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MULTILEVEL CONVERTER IN LOW POWER DRIVE APPLICATIONS

ABSTRACT *The paper describes a method of generating high quality AC voltage waveforms in low power AC drives. Voltage waveforms are generated by use of multilevel inverters. This technique can be applied particularly in small devices, servomechanisms, robotics. An approach to the synthesis of AC voltage waveforms using analytical methods like Fourier and wavelets transform is presented. The analysis of the obtained waveforms is included to the paper as well as a proposal of adequate converter circuitry.*

Keywords: *multilevel converter, electronic power devices, wavelets, waveform analysis*

1. INTRODUCTION

There are many applications i.e. AC drives, uninterruptible power sources or distributed power generation systems, where the demand is to generate 0÷50 Hz (or even more) sinusoidal voltage waveforms of relatively high quality. The voltage obtained from DC sources like storage accumulators, photovoltaic farms or fuel cells are converted to AC using power electronics converters. The quality

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of generated waveforms, especially THD (*Total Harmonic Distortion*) factor, should comply with the appropriate standards. The commonly used solution for this purpose is two level VSI (*Voltage Source Inverter*), controlled using PWM (*Pulse Width Modulation*) method. But two level inverters have serious limitations: lower power range of below 100 kW and well known disadvantages. Some of them could be reduced by use of multilevel converters and amplitude modulation method. Generally multilevel converters are destined to high power and high voltage applications. The main goal of the paper is to take full advantage of multilevel waveform synthesis and apply it to low or very low power applications, useful in e.g. micro drives. The mathematical approach to the adequate control strategy of multilevel converters has been presented in [1, 2, 3, 4, 5]. The paper presents a shortened approach of two mathematical methods. They permit to perform a stepped waveform synthesis according to the best approximation of the output sine wave and are based on Fourier series and wavelet transform. The resulting in properties create a comfortable mathematical tool which could be applied in inverter circuit designing.

2. STANDARD WAVEFORMS SYNTHESIS BASED ON FOURIER SERIES

The scaling function $\varphi_n(x)$ is defined as follows:

$$\varphi_n(x) = \varphi(x - n\alpha) \quad \text{for} \quad n = \ldots,-2,-1,0,1,2,\ldots \tag{1}$$

The equation defines a set of rectangular pulses of unitary amplitude and angle duration equal to α. The pulse position on x axis depends on parameter n. A few examples of the scaling functions are presented in Fig. 1.

In an interval $x \in \langle a,b)$, which length is $k\alpha$ ($k \geq 1$), functions $\varphi_n(x)$ satisfy two conditions:

$$\|\varphi_n\|^2 = \int_a^b \varphi_n{}^2(x)\mathrm{d}x = \alpha \qquad \text{and} \quad \int_a^b \varphi_k(x)\varphi_m(x)\mathrm{d}x = 0 \quad \text{if} \quad k \neq m \tag{2}$$

therefore the notation (1) creates a set of orthogonal functions called an orthogonal base. Since all functions $\varphi_n(x)$ have their norms $\|\varphi_n\|^2$ equal to α, this base is called an orthonormal base.

The expansion of a function $f(x)$ in a generalized Fourier series related to the set of scaling functions (φ_n) is as follows:

$$f(x) = \sum_{n=0}^{\infty} c_n \varphi_n(x) \tag{3}$$

where

$$c_n = \frac{(f, \varphi_n)}{\|\varphi\|^2} = \frac{\int_a^b f(x)\varphi_n(x)\mathrm{d}x}{\alpha}$$

The expansion (3) is valid for any function $f(x) \in L^2_{<a,b>}$ The Fourier series contains an infinite number of elements and makes it possible to approximate a function $f(x)$ by use of an infinite set (a sum) of adequately scaled functions $\varphi_n(x)$. Particularly it is possible to expand a function $f(x) = \sin(x)$ by summing an infinite set of rectangular pulses. It is in contradistinction to typical application of the Fourier series where any function $f(x)$ is expanded as a set of harmonics.

