Pavel Záskalický
Technical University of Košice, Slovakia

TORQUE RIPPLE CALCULATION OF A PERMANENT MAGNET SYNCHRONOUS MOTOR SUPPLIED BY A THREE PHASE INVERTER WITH PWM

Abstract: A steady-state analysis of a permanent magnet synchronous motor drive with a voltage-source inverter is presented. The torque-speed profile required of the drive is a constant torque region from zero to the base speed and a constant power region above base speed. A Fourier series approach is used to predict the line current waveform. Assuming the output inverter's voltage is controlled by a pulse-width notching. Suppose the notching frequency to be constant for all frequency of the fundamental squarewave and to be fixed on 500 Hz. The permanent magnet synchronous motor model is obtained from its analogy to the fixed-excited synchronous motor. From the induced voltage and line current waveforms the electromagnetic torque ripple waveform is calculated.

1. Introduction

DC motors are known for their versatile characteristics but the commutation process involved in such motors makes them unsuitable for high-speed operation. DC motors also require regular maintenance due to the constant wear of the brushes. Considering these shortcomings, DC drives are increasingly being replaced by inverter-fed AC motors.

Induction motors are rugged, physically small, cheap, and require little maintenance. They are suitable for many drives. Synchronous motors are, however, preferred for drives requiring accurate position control or precise speed matching between different motors. Synchronous motors also have somewhat better efficiency than induction motors. An advantage of synchronous motors lies in their ability to operate at a leading power factor, simplifying the converter used to drive the motor.

As quality and strength are improved, permanent magnets are being increasingly used in synchronous motors to provide excitation, eliminating the need for slip rings and field supply. The motor magnet must, however, be designed to withstand the demagnetizing force of the stator currents.

This paper presents such a steady-state analysis assuming that the mode of operation of the voltage-source inverter is 180° conduction mode. Control strategies are developed for torque control. A Fourier series approach is used to analyze the time harmonics in the motor.

In the drive analysis, the following assumptions are made with respect to the inverter and motor.

- All parameters of the motor are assumed to be constant and frequency independent. Magnetic saturation and core losses are neglected.
- The system is in steady-state. All harmonic torques resulting from supply harmonics are considered.
- The inverter is assumed to be ideal with no losses and has instantaneous switching devices.

2. System block diagram

The block diagram of the permanent magnet synchronous motor drive is given in Fig.1.

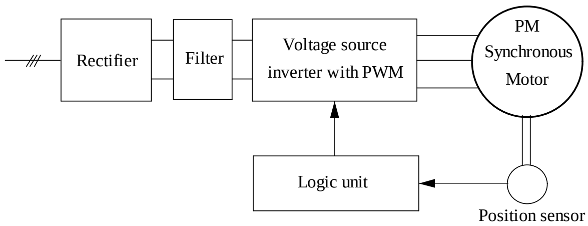

Fig.1. Block diagram of PM synchronous motor drive

The system consists of a non controlled rectifier supplying through filter a DC voltage to the input of the voltage source inverter. The inverter supplies a three-phase variable frequency and voltage to the stator synchronous motor terminals. The feedback loop consists of a shaft position sensor and logic unit which produces the three-phase gating signals and also controls the power angle. Since the inverter frequency is derived from the rotor, the motor cannot lose synchronism. Also, with position feedback control, no hunting occurs.

3. Inverter mathematical model

In Fig.2 a basic three-phase bridge inverter connection is shown. The basic output relations of the inverter output voltage were developed in [1] and [2]. Assuming the inverter's output voltage is controlled by PWM of the constant unipolar modulation frequency.

