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COMPARISON OF SELECTED PROPERTIES OF THREE- AND FIVE-PHASE INDUCTION MOTORS

Abstract: The paper analyses the rise of higher space harmonics in three-phase and five-phase induction machines. The results of the analysis show that there is a group of harmonics which are associated with symmetrical components of phase currents. In five-phase machines, the spectrum of higher harmonics assigned to the component which generates the operating torque is shifted to higher orders. Due to this fact, the five-phase machines have several advantages compared with the three-phase machines. These advantages are apparent mainly in a steady-state operation at symmetrical feeding. The five-phase machines have, however, also improved properties in transient and fault states. This fact is shown at the example of operation of three-phase and five-phase machines after the two-phase short circuit at the terminals of two neighbouring phase stator windings. In this case, the five-phase machine can, for a short time, operate with a rated load, while a three-phase machine is not able to generate sufficient torque and the rotor speed rapidly decreases to the standstill.

1. Introduction

In the literature, a considerable attention is at present paid to multi-phase machines. Several advantages of these machines compared with three-phase machines are mentioned in [1], [2] and [3]. Due to their improved properties, these machines are used in several special applications, e.g. drives in aircrafts and ships. One of their advantages is the improved fault tolerance. The presented paper deals with comparison of properties of the three- and five-phase induction machines both in steady and transient states. A two-phase short circuit has been chosen as an example of a fault operation. The comparison of these machines cannot be carried out without considering higher space harmonics of a current layer along the air gap and the yoke flux. For this reason, a method of space vectors and symmetrical components of instantaneous values described in [4] has been chosen for solving the given problem.

2. Higher Harmonics in Three-Phase and Five-Phase Induction Machines

The comparison of the properties of three-phase and five-phase induction machines is based on the analysis of the rise of higher space harmonics of the current layer and the yoke flux. These harmonics, mainly in the case of multi-phase windings, can, under certain circumstances, essentially influence the properties of these machines.

As shown in literature [4], a group of coils composing a single phase winding gives rise to individual space harmonics of the yoke flux. These harmonics can be respected by means of a space vector $\boldsymbol{\Phi}_{j\nu}$ which can be written as

$$\boldsymbol{F}_{j\nu} = L_\nu N \mathrm{k}_\nu \boldsymbol{v}_\nu i \tag{1}$$

where ν is an order of a harmonic, i is an instantaneous value of the winding current, L_ν is the inductivity of a single conductor, N is a number of conductors of the considered winding, κ_ν is a factor of this winding for the ν-th harmonic, **v** is a unit vector determining the position of the winding in the complex plane of the considered harmonic. In the case of a two-pole machine and the first space harmonic, this plane is identical to the plane perpendicular to the shaft of the machine. The quantity

$$\boldsymbol{\iota}_{N\nu} = N \mathrm{k}_\nu \boldsymbol{v}_\nu i \tag{2}$$

is called [4] the space vector of the current layer of the considered winding. This vector lies in the axis of the considered winding and its magnitude is proportional to the instantaneous value of the current i. In the case of the three-phase winding, the resultant space vector of the stator currents will be given by the sum of the current vectors of the particular windings. If the axis of a single winding (e.g. of the phase A) coincides with the real axis of the considered complex plane, the axis of the phase B will be shifted by the angle of $e^{j2\pi/3}$ and the axis of the phase C by the angle of $e^{j4\pi/3}$. Then the space vector of the ν-th wave of the current layer is

$$\boldsymbol{i}_{SN\nu} = N_S \mathrm{k}_{\nu S} \left(i_{SA} + i_{SB} \boldsymbol{a}_3^{\nu} + i_{SC} \boldsymbol{a}_3^{2\nu} \right) \tag{3}$$

where i_{SA}, i_{SB} and i_{SC} are instantaneous values of phase currents.

$$\boldsymbol{a}_3 = e^{j\frac{2\pi}{3}} \quad (4)$$

Due to the periodically repeating values $\boldsymbol{a}^{\nu}$, the expression in the parentheses in equation (3) can gain only three different values which are proportional to the symmetrical components $\boldsymbol{i}_{1S}$, $\boldsymbol{i}_{2S}$, and $\boldsymbol{i}_{3S}$ of the instantaneous values of the stator currents.