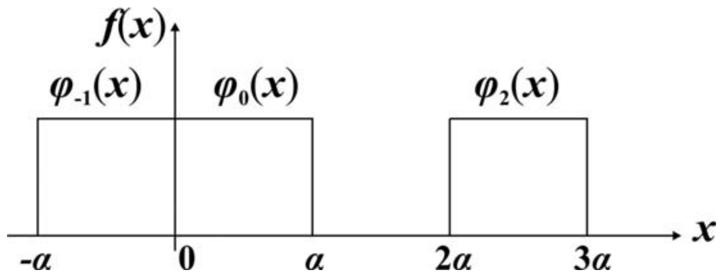

Fig. 1. Examples of the scaling functions:
$\varphi_0(x) = \varphi(x)$, $\varphi_{-1}(x)$, $\varphi_2(x)$

According to (3) and (4) the expansion of $\sin(x)$ in the interval $x \in < a, b)$ is given as:

$$\sin(x) = \sum_{n=0}^{n=\infty} \left\{ \frac{\int_a^b \sin(x)\varphi_n(x)\mathrm{d}x}{\alpha} \varphi_n(x) \right\} \quad x \in < a, b > \tag{5}$$

The expression (5) defines a series of consecutive rectangular pulses represented by functions $\varphi_n(x)$. Amplitudes of pulses are different and determined after calculation of the integral. This statement can be applied for composition of sine waveforms in power electronics. Rectangular pulses represent an essential shape of output voltage (current) of an inverter. Naturally the composition of stepped waveforms using rectangular pulses has been applied in many applications but it concerned a "vertical" addition of waveforms. The resulting phase voltage was synthesized by the addition of the voltages generated by different cells of a cascaded inverter. The presented proposal relates to the addition of pulses "along *x axis*" or in a time scale. Consecutive pulses form the resulting voltage or current of the converter.

Practically in power electronics applications, the approximation of a sine wave can be realized using a finite number N of the series members and natural aspiration of designers is to utilize the possibly lowest N number. The accuracy of approximation depends on it. Its numerical value can be measured in different ways. In mathematics the accuracy of approximation is defined as the average square error δ, a very useful criterion destined to that purpose.

In power electronics the most important criterion of the accuracy or rather the quality of approximated waveforms is THD factor. The example of a very simple approximation has been presented in Fig. 2. The presented stepped waveform has been obtained after approximation based on the set defined according to (1) and (2).

The results of Fourier approximation for selected N are collected in Table 1. The results of average square errors are presented in the column denoted by δ_N. The symbol F_N denotes the number of steps in the scanned interval – herein it is the interval $<0,2\pi>$. The parameter $N_{|F_N|}$ denotes the number of demanded supply DC voltage or current sources. This value $N_{|F_N|}$ is a very important parameter of multilevel converters.

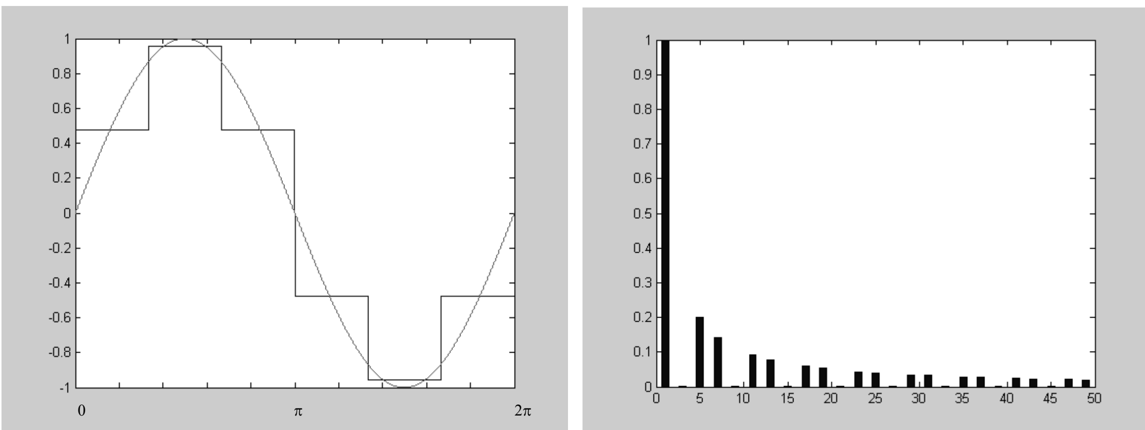

Fig. 2. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 6 ($\alpha = \pi/3$) and the spectrum analysis of the waveform