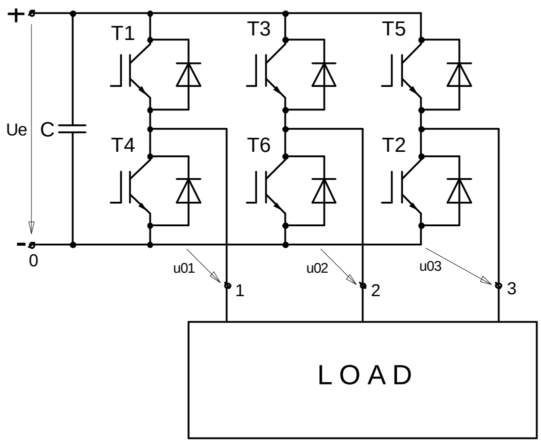

Fig.2. Basic three-phase bridge inverter

To build the mathematical model of the inverter, the Fourier complex series were used. The waveform of the fundamental inverter branch voltage u_{01} can be expressed as:

$$u_{01} = \frac{U_e}{2} + U_e \sum_{k=1}^{\infty} a_k \left(e^{jk\omega t} - e^{-jk\omega t} \right) \tag{1}$$

a_k present the Fourier coefficient of the form

$$a_k = \frac{\left[1 - (-1)^k\right]}{j2k\pi}$$

For the other two branch voltages shifted by $T/3$ or $2T/3$ respectively.

Let's denote:

$$e^{-j2\pi/3} = c_k; \quad e^{j2\pi/3} = c_k^{-1};$$

and

$$e^{-ja_k 4\pi/3} = c_k^2; \quad e^{ja_k 4\pi/3} = c_k^{-2};$$

For the other two branches the following relations are valid

$$\begin{aligned} u_{02} &= \frac{U_e}{2} + U_e \sum_{k=1}^{\infty} a_k \left(c_k e^{jk\omega t} - c_k^{-1} e^{-jk\omega t} \right) \\ u_{03} &= \frac{U_e}{2} + U_e \sum_{k=1}^{\infty} a_k \left(c_k^2 e^{jk\omega t} - c_k^{-2} e^{-jk\omega t} \right) \end{aligned} \tag{2}$$

These branch voltages are modulated by a modulation signal of the constant frequency f_1 and variable the on/off ratio. The modulation signal can be expressed as

$$u_m = a_0 + \sum_{m=1}^{\infty} \left[A_m e^{jk\omega_1 t} + B_m e^{jk\omega_1 t} \right] \tag{3}$$

The Fourier coefficients are expressed as follow

$$a_0 = \frac{\alpha}{2\pi}$$

$$A_m = \frac{\sin(k\alpha)}{2k\pi} + \frac{1 - \cos(k\alpha)}{j2k\pi}$$

$$B_m = \frac{\sin(k\alpha)}{2k\pi} - \frac{1 - \cos(k\alpha)}{j2k\pi}$$

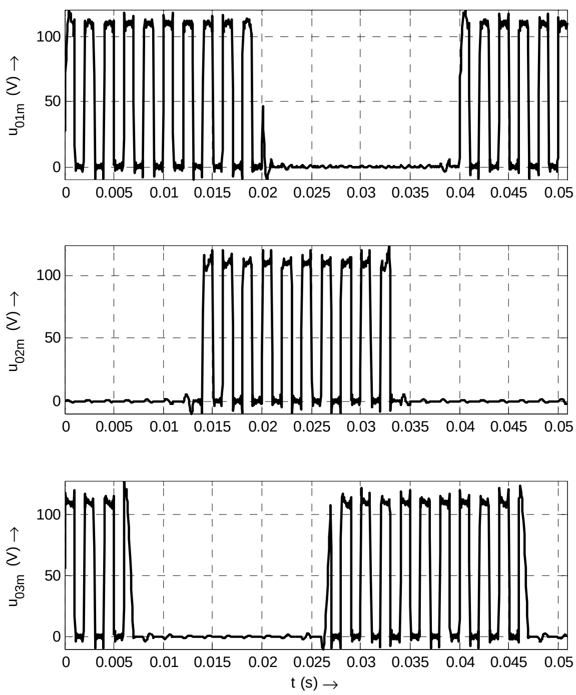

Fig.3. The branch modulated voltages

In the Fig.3 are given the plot of modulated branch voltages for fundamental frequency of $f = 50Hz$ and modulation frequency of $f_1 = 500Hz$. The waveforms were calculated on the base of equations (4).