The symmetrical components are defined as

$$\boldsymbol{i}_{1S} = h\left(i_{SA} + i_{SB}\boldsymbol{a} + i_{SC}\boldsymbol{a}^2\right) \quad (5)$$

$$\boldsymbol{i}_{2S} = h\left(i_{SA} + i_{SB}\boldsymbol{a}^2 + i_{SC}\boldsymbol{a}\right) \quad (6)$$

$$\boldsymbol{i}_{3S} = h\left(i_{SA} + i_{SB} + i_{SC}\right) \quad (7)$$

The symmetrical components of the voltages can be defined by analogy. The constant h can be chosen quite arbitrarily. In the following text, the constant h is considered as 1/3.

The first and second symmetrical components of instantaneous values of voltage currents are complex conjugated quantities. They must be strictly distinguished from the symmetrical components of phasors (positive-, negative-, and zero sequence components). Equations (3), (5), (6) and (7) show that the first and the second components give rise to the space harmonics of the current layer and the yoke flux of the order $\nu = |1 \pm 3k|$ and $\nu = |2 \pm 3k|$, where k is a positive integer including zero. The third component gives rise to the harmonics of the orders $\nu = |3 \pm 3k|$. Space harmonic waves of the lowest orders corresponding to particular components are in Tab. 1. In Tab. 1, there are not mentioned harmonics of the even orders which are in commonly used windings suppressed by the winding factors.

Tab. 1

component	ν						
1,2	1	5	7	11	13	17	19
3	3	9	15	21	27	33	39

Analogically, in the case of a five-phase machine, the space vectors of individual harmonics of a current layer are

$$\boldsymbol{i}_{SN\nu} = N_S \kappa_{\nu S}\left(i_{SA} + i_{SB}\boldsymbol{a}_5^{\nu} + i_{SC}\boldsymbol{a}_5^{2\nu} + i_{SD}\boldsymbol{a}_5^{3\nu} + i_{SE}\boldsymbol{a}_5^{4\nu}\right) \quad (8)$$

where

$$\boldsymbol{a}_5 = e^{j\frac{2p}{5}} \quad (9)$$

The expression in the parentheses on the left can be for individual groups of harmonics substituted by symmetrical components of instantaneous values of a five-phase system. These components can be written as

$$\boldsymbol{i}_{1S} = h\left(i_{SA} + i_{SB}\boldsymbol{a}_5 + i_{SC}\boldsymbol{a}_5^2 + i_{SD}\boldsymbol{a}_5^3 + i_{SE}\boldsymbol{a}_5^4\right) \quad (10)$$

$$\boldsymbol{i}_{2S} = h\left(\mathrm{i}_{SA} + \mathrm{i}_{SB}\boldsymbol{a}_5^2 + \mathrm{i}_{SC}\boldsymbol{a}_5^4 + \mathrm{i}_{SD}\boldsymbol{a}_5 + \mathrm{i}_{SE}\boldsymbol{a}_5^3\right) \quad (11)$$

$$\boldsymbol{i}_{3S} = h\left(i_{SA} + i_{SB}\boldsymbol{a}_5^{-2} + i_{SC}\boldsymbol{a}_5^{-4} + i_{SD}\boldsymbol{a}_5^{-1} + i_{SE}\boldsymbol{a}_5^{-3}\right) \quad (12)$$

$$\boldsymbol{i}_{4S} = h\left(i_{SA} + i_{SB}\boldsymbol{a}_5^{-1} + i_{SC}\boldsymbol{a}_5^{-2} + i_{SD}\boldsymbol{a}_5^{-3} + i_{SE}\boldsymbol{a}_5^{-4}\right) \quad (13)$$

