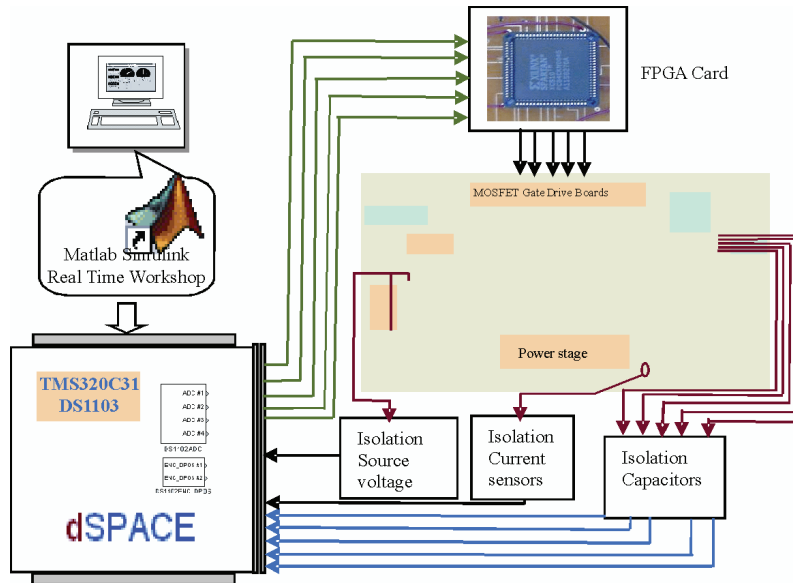


Fig. 1. General structure of the dSPACE observer

The proposed observation scheme is validated and experimental results are shown. Finally, conclusion is reported in Section 7.

### 2. *P*-cell converter model

Throughout this paper, the *p*-cell converter connects in series *p* elementary cells and a passive load *R* and *L* as shown in Figure 2. Each switching cell is controlled by a binary input signal  $S_k(t)$  for  $k = 1, \dots, p$ .

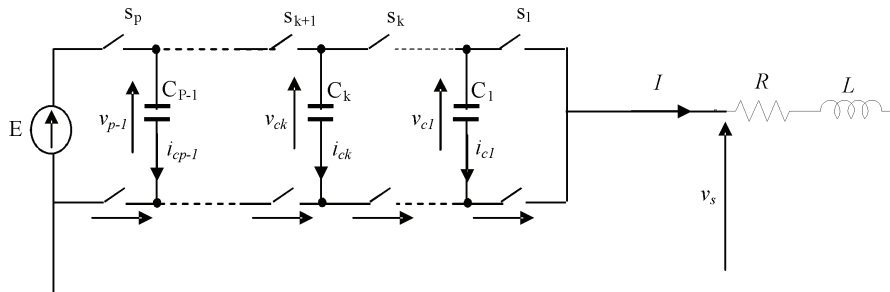


Fig. 2. A *p*-cells converter

This signal  $S_k(t)$  is equal to 1 when the upper switch of the cell is conducting and 0 when the lower complementary switch of the cell is conducting.

The mathematical model describing the behavior of a  $p$ -cell converter is given by:

$$\sum_{p\text{-cell}} : \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_p - \frac{v_{cp-1}}{L}(S_p - S_{p-1}) - \frac{v_{c1}}{L}(S_2 - S_1) \\ \frac{dv_{c1}}{dt} = \frac{1}{c_1}(S_2 - S_1)I \\ \frac{dv_{c2}}{dt} = \frac{1}{c_2}(S_3 - S_2)I \\ \vdots \\ \frac{dv_{cp-1}}{dt} = \frac{1}{c_{p-1}}(S_p - S_{p-1})I \\ y = I \end{cases} \quad (1)$$

where  $v_{ck}$  is the  $k$ -th flying capacitor voltage and  $I$  is the output load current, which is the only measurable output.  $c_k$  for  $k = 1, \dots, p$ ; are the capacitors,  $E$  is the voltage of the source,  $R$  is the resistance and  $L$  is the inductance.

Now from the instantaneous state model of the  $p$ -cell converter given in (1), we will analyze the observability properties of such system in order to construct an observer. It is well known that the observability of a nonlinear system depends on the applied input, and a study of different classes of inputs which render the system observable or unobservable is given in [9-10].