For the modulated branch voltages the following equation are valid:

$$u_{01m} = U_e \left\{ \frac{1}{2} a_0 + \frac{1}{2} \sum_{m=1}^{\infty} \left(A_m e^{jm\omega_1 t} + B_m e^{-jm\omega_1 t} \right) \right.$$
$$+ a_0 \sum_{k=1}^{\infty} a_k \left(e^{jk\omega t} + e^{-jk\omega t} \right) +$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\infty} a_k A_m \left[e^{j(k\omega + m\omega_1)t} - e^{-j(k\omega - m\omega_1)t} \right] +$$
$$\left. a_k B_m \left[e^{j(k\omega - m\omega_1)t} - e^{-j(k\omega + m\omega_1)t} \right] \right\}$$
$$u_{02m} = U_e \left\{ \frac{1}{2} a_0 + \frac{1}{2} \sum_{m=1}^{\infty} \left(A_m e^{jm\omega_1 t} + B_m e^{-jm\omega_1 t} \right) \right.$$
$$+ a_0 \sum_{k=1}^{\infty} a_k \left(c_k e^{jk\omega t} + c_k^{-1} e^{-jk\omega t} \right) +$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\infty} a_k A_m \left[c_k e^{j(k\omega + m\omega_1)t} - c_k^{-1} e^{-j(k\omega - m\omega_1)t} \right] +$$
$$\left. a_k B_m \left[c_k e^{j(k\omega - m\omega_1)t} - c_k^{-1} e^{-j(k\omega + m\omega_1)t} \right] \right\}$$
$$u_{03m} = U_e \left\{ \frac{1}{2} a_0 + \frac{1}{2} \sum_{m=1}^{\infty} \left(A_m e^{jm\omega_1 t} + B_m e^{-jm\omega_1 t} \right) \right.$$
$$+ a_0 \sum_{k=1}^{\infty} a_k \left(c_k^2 e^{jk\omega t} + c_k^{-2} e^{-jk\omega t} \right) +$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\infty} a_k A_m \left[c_k^2 e^{j(k\omega + m\omega_1)t} - c_k^{-2} e^{-j(k\omega - m\omega_1)t} \right] + \tag{4}$$
$$\left. a_k B_m \left[c_k^2 e^{j(k\omega - m\omega_1)t} - c_k^{-2} e^{-j(k\omega + m\omega_1)t} \right] \right\}$$

The line voltages on the inverter's terminal are given by relations:

$$u_{12} = u_{01m} - u_{02m}$$
$$u_{23} = u_{02m} - u_{03m} \tag{5}$$
$$u_{31} = u_{03m} - u_{01m}$$

Assuming the stator winding is wye connected with insulated neutral node. The sum of the phase currents must be equal to zero. If the load is balanced, for the phase voltages, the following equations are valid:

$$u_1 = \frac{1}{3}(u_{12} - u_{31}) = \frac{1}{3}(2u_{01m} - u_{02m} - u_{03m})$$
$$u_2 = \frac{1}{3}(u_{23} - u_{12}) = \frac{1}{3}(2u_{02m} - u_{03m} - u_{01m}) \tag{6}$$
$$u_3 = \frac{1}{3}(u_{31} - u_{23}) = \frac{1}{3}(2u_{03m} - u_{01m} - u_{02m})$$

On the base of the equations (4) and (6), the following relations can be obtained:

$$u_1 = \frac{U_e}{3} \left\{ a_0 \sum_{k=1}^{\infty} a_k \left(A_{k1} e^{jk\omega t} - B_{k1} e^{-jk\omega t} \right) + \right.$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\omega} a_k A_m \left[A_{k1} e^{j(k\omega + m\omega_1)t} - B_{k1} e^{-j(k\omega - m\omega_1)t} \right] +$$
$$\left. a_k B_m \left[A_{k1} e^{j(k\omega - m\omega_1)t} - B_{k1} e^{-j(k\omega + m\omega_1)t} \right] \right\}$$
$$u_2 = \frac{U_e}{3} \left\{ a_0 \sum_{k=1}^{\infty} a_k \left(A_{k2} e^{jk\omega t} - B_{k2} e^{-jk\omega t} \right) + \right.$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\omega} a_k A_m \left[A_{k2} e^{j(k\omega + m\omega_1)t} - B_{k2} e^{-j(k\omega - m\omega_1)t} \right] +$$
$$\left. a_k B_m \left[A_{k2} e^{j(k\omega - m\omega_1)t} - B_{k2} e^{-j(k\omega + m\omega_1)t} \right] \right\}$$
$$u_3 = \frac{U_e}{3} \left\{ a_0 \sum_{k=1}^{\infty} a_k \left(A_{k3} e^{jk\omega t} - B_{k3} e^{-jk\omega t} \right) + \right.$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\omega} a_k A_m \left[A_{k3} e^{j(k\omega + m\omega_1)t} - B_{k3} e^{-j(k\omega - m\omega_1)t} \right] +$$
$$\left. a_k B_m \left[A_{k3} e^{j(k\omega - m\omega_1)t} - B_{k3} e^{-j(k\omega + m\omega_1)t} \right] \right\} \tag{7}$$

With the coefficients:

$$A_{k1} = 2 - c_k - c_k^2; \quad B_{k1} = 2 - c_k^{-1} - c_k^{-2};$$
$$A_{k2} = 2c_k - c_k^2 - 1; \quad B_{k2} = 2c_k^{-1} - c_k^{-2} - 1;$$
$$A_{k3} = 2c_k^2 - 1 - c_k; \quad B_{k3} = 2c_k^{-2} - 1 - c_k^{-1};$$

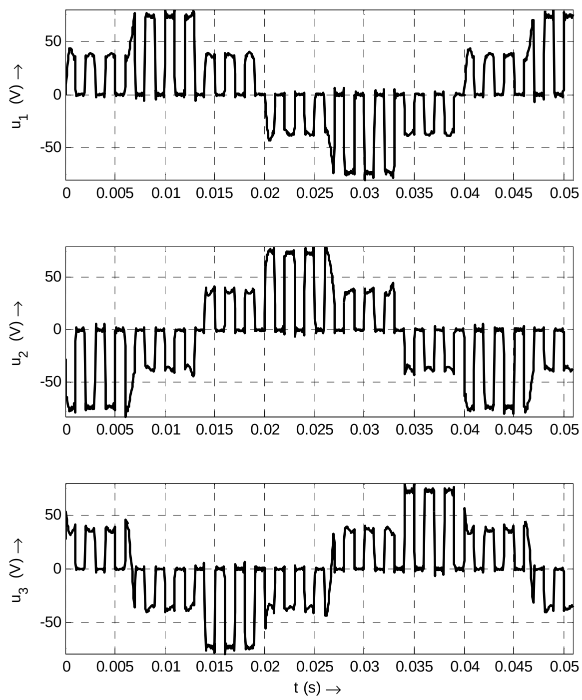

Fig.4. Plot of the phase voltages

In the Fig.4 are shown the waveforms of the phase voltages, calculated on the base of equations (9).

4. Synchronous motor model

The permanent magnet synchronous motor can be approximately modeled by means of the single-phase equivalent circuit given in Fig. 5.

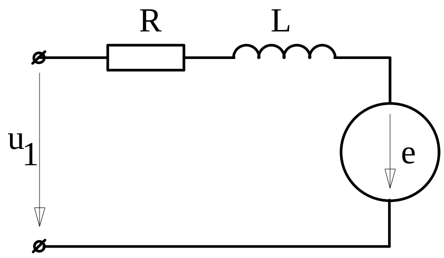

Fig.5 Single-phase simplified equivalent circuit

Input supply voltage u_1 presents output inverter phase voltage. Induced electromagnetic force e is supposed to be sinusoidal with the same pulsation as fundamental of the supply voltage. Stator winding resistance R and synchronous inductance L are assumed to be constant.
For the instantaneous value of the induced electromagnetic force the following equation is valid:

$$e = E\sin\left(\omega t-\beta\right)=\frac{E}{2j}\left[e^{j(\omega t-\beta)}-e^{-j(\omega t-\beta)}\right] \quad (8)$$

Where: E – is the magnitude of the induced voltage; β – is the power angle;

The other two induced voltages are shifted by $T/3$ or $2T/3$ respectively.