$$i_{5S} = h\left(i_{SA} + i_{SB} + i_{SC} + i_{SD} + i_{SE}\right) \quad (14)$$

In the following, the constant h will be considered as 1/5. The first and the fourth components are complex conjugated and give rise to a group of harmonics of a current layer of the order $\nu = |1 \pm 5k|$ or $\nu = |4 \pm 5k|$. The second and the third components constitute harmonics $\nu = |2 \pm 3k|$ or $\nu = |3 \pm 3k|$. The orders of space harmonics associated with individual components are in Tab. 2.

Tab. 2

component	ν						
1,4	1	9	11	19	21	29	31
2,3	3	7	13	17	23	27	33
5	5	15	25	35	45	55	65

In the case of the pairs of complex conjugated quantities, only one of these components may be considered. If a five-phase stator is fed by a symmetrical system of phase voltages, it can be easily proved that components 2, 3 and 5 of voltages and currents do not arise. The comparison of the orders of harmonics assigned to the first symmetrical component of a five-phase stator and the first symmetrical component of a three-phase stator (see Tab. 1 and Tab. 2) shows that in case of a five-phase machine the spectrum of harmonics is shifted to higher orders. It is known that the significance of higher harmonics considerably decreases with the order. It can be assumed with high probability that a five-phase induction machine will have smaller additional losses due to the reduced differential leakage, lowered noise and increased efficiency in comparison with the three-phase machine of the same power output. If the five-phase machine is fed by an unbalanced system of phase voltage, the second and the third component may significantly influence the situation in the machine. The feeding by unbalanced voltages occurs mainly in various fault states, e.g. in various types of short circuits. The behaviour of machines during various faults can be analysed by means of differential equations, which represent mathematical description of these machines. In the following text, a short circuit on the terminals of two neighbouring phase windings will be considered as an example of a fault.

3. Simulation of a short circuit between two neighbouring stator terminals and conclusions

According to [5] and [6], a three-phase induction machine is described by equations

$$\boldsymbol{u}_{1S} = R_S \boldsymbol{i}_{1S} + L_{1S}\frac{d\boldsymbol{i}_{1S}}{dt} + L_{1h}\frac{d\boldsymbol{i}_{1R\lambda}}{dt} \tag{15}$$

$$0 = R_{1R}\boldsymbol{i}_{1R\lambda} + L_{1R}\frac{d\boldsymbol{i}_{1R\lambda}}{dt} + L_{1h}\frac{d\boldsymbol{i}_{1S}}{dt} - \\ - jp\omega_m\left(L_{1R}\boldsymbol{i}_{1R\lambda} + L_{1h}\boldsymbol{i}_{1S}\right) \tag{16}$$

$$u_{3S} = R_S i_{3S} + L_{3S}\frac{di_{3S}}{dt} + \mathrm{Re}\left[L_{3h}\frac{d\boldsymbol{i}_{3R\lambda}}{dt}\right] \tag{17}$$

$$0 = R_{3R}\boldsymbol{i}_{3R\lambda} + L_{3R}\frac{d\boldsymbol{i}_{3R\lambda}}{dt} + L_{3h}\frac{di_{3S}}{dt} - \\ - j3p\omega_m\left(L_{3R}\boldsymbol{i}_{3R\lambda} + L_{3h}i_{3S}\right) \tag{18}$$