Rewriting the model (1) in a state affine form, we have:

$$\sum : \begin{cases} \dot{X} = \bar{A}(u)X + \bar{B}(u) \\ y = \bar{C}X \end{cases} \quad (2)$$

where  $X = (I, v_{c1}, \dots, v_{cp-1})$  is the state vector,  $u = \{S_1, \dots, S_p\}$  is the input sequence applied to the converter,

$$\bar{A}(u) = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L}(S_2 - S_1) & \dots & -\frac{1}{L}(S_p - S_{p-1}) \\ -\frac{1}{c_1}(S_2 - S_1) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ -\frac{1}{c_{p-1}}(S_p - S_{p-1}) & 0 & \dots & 0 \end{pmatrix},$$

$$\bar{B}(u) = \begin{pmatrix} \frac{E}{L} S_p \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{and} \quad \bar{C} = (1, 0, \dots, 0).$$

Regarding the instantaneous model of the multi-cell converter (2), we can see that there are several operating switching modes  $(S_k, S_{k+1})$  which render the system unobservable for the following operating switching modes

$$(S_k, S_{k+1}) = (1, 1) \quad \text{and} \quad (S_k, S_{k+1}) = (0, 0) \quad \text{for} \quad k = 1, \dots, p - 1.$$

These operating switching modes are not affected by the capacitor voltages. However, these cases occur only for a part of control sequence. If it occurs for all the control sequences then it is not of physical interest because they represent particular situations in which the cell chopper is not operating.

For any sequence of the corresponding input  $u = \{u_1, \dots, u_{p-1}\}$ , where  $u_k = S_{k+1} - S_k$  is applied to the system (1), the control sequence becomes sufficiently periodic. Furthermore, assuming that the current  $I$  is the only measurable variable of the system (2) from the observability rank condition then it can be written as:

$$\text{Rank} \left( \bar{C}, \bar{C} \bar{A}(u), \dots, \bar{C} \bar{A}^{p-1}(u) \right)^T = 2. \quad (3)$$

It is clear that the system is not of the full rank, i.e. the system is not observable. In order to overcome this difficulty, we consider a new representation of the multi-cell converter, which is constituted of a set of 2D subsystems. These subsystems are represented as an interconnected structure such as a whole system. An analysis of each subsystem observability is required and is given in the following Section.

### 3. Observer design for a $p$ -cell chopper

In this section, the design of  $p - 1$  interconnected observers for  $p$ -cell chopper is given. We will consider a different representation of system (1) such that the original system can be splitted into a suitable set of  $p - 1$  subsystems for which it will be possible to design an observer for estimating the capacitor voltages  $v_{cj}$ ,  $j = 1, \dots, p - 1$ . Next, considering that the system (1) can be splitted into  $p - 1$  interconnected subsystems of the form (4):

$$\sum_k : \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L} S_p - \frac{1}{L} \sum_{j=1}^{p-1} (S_{j+1} - S_j) v_{cj} \\ \frac{dv_{ck}}{dt} = \frac{1}{c_k} (S_{k+1} - S_k) i \\ y = I \end{cases} \quad (4)$$

The above mentioned system can be represented, for  $k = 1, \dots, p - 1$  in a compact form as:

$$\sum_k : \begin{cases} \dot{X}_k = A_k(u_k) X_k + B_k(\bar{u}_k, \bar{X}_k) \\ y = C_k X_k \end{cases}, \quad (5)$$

where  $X_k = (I, v_{ck})^T$  is the state vector of the subsystem (5),  $X = (I, v_{c1}, \dots, v_{cp-1})^T$  is the state vector of the system (1),  $\bar{X}_k = (v_{c1}, \dots, v_{ck-1}, v_{ck+1}, \dots, v_{cp-1})^T$ ,  $u_k = (S_{k+1} - S_k)$  for  $k = 1, \dots, p - 1$  and  $\bar{u}_k = (u_1, \dots, u_{k-1}, u_{k+1}, \dots, u_p)^T$  are the inputs. Furthermore,  $y = C_k X_k = I$  is the output of the subsystem (5) with  $C_k = (1 \ 0)$  for  $k = 1, \dots, p - 1$ ; and

$$A_k(u_k) = \begin{pmatrix} -\frac{R}{L} & -\frac{(u_k)}{L} \\ \frac{(u_k)}{c_k} & 0 \end{pmatrix}, \quad (6)$$

and for  $k = 1, 2, \dots, p - 1$

$$B_k(\bar{u}_k, \bar{X}_k) = \begin{pmatrix} -\frac{1}{L} \sum_{\substack{j=1 \\ j \neq k}}^{p-1} (S_{j+1} - S_j) v_{cj} + \frac{E}{L} S_p \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{L} \bar{u}_k^T \bar{X}_k + \frac{E}{L} S_p \\ 0 \end{pmatrix}. \quad (7)$$

It is clear that for  $u_k = 0$ , the system becomes unobservable. However, each subsystem  $k$ -th, which is of dimension 2 is observable for an appropriate input  $u_k$  and it's rank is equal to 2. In order to estimate the unmeasurable variables, no feedback is applied to excite the converter as it has been proposed in other works. In this work, we propose a similar concept which is a well-known concept of regularly persistent input (see appendix). More precisely, a regularly persistent input is applied to the system which allows exciting it sufficiently to obtain the information necessary to reconstruct the unmeasurable variables by means of an observer. If the input is not sufficiently persistent then it is not possible to reconstruct the state of the system from the measured output. The observer works correctly by avoiding the applied input.