This voltage depends on the permanent magnet flux ϕ, the frequency or speed of rotor rotation ω_m and the machine construction.
The induced electromagnetic force magnitude can be expressed

$$E = K\phi\omega_m \quad (9)$$

For each phase of the motor the following voltage equations are valid:

$$u_1 = Ri_1 + L\frac{di_1}{dt}+\frac{E}{2j}\left[e^{j(\omega t-\beta)}-e^{-j(\omega t-\beta)}\right]$$

$$u_2 = Ri_2 + L\frac{di_2}{dt}+\frac{E}{2j}\left[e^{j(\omega t-2\pi/3-\beta)}-e^{-j(\omega t-2\pi/3-\beta)}\right]$$

$$u_3 = Ri_3 + L\frac{di_3}{dt}+\frac{E}{2j}\left[e^{j(\omega t-4\pi/3-\beta)}-e^{-j(\omega t-4\pi/3-\beta)}\right]$$

(10)

The supply voltages u_1, u_2, u_3 are given by the equations (7).

5. Phase current calculation

The motor line current waveforms can be calculated on the base of equations (10) and (7).

$$i_1 = \frac{U_e}{3}\left\{a_0\sum_{k=1}^{\infty}a_k\left(\frac{A_{k1}e^{jk\omega t}}{R+jk\omega L}-\frac{B_{k1}e^{-jk\omega t}}{R-jk\omega L}\right)+\right.$$

$$\sum_{k=1}^{\infty}\sum_{m=1}^{\omega}a_k A_m\left[\frac{A_{k1}e^{j(k\omega+m\omega_1)t}}{R+j\left(k\omega+m\omega_1\right)L}-\right.$$

$$\left.\frac{B_{k1}e^{-j(k\omega-m\omega_1)t}}{R-j\left(k\omega-m\omega_1\right)L}\right]+$$

$$\left.a_k B_m\left[\frac{A_{k1}e^{j(k\omega-m\omega_1)t}}{R+j\left(k\omega-m\omega_1\right)L}-\frac{B_{k1}e^{-j(k\omega+m\omega_1)t}}{R-j\left(k\omega+m\omega_1\right)L}\right]\right\}-$$

$$\frac{E}{2j}\left[\frac{e^{j(\omega t-\beta)}}{R+j\omega L}+\frac{e^{-j(\omega t-\beta)}}{R-j\omega L}\right]$$

$$i_2 = \frac{U_e}{3}\left\{a_0\sum_{k=1}^{\infty}a_k\left(\frac{A_{k2}e^{jk\omega t}}{R+jk\omega L}-\frac{B_{k2}e^{-jk\omega t}}{R-jk\omega L}\right)+\right.$$

$$\sum_{k=1}^{\infty}\sum_{m=1}^{\omega}a_k A_m\left[\frac{A_{k2}e^{j(k\omega+m\omega_1)t}}{R+j\left(k\omega+m\omega_1\right)L}-\right.$$

$$\left.\frac{B_{k2}e^{-j(k\omega-m\omega_1)t}}{R-j\left(k\omega-m\omega_1\right)L}\right]+$$

$$\left.a_k B_m\left[\frac{A_{k2}e^{j(k\omega-m\omega_1)t}}{R+j\left(k\omega-m\omega_1\right)L}-\frac{B_{k2}e^{-j(k\omega+m\omega_1)t}}{R-j\left(k\omega+m\omega_1\right)L}\right]\right\}-$$