Equations for the torque components and the total torque are

$$T_1 = 6pL_{1h}\,\mathrm{Re}\left[j\boldsymbol{i}_{1S}^{*}\boldsymbol{i}_{1R\lambda}\right] \tag{19}$$

$$T_3 = 9pL_{3h}\,\mathrm{Re}\left[j\boldsymbol{i}_{3S}^{*}\boldsymbol{i}_{3R\lambda}\right] \tag{20}$$

$$T = T_1 + T_3 \tag{21}$$

According to [7], a five-phase machine is described by equations

$$\boldsymbol{u}_{1S} = R_S \boldsymbol{i}_{1S} + L_{1S}\frac{d\boldsymbol{i}_{1S}}{dt} + L_{1h}\frac{d\boldsymbol{i}_{1R\lambda}}{dt} \tag{22}$$

$$0 = R_{1R}\boldsymbol{i}_{1R\lambda} + L_{1R}\frac{d\boldsymbol{i}_{1R\lambda}}{dt} + L_{1h}\frac{d\boldsymbol{i}_{1S}}{dt} - \\ - jp\omega_m\left(L_{1R}\boldsymbol{i}_{1R\lambda} + L_{1h}\boldsymbol{i}_{1S}\right) \tag{23}$$

$$\boldsymbol{u}_{3S} = R_S \boldsymbol{i}_{3S} + L_{3S}\frac{d\boldsymbol{i}_{3S}}{dt} + L_{3h}\frac{d\boldsymbol{i}_{3R\lambda}}{dt} \tag{24}$$

$$0 = R_{3R}\boldsymbol{i}_{3R\lambda} + L_{3R}\frac{d\boldsymbol{i}_{3R\lambda}}{dt} + L_{3h}\frac{d\boldsymbol{i}_{3S}}{dt} - \\ - j3p\omega_m\left(L_{3R}\boldsymbol{i}_{3R\lambda} + L_{3h}\boldsymbol{i}_{3S}\right) \tag{25}$$

$$u_{5S} = R_S i_{5S} + L_{5S}\frac{di_{5S}}{dt} + L_{5h}\,\mathrm{Re}\left[\frac{d\boldsymbol{i}_{5R\lambda}}{dt}\right] \tag{26}$$

$$0 = R_{5R}\boldsymbol{i}_{5R\lambda} + L_{5R}\frac{d\boldsymbol{i}_{5R\lambda}}{dt} + L_{5h}\frac{di_{5S}}{dt} - \\ - j5p\omega_m\left(L_{5R}\boldsymbol{i}_{5R\lambda} + L_{5h}i_{5S}\right) \tag{27}$$

$$T_1 = 10pL_{1h}\,\mathrm{Re}\left[j\boldsymbol{i}_{1S}^{*}\boldsymbol{i}_{1R\lambda}\right] \tag{28}$$

$$T_3 = 30pL_{3h}\,\mathrm{Re}\left[j\boldsymbol{i}_{3S}^{*}\boldsymbol{i}_{3R\lambda}\right] \tag{29}$$

$$T_5 = 25pL_{5h}\,\mathrm{Re}\left[j\boldsymbol{i}_{5S}^{*}\boldsymbol{i}_{5R\lambda}\right] \tag{30}$$

$$T = T_1 + T_2 + T_3 \tag{31}$$

Rotor parameters and variables are rated to the effective number of conductors of a single stator phase for the considered component (index 1, 3, 5). Rotor current components are transferred into the stator coordinate system.

For the sake of simplicity, only the strongest waves of particular groups of harmonics were considered while deriving the above mentioned equations. The strongest waves are the harmonics of the lowest order from the group of harmonics corresponding to the considered component. Equations (15) and (16) for the first component of a three-phase machine are identical to Eqs. (22) and (23) for the first component of a five-phase machine. It can thus be supposed that the properties of the three- and five-phase machines will be similar in the case of symmetrical feeding. Analogical similarity is between Eqs. (17) and (18) for the third component of a three-phase machine and Eqs. (26) and (27) for the fifth component of a five-phase machine. These components do not arise if a node of stator winding is not connected with a node of the source of feeding voltage.

Equations (24) and (25) for the third component of a five-phase machine do not have an analogy in the case of a three-phase machine. However, in the case of unbalanced feeding, the third component can substantially influence the situation in the machine and must not be neglected in any case.