The function  $B_k(\bar{u}_k, \bar{X}_k)$  is the interconnection term depending on inputs and states of each subsystem [15-17]. The output is the current  $I(t)$ ; it is the same for each subsystem (see (8))

$$O_k : \begin{cases} \dot{Z}_k = A_k(u_k)Z_k + B_k(\bar{u}_k, \bar{Z}_k) - P_k^{-1} C_k^T (y_k - \hat{y}_k) \\ \dot{P}_k = -\theta_k P_k - A_k^T(u_k)P_k - P_k A_k(u_k) + C_k^T C_k \end{cases} \quad (8)$$

$O_k$  is an observer for the subsystem (5) for  $k = 1, 2, \dots, p-1$  where  $\theta_k > 0$ ,  $\hat{y}_k = C_k X_k = \hat{I}$  and  $P_k^{-1} C_k^T$  is the gain of the observer which depends on the solution of the second equation of (8) for each subsystem with  $Z_k = (\hat{I}, \hat{v}_{ck})^T$ ,  $\bar{Z}_k = (\hat{v}_{c1}, \dots, \hat{v}_{ck-1}, \hat{v}_{ck+1}, \dots, \hat{v}_{cp-1})^T$  and  $A_k(u_k)$  is given in (6)

Now, consider that the system (1) can be represented as follows:

$$\Sigma : \begin{cases} \dot{X}_1 = A_1(u_1)X_1 + B_1(\bar{u}_1, \bar{X}_1) \\ \dot{X}_2 = A_2(u_2)X_2 + B_2(\bar{u}_2, \bar{X}_2) \\ \vdots \\ \dot{X}_{p-1} = A_{p-1}(u_{p-1})X_{p-1} + B_{p-1}(\bar{u}_{p-1}, \bar{X}_{p-1}). \end{cases} \quad (9)$$

Noting that the output is the current  $I(t)$  and is the same for each subsystem. The main idea of this paper is to construct an observer for the whole system (1) from the separate observer design of each subsystem (5). In general, if each (8) is an exponential observer for (5), for  $k = 1, 2, \dots, p-1$ ; then the following interconnected system can be represented as:

$$O : \begin{cases} \dot{Z}_1 = A_1(u_1)Z_1 + B_1(\bar{u}_1, \bar{Z}_1) - P_1^{-1} C_1^T (y - \hat{y}) \\ \dot{Z}_2 = A_2(u_2)Z_2 + B_2(\bar{u}_2, \bar{Z}_2) - P_2^{-1} C_2^T (y - \hat{y}) \\ \vdots \\ \dot{Z}_{p-1} = A_{p-1}(u_{p-1})Z_{p-1} + B_{p-1}(\bar{u}_{p-1}, \bar{Z}_{p-1}) - P_{p-1}^{-1} C_{p-1}^T (y - \hat{y}) \\ \dot{P}_1 = -\theta_1 P_1 - A_1^T(u_1)P_1 - P_1 A_1(u_1) + C_1^T C_1 \\ \dot{P}_2 = -\theta_2 P_2 - A_2^T(u_2)P_2 - P_2 A_2(u_2) + C_2^T C_2 \\ \vdots \\ \dot{P}_{p-1} = -\theta_{p-1} P_{p-1} - A_{p-1}^T(u_{p-1})P_{p-1} - P_{p-1} A_{p-1}(u_{p-1}) + C_{p-1}^T C_{p-1}. \end{cases} \quad (10)$$

The equation (10) is an observer for the interconnected system (8).

**Remark 1.** The proposed observer (10) works for inputs satisfying the regularly persistent condition, which is equivalent to each subsystem (5) being observable, and hence, observer (8) works at the same time while the other subsystems become observable when their corresponding input satisfies the regularly persistent condition.

Now, we will give the sufficient conditions which ensure the convergence of the interconnected observer (10). For that, we introduce the following assumptions.

**Assumption 1.** Assume that the input  $u_k = (S_{k+1} - S_k)$ , for  $k = 1, 2, \dots, p - 1$  is regularly persistent input for subsystem (5), and admits an exponential observer (8). The estimation error, defined as  $\varepsilon_k = Z_k - X_k$  is bounded.