$$\frac{E}{2j}\left[\frac{e^{j(\omega t-2\pi/3-\beta)}}{R+j\omega L}+\frac{e^{-j(\omega t-2\pi/3-\beta)}}{R-j\omega L}\right]$$

$$i_3 = \frac{U_e}{3}\left\{a_0\sum_{k=1}^{\infty}a_k\left(\frac{A_{k3}e^{jk\omega t}}{R+jk\omega L}-\frac{B_{k3}e^{-jk\omega t}}{R-jk\omega L}\right)+\right.$$

$$\sum_{k=1}^{\infty}\sum_{m=1}^{\omega}a_k A_m\left[\frac{A_{k3}e^{j(k\omega+m\omega_1)t}}{R+j\left(k\omega+m\omega_1\right)L}\right.$$

$$\left.-\frac{B_{k3}e^{-j(k\omega-m\omega_1)t}}{R-j\left(k\omega-m\omega_1\right)L}\right]+$$

$$\left.a_k B_m\left[\frac{A_{k3}e^{j(k\omega-m\omega_1)t}}{R+j\left(k\omega-m\omega_1\right)L}-\frac{B_{k3}e^{-j(k\omega+m\omega_1)t}}{R-j\left(k\omega+m\omega_1\right)L}\right]\right\}-$$

$$\frac{E}{2j}\left[\frac{e^{j(\omega t-4\pi/3-\beta)}}{R+j\omega L}+\frac{e^{-j(\omega t-4\pi/3-\beta)}}{R-j\omega L}\right]$$

(11)

6. Torque ripples calculation

With the simplified equivalent circuit shown in Fig.5, the instantaneous phase electromagnetic torque waveforms can be calculated.

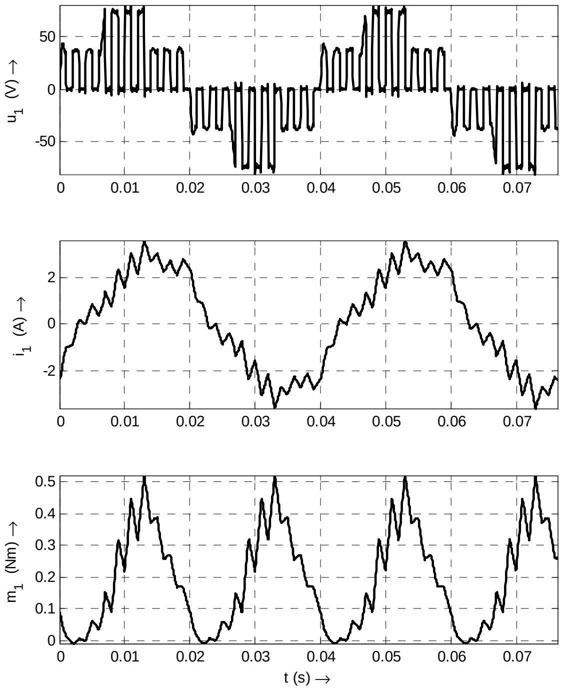

Fig.6 Calculated quantities

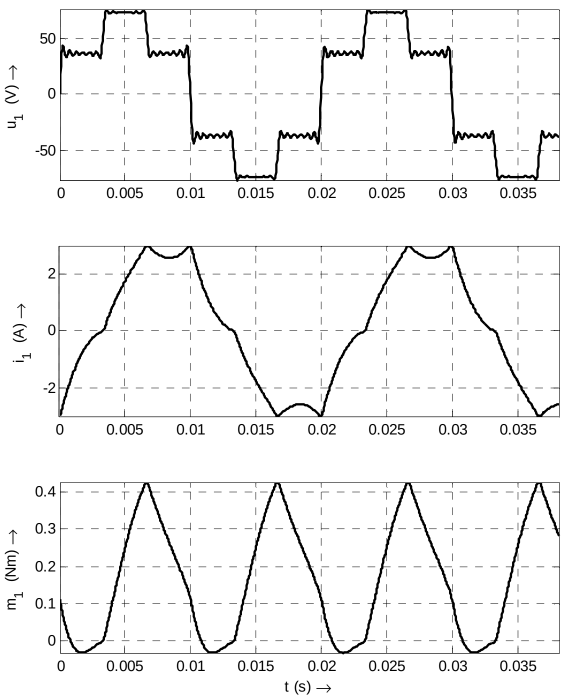

Fig.7 Calculated quantities

The phase electromagnetic torque is the ratio of the phase electromagnetic powers and the synchronous mechanical speed for each of the phases.