Based on the above mentioned equations, numerical models of three-phase and five-phase induction machine have been developed in the Matlab-Simulink system. Simulations for the four-pole machine with the output of 7.5 kW with the stator voltage of 220 V rms have been carried out. The parameters of this machine are R_S = 0.66 Ω, R_R = 0.44 Ω, R_{3R} = 0.12 Ω, L_{1S} = 0.119 H, L_{1R} = 0.114 H, L_{1h} = 0.117 H, L_{3S} = 0.009H, L_{3R} = 0.0044 H, L_{3h} = 0.0035 H, p = 2. The torque of inertia is 0.1 kgm/sec and the load torque 50 Nm. The parameters of the five-phase machine were estimated from the parameters of the above considered three-phase machine: R_S = 0.396 Ω, R_R = 0.265 Ω, R_{3R} = 0.175 Ω, R_{5R} = 0.069 Ω, L_{1S} = 0.072 H,

L_{1R} = 0.070 H, L_{1h} = 0.0683 H, L_{3S} = 0.0084 H, L_{3R} = 0.0064 H, L_{3h} = 0.005 H, L_{5S} = 0.007 H, L_{5R} = 0.0041 H, L_{5h} = 0.0036 H. The moment of inertia and the load torque were considered the same as in the case of a three-phase machine. A short circuit was considered across the terminals of the phase *A* and *B*. The waveforms of phase voltages of a three-phase machine are shown in Fig. 1. In time t = 0.03 sec a short circuit occurred. The waveforms of the real part $u_{1S\alpha}$ and the imaginary part $u_{1S\beta}$ of the vector $\boldsymbol{u}_{1S}$ are shown in Fig. 2. The third component of voltage and thus the third component of current will not arise in this case. Fig. 3 shows the waveforms of the real part $i_{1S\alpha}$ and the imaginary part $i_{1S\beta}$ of the vector of the first component of the currents.

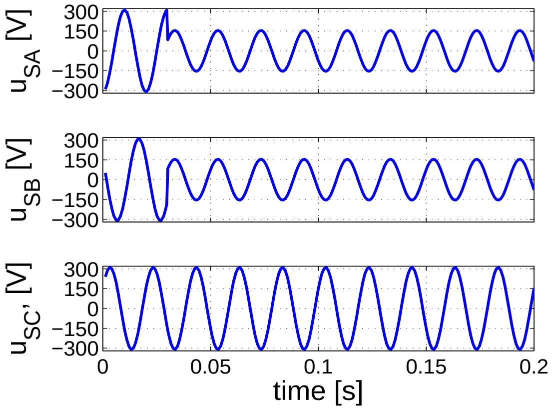

Fig.1. Phase voltages

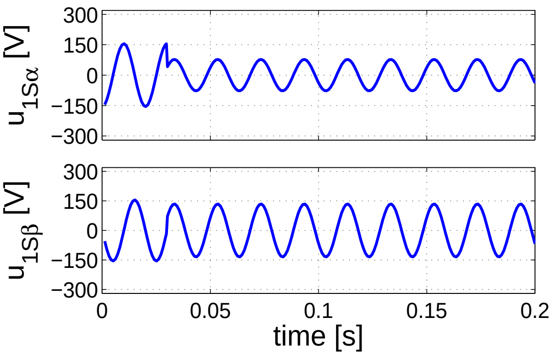

Fig.2. Real and imaginary parts of $\boldsymbol{u}_{1S}$

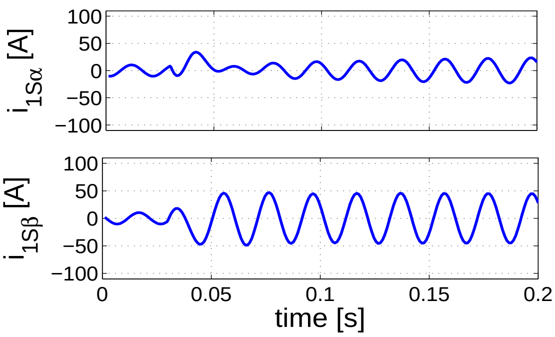

Fig.3. Real and imaginary parts of $\boldsymbol{i}_{1S}$

The phase currents are shown in Fig. 4, the torque and the speed are shown in Fig. 5. During the considered load in the two-phase short circuit, a three-phase induction motor is not able to generate an additional torque and the speed rapidly decreases.