**Assumption 2.** The term  $B_k(\bar{u}_k, \bar{X}_k)$  does not destroy the observability property of the subsystem (5), under the action of the regularly persistent input  $u_k = (S_{k+1} - S_k)$ , for  $k = 1, 2, \dots, p - 1$ . Moreover,  $B_k(\bar{u}_k, \bar{X}_k)$  is Lipschitz with respect to  $k$  and uniformly with respect to  $\bar{u}_k$ , for  $k = 1, 2, \dots, p - 1$ .

The observer convergence can be proved only if the inputs  $u_k$  are regularly persistent, i.e. it is a class of admissible inputs that allows to observe the system (for more details see [12-13]). This assumption guarantees that the observer works and its gain is well-defined, i.e. the matrices  $P_k$ , for  $k = 1, 2, \dots, p - 1$ ; are non-singular (see appendix).

The following result can be established.

**Proposition 1.** Consider the system (1) can be represented in the form of system (9), where each subsystem (5) satisfies the assumption 1 and assumption 2, for  $k = 1, 2, \dots, p - 1$ . Furthermore, the system (10) is an exponential observer for system (9), thus the estimation error, defined as  $\varepsilon = Z - X$ , converges exponentially to zero.

**Proof.** In order to prove the convergence of the observer (10), first we consider the dynamics of the subsystem (5), for which an observer of the form (8) can be designed. Then, defining the estimation error  $\varepsilon_k = Z_k - X_k$  whose dynamics is given by

$$\dot{\varepsilon}_k = \left\{ A(u_k) - P_k^{-1} C_k^T C_k \right\} \varepsilon_k + \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k), \quad (11)$$

where  $\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) = B_k(\bar{u}_k, \bar{Z}_k) - B_k(\bar{u}_k, \bar{X}_k)$ , for  $k = 1, \dots, p - 1$ .

From assumption 1 and lemma 1 (see appendix), we can define

$$V = \sum_{l=1}^{p-1} V_l,$$

as a Lyapunov function for the interconnected system (9), where  $V(\varepsilon_k) = \varepsilon_k^T P_k \varepsilon_k$  is a Lyapunov function for subsystem (5). It is clear that these functions are well defined because the matrices  $P_k$  are non-singular.

Taking the time derivative of  $V(\varepsilon_k)$ , it follows that

$$\dot{V}(\varepsilon_k) \leq -\theta_k V(\varepsilon_k) + \varepsilon_j^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) \text{ for } k = 1, \dots, p - 1. \quad (12)$$

Now, adding and subtracting the term  $\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)$ , we have



$$\begin{aligned} \dot{V}(\varepsilon_k) \leq & -\theta_k V(\varepsilon_k) + 2\varepsilon_k^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) \pm \\ & \pm \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k). \end{aligned} \quad (13)$$

Next, regrouping the appropriate terms

$$\begin{aligned} \dot{V}(\varepsilon_k) \leq & -(\theta_k - 1) \|\varepsilon_k\|^2 P_k - \|\varepsilon_k\|^2 P_k + 2\varepsilon_k^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) + \\ & - \|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|^2 P_k + \|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|^2 P_k. \end{aligned} \quad (14)$$

It follows that

$$\dot{V}(\varepsilon_k) \leq -(\theta_k - 1) \|\varepsilon_k\|^2 P_k + \|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|^2 P_k. \quad (15)$$

Now, from assumption 2,  $B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)$  is Lipschitz, it follows that

$$\|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|^2 P_k < \sum_{l=1, l \neq k}^{p-1} \lambda_l \|\varepsilon_l\|^2 P_k, \quad (16)$$

we get then,

$$\dot{V}(\varepsilon_k) \leq -(\theta_k - 1) \|\varepsilon_k\|^2 P_k + \lambda_l \|\varepsilon_l\|^2 P_k, \quad (17)$$

the time derivative of  $V$  is given by

$$\dot{V}(\varepsilon) = \sum_{k=1}^{p-1} \dot{V}(\varepsilon_k), \quad (18)$$

$$\dot{V}(\varepsilon) \leq \sum_{k=1}^{p-1} \left\{ -(\theta_k - 1) \|\varepsilon_k\|^2 P_k + \sum_{l=1, l \neq k}^p \lambda_l \|\varepsilon_l\|^2 P_k \right\}. \quad (19)$$

Using the lemma on equivalence of norms, i.e. it exists a positive constant  $\mu_l$  such that

$$\|\varepsilon_l\|^2 P_k \leq \mu_l \|\varepsilon_l\|^2 P_l, \quad \forall l = 1, \dots, p-1. \quad (20)$$