$$m_1 = \frac{e_1 i_1}{\omega_m}; \quad m_2 = \frac{e_2 i_2}{\omega_m}; \quad m_3 = \frac{e_3 i_3}{\omega_m}; \tag{12}$$

The induced electromagnetic torque of the motor is given as a sum of the phase's torque.

$$m = \sum_{k=1}^{3} m_k = \sum_{k=1}^{3} \frac{e_k i_k}{\omega_m} \tag{13}$$

In the Fig.6 is given a plot of phase voltage, current and torque for voltage frequency of $f = 25Hz$ and modulation frequency of $f_1 = 500Hz$. The DC supply voltage has a value $U_e = 110V$. Fig. 8 shows the plot of total induced electromagnetic torque.

In the Fig.7 is given a plot for voltage frequency of $f = 50Hz$ and full voltage. In the Fig. 9 is given a plot of total electromagnetic torque.

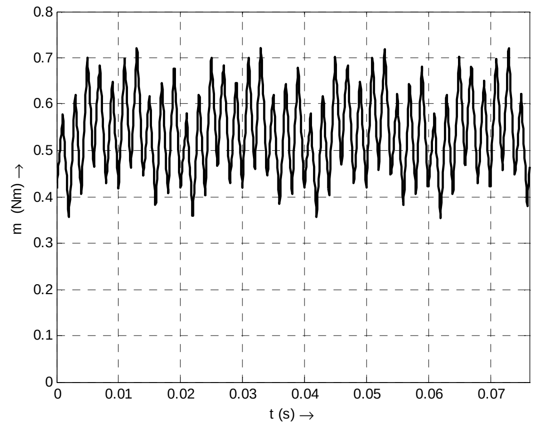

Fig.8 Plot of electromagnetic torque

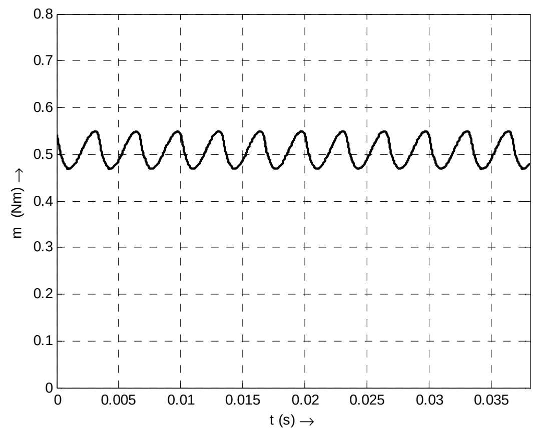

Fig.9 Plot of electromagnetic torque

7. Conclusion

A steady-state analysis of a permanent magnet synchronous motor drive with voltage-source inverter anf PWM of the input voltage has been presented. The harmonic distortion in the line current of the motor is a strong function of the power angle β . An analytical expression has been obtained for the line currents of the motor. The output electromagnetic torque ripples waveform has been calculated.

Acknowledgments

The financial support of the Slovak Research and Development Agency under the contract No. APVV -0510-06 is acknowledged.

Special thanks to my wife RNDr. Mária Záskalicka for voluntary help in mathematical problems.

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Author

Prof. Ing. Pavel Záskalický, PhD., Department of Electical, Mechatronic and Industrial Engineering, Faculty of Electrical Engineering and Informatics, Technical University o Košice, Letná 9, 04001 Košice, Slovakia. Tel: +421556022272, email:pavel.zaskalicky@tuke.sk

Reviewer

Prof. dr hab. inż. Mieczysław Ronkowski