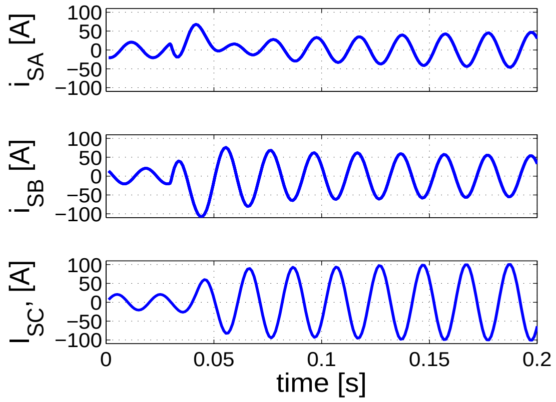

Fig.4. Phase currents

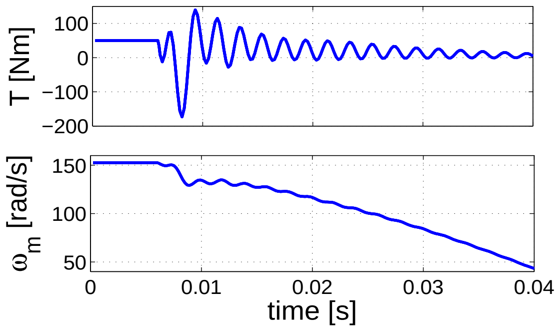

Fig.5. Torque and speed

The results of the simulation of the same type of a short circuit on the terminals of the five-phase machine are presented in Figs. 6 to 12.

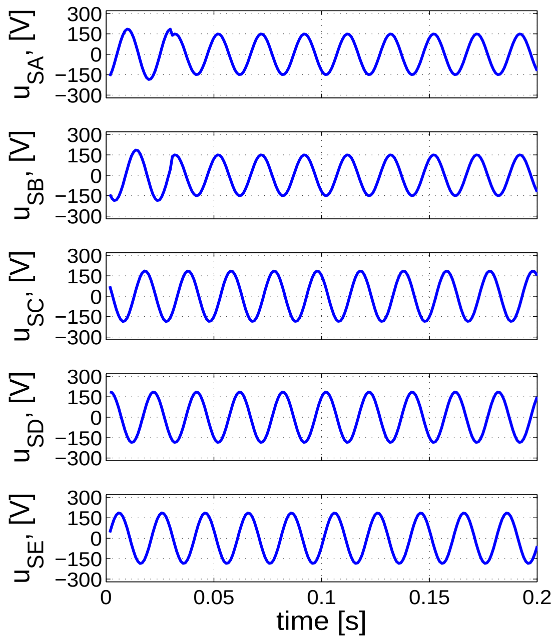

Fig.6. Phase voltages

The waveforms of the real parts α and the imaginary parts β of the components $\boldsymbol{i}_{1S}$ and $\boldsymbol{i}_{3S}$ are in Figs. 9 and 10. Even though the magnitude of the quantity u_{3S} is considerably smaller than the magnitude of the quantity $\boldsymbol{u}_{1S}$ (see Figs. 7 and 8), the magnitudes of the vectors $\boldsymbol{i}_{1S}$ and i_{3S} are comparable (see Figs. 9 and 10).