Then, it follows that

$$\dot{V}(\varepsilon) \leq \sum_{k=1}^{p-1} \left\{ -(\theta_k - 1) \|\varepsilon_k\|^2 P_k + \sum_{l=1, l \neq k}^{p-1} \lambda_l \mu_l \|\varepsilon_l\|^2 P_k \right\} \quad (21)$$

or

$$\dot{V}(\varepsilon) \leq \sum_{k=1}^{p-1} - \{(\theta_k - 1) - (p-1) \lambda_l \mu_l\} \|\varepsilon_j\|^2 P_k. \quad (22)$$

Finally, we have  $V(\varepsilon) \leq V(\varepsilon(t_0))e^{-\gamma(t-t_0)}$ , for  $\gamma = \min(\gamma_1, \dots, \gamma_{p-1})$  where  $\gamma_k = (\theta_k - 1) - (p-1)\lambda_k\mu_k$ . Taking  $\varepsilon = \text{col}(\varepsilon_1, \dots, \varepsilon_{p-1})$ , it is easy to see that

$$\|\varepsilon(t)\| \leq K \|\varepsilon(t_0)\| e^{-\gamma(t-t_0)}. \quad (23)$$

This ends the proof.

#### 4. Observer 5 cell chopper

In this section we present the proposed methodology which is applied to a model of 5 Cell Chopper converter. For that, consider the following model of 5 cell chopper

$$\sum_{5 \text{ cell}} : \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2 - S_1)}{L}v_{c1} - \frac{(S_3 - S_2)}{L}v_{c2} - \frac{(S_4 - S_3)}{L}v_{c3} - \frac{(S_5 - S_4)}{L}v_{c4} \\ \frac{dv_{c1}}{dt} = \frac{1}{c_1}(S_2 - S_1)I \\ \frac{dv_{c2}}{dt} = \frac{1}{c_2}(S_3 - S_2)I \\ \frac{dv_{c3}}{dt} = \frac{1}{c_3}(S_4 - S_3)I \\ \frac{dv_{c4}}{dt} = \frac{1}{c_4}(S_5 - S_4)I \end{cases} \quad (24)$$

Following the ideas of this original methodology, the model can be rewritten in the following form:

$$\begin{aligned} \sum_1 : & \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2 - S_1)}{L}v_{c1} - \frac{(S_3 - S_2)}{L}v_{c2} - \frac{(S_4 - S_3)}{L}v_{c3} - \frac{(S_5 - S_4)}{L}v_{c4} \\ \frac{dv_{c1}}{dt} = \frac{1}{c_1}(S_2 - S_1)I \end{cases} \\ \sum_2 : & \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2 - S_1)}{L}v_{c1} - \frac{(S_3 - S_2)}{L}v_{c2} - \frac{(S_4 - S_3)}{L}v_{c3} - \frac{(S_5 - S_4)}{L}v_{c4} \\ \frac{dv_{c2}}{dt} = \frac{1}{c_2}(S_3 - S_2)I \end{cases} \\ \sum_3 : & \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2 - S_1)}{L}v_{c1} - \frac{(S_3 - S_2)}{L}v_{c2} - \frac{(S_4 - S_3)}{L}v_{c3} - \frac{(S_5 - S_4)}{L}v_{c4} \\ \frac{dv_{c3}}{dt} = \frac{1}{c_3}(S_4 - S_3)I \end{cases} \\ \sum_4 : & \begin{cases} \frac{dI}{dt} = -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2 - S_1)}{L}v_{c1} - \frac{(S_3 - S_2)}{L}v_{c2} - \frac{(S_4 - S_3)}{L}v_{c3} - \frac{(S_5 - S_4)}{L}v_{c4} \\ \frac{dv_{c4}}{dt} = \frac{1}{c_4}(S_5 - S_4)I \end{cases} \end{aligned} \quad (25)$$

This set of subsystems can be represented in an interconnected compact form as follows:

$$\sum_i : \begin{cases} \dot{X}_i = A_i(u_i)X_i + B_i(\bar{u}_i, \bar{X}_i) \\ y = C_i X_i = I \end{cases} \quad \text{for } i = 1, \dots, 4.$$

It can be assumed that the control sequence of inputs provides the sufficient persistency to guarantee that the observer works correctly (see appendix and assumption 1). Using this assumption, an observer for the above interconnected subsystems are given by:

$$O_i : \begin{cases} \dot{\hat{Z}}_i = A_i(u_i)\hat{X}_i + B_i(\bar{u}_i, \bar{Z}_i) + P_i^{-1} C_i^T (y - \hat{y}) \\ \dot{P}_i = -\theta_i P_i - A_i^T(u_i)P_i + P_i A_i(u_i) + C_i^T C_i \end{cases} \quad \text{for } i = 1, \dots, 4.$$

#### 4.1. Experimental results

In this section, we show some experimental results obtained by using the proposed interconnected observer. In order to illustrate the performance of this observer, where the estimates states converge to the real states, the instantaneous converter model of 5 cells (24) is used for the observer design, where the capacitor voltages have been considered unmeasurable. The parameters of the model were chosen as follows:

$$f_d = 16 \text{ kHz}, \quad C = 40 \text{ } \mu\text{F}, \quad L = 1 \text{ mH}, \quad R = 100 \text{ } \Omega, \quad E = 120 \text{ V}.$$

Furthermore, to carry out the experimentation and show the efficiency of the proposed observer, we use a trajectory for the input voltage as given in Figure 3.

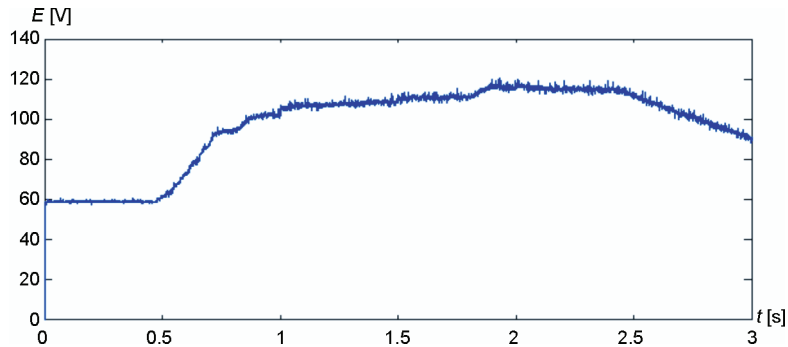


Fig. 3. The input voltage  $E$

Finally, the following initial conditions of the system and the observer were selected as follows: for the system:  $X_k = (I, v_{ck})^T = (0, 0)^T$  and for the observer:  $Z_k = (\hat{i}, \hat{v}_{ck})$  are given as (1, 20), (1, 30), (1, 35) and (1, 40) for  $k = 1, \dots, 4$ .

The parameters  $\theta_k$ , for  $k = 1, \dots, 4$  which are the design parameters used to control the rate of convergence of each observer, were chosen as follows:  $\theta_1 = 30$ ,  $\theta_2 = 40$ ,  $\theta_3 = 50$  and  $\theta_4 = 60$ .

## 5. Benchmark observation

The experimental setup realized based on the DS1103 dSPACE kit (Fig. 1) gives the global scheme of the experimental setup. This kit allows real time implementation of converter, it includes several functions such as analog/digital converters and digital signal filtering. In order to run the application we must write our algorithm in C language. Then, we use the RTW and RTI packages to compile and load the algorithm on processor. To visualize and adjust the control parameters in real time we use the software control-desk which allows conducting the process by the computer.

The multi-cells chopper power stage is based on the use of MOSFET. The pulse width-modulator (PWM) blocks are generated by FPGA card. The observer is first designed in Simulink/Matlab, then, the Real-Time Workshop is used to automatically generate optimized C code for real time applications. Afterward, the interface between Simulink/Matlab and the digital signal processor (DSP) (DS1103 of dSpace) allows the control algorithm to be run on the hardware.