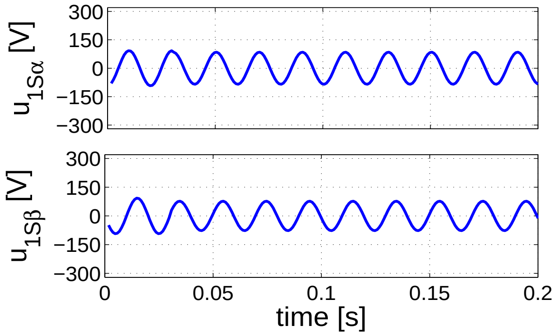

Fig.7. Real and imaginary parts of $\boldsymbol{u}_{1S}$

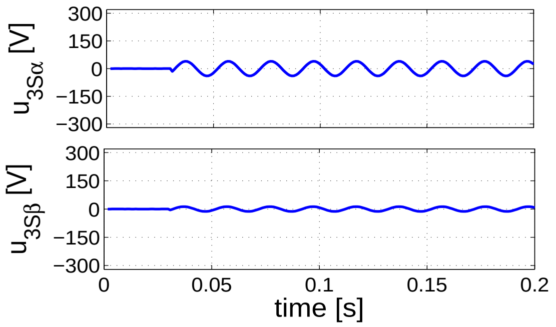

Fig.8. Real and imaginary parts of $\boldsymbol{u}_{3S}$

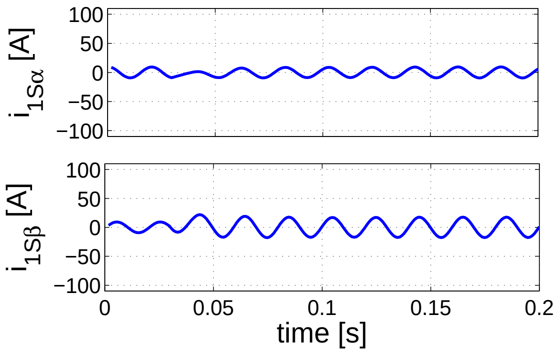

Fig.9. Real and imaginary parts of $\boldsymbol{i}_{1S}$

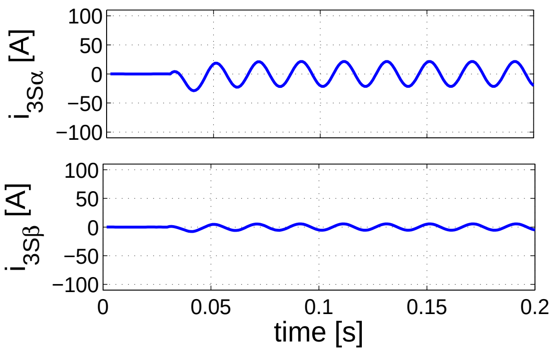

Fig.10. Real and imaginary parts of $\boldsymbol{i}_{3S}$

The third component will thus significantly influence the values of the stator currents in Fig. 11. The biggest current stress is in the phase *B*. The torques and the speed are shown in Fig. 12. The torque T_3 generated by the third component (dashed line) is relatively small in view of the fact that the machine works with the speed considerably higher than the synchronous speed of the third component. The five-phase machine generates a sufficient torque in contrary to a three-phase machine is able to operate for some time.