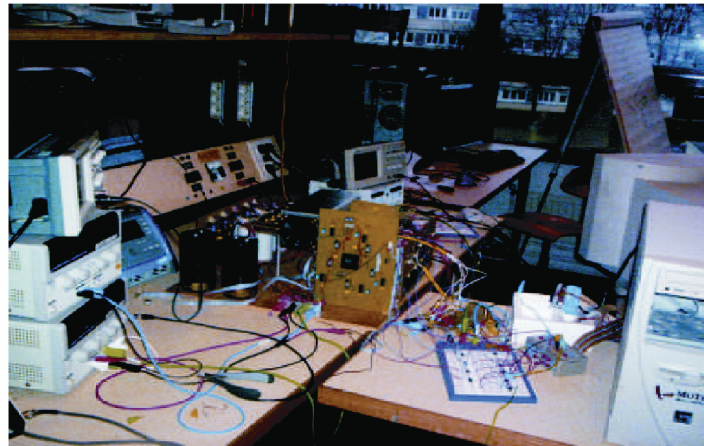
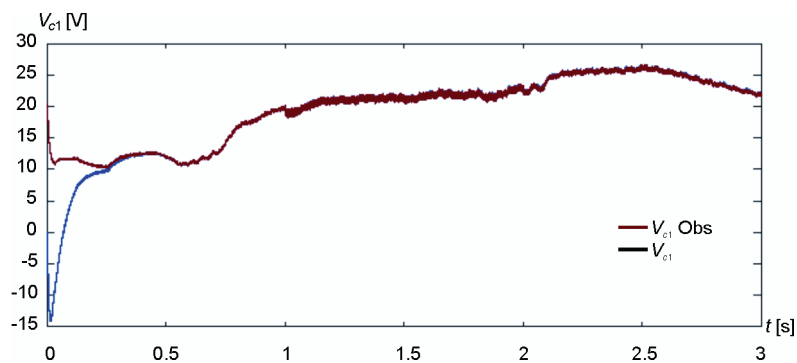
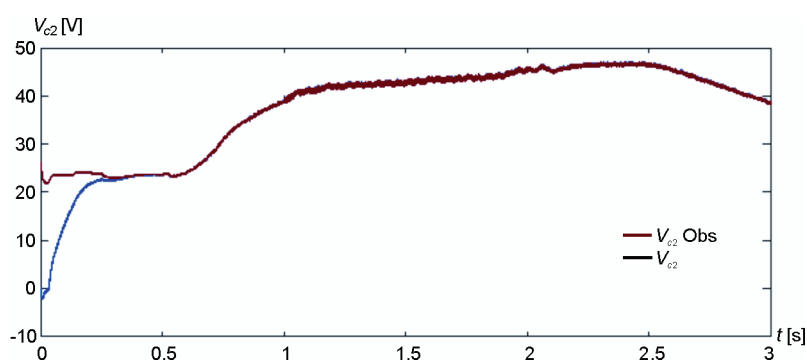
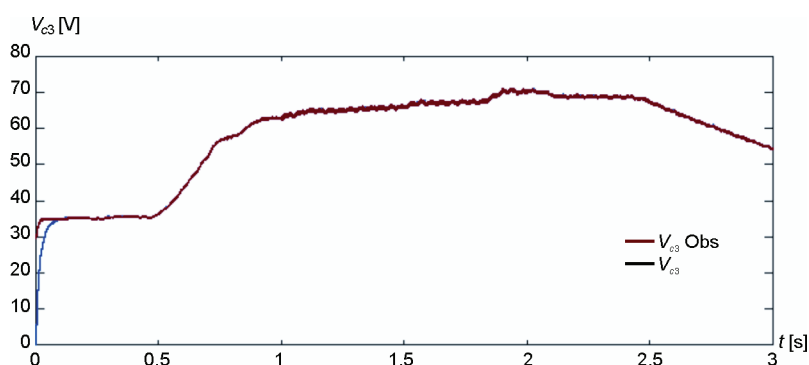


Fig. 4. Laboratory stand

The master bit I/O is used to generate the required 5 gate signals, and six analog-to-digital converters (ADCs) are used for the sensed line-currents, capacitors voltage, and output voltages. An optical interface board is also designed in order to isolate the entire DSP master bit I/O and ADCs. The block diagram of the laboratory stand is given in Fig. 4.

## 6. Experimental evaluations

The experimental results (see Figs. 5-9) are obtained under the following test conditions: The sample time was chosen equal to 50 micro-seconds, and the data acquisition is close equal to 1 sec in this experimental evaluation. We assume that all parameter are known.

Fig. 5. Capacitor voltage  $V_{c1}$  measured and its estimatedFig. 6. Capacitor voltage  $V_{c2}$  measured and its estimatedFig. 7. Capacitor voltage  $V_{c3}$  measured and its estimated

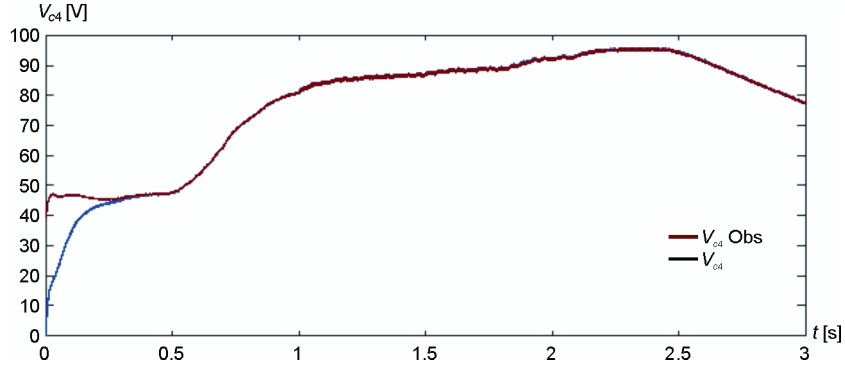
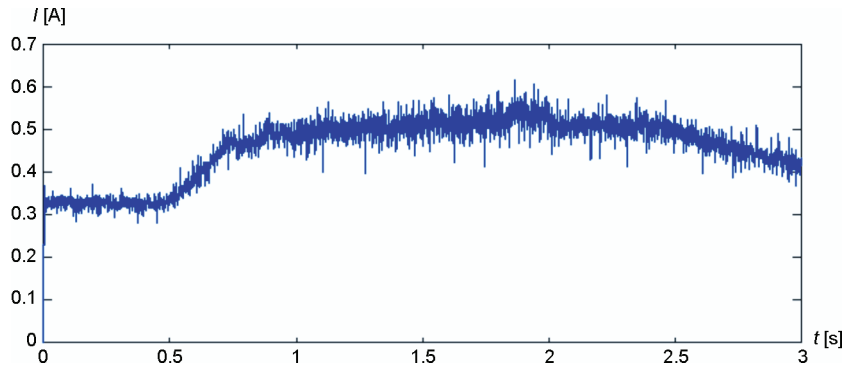
Fig. 8. Capacitor voltage  $V_{c4}$  measured and its estimated

Fig. 9. The output current load

In order to compare the real and estimated voltages, 4 sensors and an optical interface were used in this case. Furthermore, to reduce the noise in the signals, a low pass filter is required. In the figures (Figs. 5-8), we can see the convergence of the estimates and real voltage given by the observer (Obs) to the real variables; this highlights the well fader performance of the proposed observation scheme. The plots show a substantial transient of the estimated voltages which is due to the error in the initial conditions. However, these transients can be reduced by choosing suitable initial conditions of the observer. In this experiment, the initial conditions were chosen far to those of the converter to show the performance of the observer. The output current  $i$  is given in (Fig. 9).

Note that all experimental results are obtained buy using a second order filter.

## 7. Conclusion

In this paper, using an instantaneous model of a Multi-Cell converter, an original methodology of observation has been presented. An observer design has been presented and validated experimentally, to estimate the capacitor voltages from the instantaneous measurement of the

current. The practical interest of such observer has been illustrated by means of experimental results. Furthermore, sufficient conditions has been given in order to prove the exponential convergence to zero, with an arbitrary rate of convergence, of the proposed interconnected observer, which only depends on the persistence of the switching control sequence.

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## 8. Appendix: Some mathematical preliminaries

Now, we introduce some definitions related with the inputs applied to the system. Consider a state-affine controlled system of the following form

$$\begin{cases} \dot{x} = A(v) x + B(v) \\ y = C x \end{cases}, \quad (26)$$

where  $x \in R^n$   $v \in R^m$   $y \in R^p$  with  $A: R^m \rightarrow M(n, m)$ ;  $B: R^m \rightarrow M(n, 1)$  continuous, and  $C \in M(p, n)$ , where  $M(k, l)$  denotes the space of  $k \times l$  matrices with coefficients in  $R$ ;  $k$  (resp.  $l$ ) is the number of rows (resp. columns).

**Notation.** Let  $\Phi_v(\tau, t)$  denotes the transition matrix of:

$$\begin{aligned} \frac{d}{dt} \Phi_v(\tau, t) &= A(v(\tau)) \Phi_v(\tau, t) \\ \Phi_v(t, t) &= I, \end{aligned} \quad (27)$$

with the classical relation:  $\Phi_v(t_1, t_2) \Phi_v(t_2, t_3) = \Phi_v(t_1, t_3)$ .

We then define:

- The Observability Grammian:

$$\Gamma(t, T, v) = \int_t^{t+T} \Phi_v^T(\tau, t) C^T C \Phi_v(\tau, t) d\tau. \quad (28)$$

- The Universality index:

$$\gamma(t, T, v) = \min(\lambda_i(\Gamma(t, T, v))), \quad (29)$$

where the  $\lambda_i(M)$  stand for the eigenvalues of a given matrix  $M$ .

The input functions are assumed to be measurable and such that  $A(v)$  is bounded on the set of admissible inputs of  $R^+$ . We recall below some required results of input functions ensuring the existence of an observer for (5).

**Definition 1.** (Regular Persistence). A measurable bounded input  $v$  is said to be regularly persistent for the state-affine system (5) if there exist  $T > 0$ ,  $\alpha > 0$  and  $t_0 > 0$  such that  $\gamma(t, T, v) > \alpha$  for every  $t \geq t_0$ .

Now, a further result based on regular persistence is introduced.

**Lemma 1.** Assume that the input  $u_k$  is regularly persistent for system (2) and consider the following Lyapunov differential equation:

$$\dot{P}_k = -\theta_k P_k - A^T(u_k) P_k - P_k A(u_k) + C_k^T C_k, \quad (30)$$

with  $P_k(0) > 0$ . Then,  $\exists \theta_{k0} > 0$  such that for any symmetric positive definite matrix  $P_k(0)$ ,  $\exists \theta_k \geq \theta_{k0}$ ,  $\exists \alpha_k, \beta_k > 0$ ,  $t_0 > 0$ :  $\forall t > t_0$

$$\alpha_k I < P_k(t) < \beta_k I, \quad (31)$$

where  $I$  is the identity matrix.