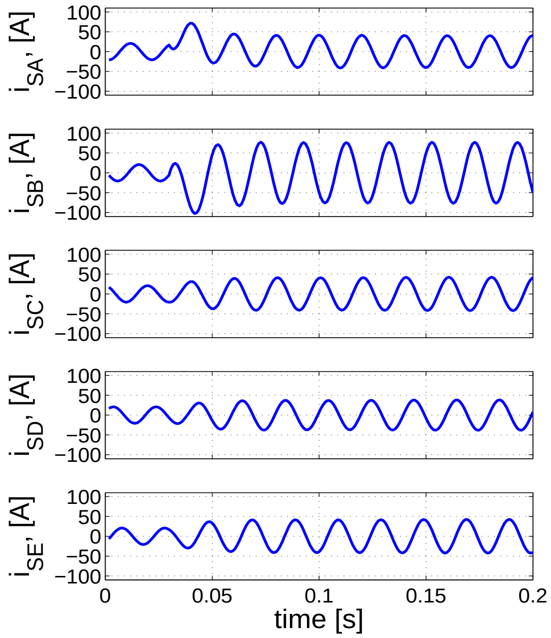

Fig.11. Phase currents

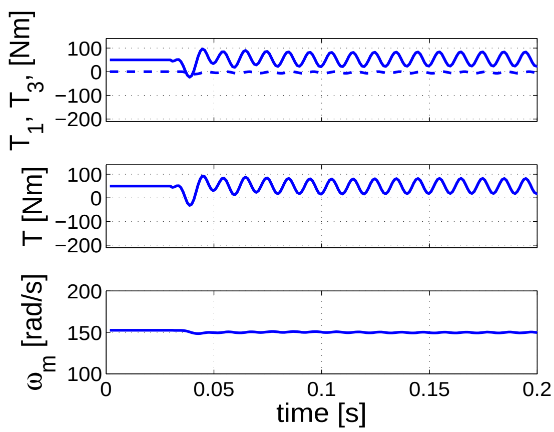

Fig.12. Torque component, resulting torque, and speed

In Figs. 13 to 15, the waveforms of the stator current vector, phase currents, the torque and the speed of the three-phase machine during a short circuit in no-load operation are shown. In this case the permanent drop in speed will occur. The torque ripples during the short circuit and in a steady-state operation after the short circuit are substantially bigger than in the case of a five-phase machine.

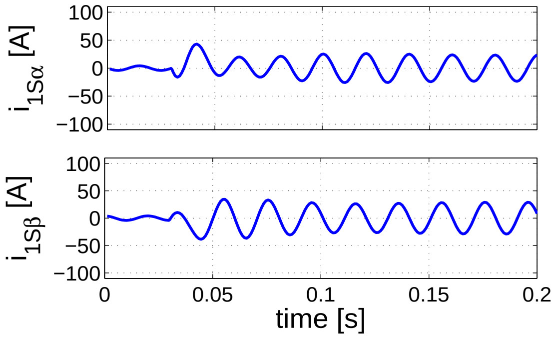

Fig.13. Real and imaginary parts of $\boldsymbol{i}_{1S}$

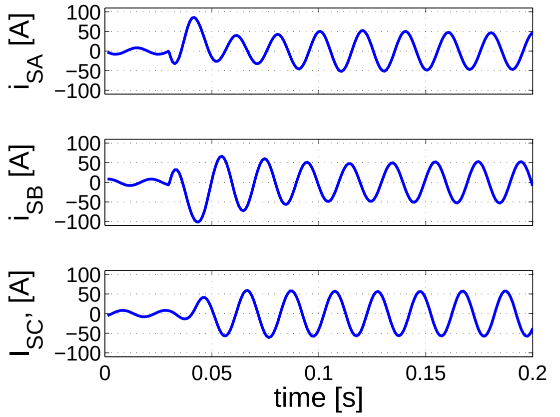

Fig.14. Phase currents

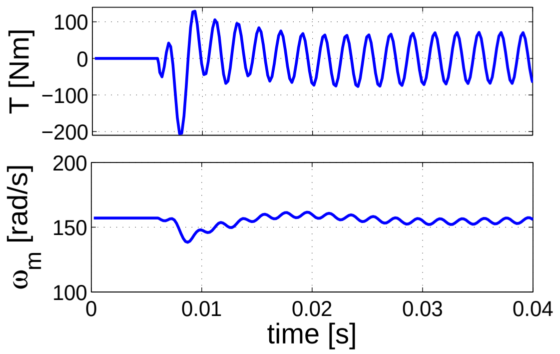

Fig.15. Torque and speed

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