Figure 2 presents the stepped waveform obtained using a very low approximation level. The ratio of the measures of steps is equal to 2 and it is the same as in a three phase inverter with connected load and controlled by an adequate set of rectangular waves. In order to produce such a waveform in one phase applications the converter needs only two DC sources easy to get e. g. by dividing one DC supply voltage.

Another example of a much more better approximation has been presented in Fig. 3. The figure presents the calculated stepped waveform obtained for $f_{N=24}(x)$ (24 pulses in one period). The THD factor has been decreased up to value of 7,46%.

a) b) **THD = 7,46%**

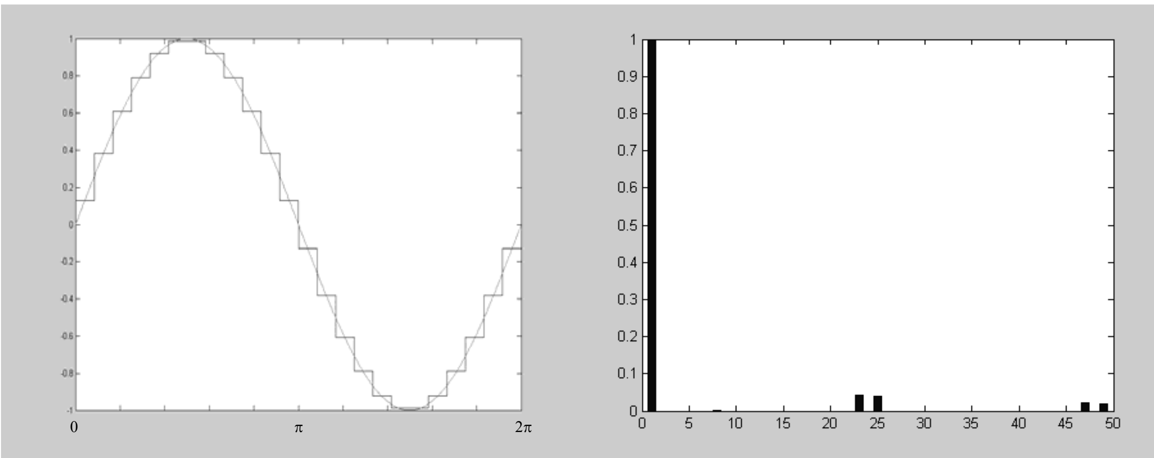

Fig. 3. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 24 ($\alpha = \pi/12$) and the spectrum analysis of the waveform

TABLE 1
The parameters of standard Fourier approximation for different N

F_N	α	$N_{\lvert F_N \rvert}$	σ_N	THD
$F_{N=2}$	π	1	0,0947	48,37%
$F_{N=6}$	π/3	2	0,0440	31,09%
$F_{N=12}$	π/6	3	0,0113	15,23%
$F_{N=16}$	π/8	4	0,0064	11,41%
$F_{N=24}$	π/12	6	0,0028	7,63%

3. WAVEFORMS SYNTHESIS BASED ON WAVELETS

The wavelets is a term for mathematical functions, which allow the analysis of signals in different time scale and with different resolution. Thanks to this adjustable „scope of the view” the wavelets can be used to distinguish and analyse as well small and big details of the investigated process. Especially they are useful in analyse of discontinuous processes or processes with step level changing. The wavelets application are in many not directly related areas like seismology, video analysis, quantum mechanics or electronic. The term „wawelets” is the direct translation of french term *„ondelettes”* or *„petites ondes”*, which means „little waves”.

Till now the wavelets have been used mainly for analysis of processes or signals based on decomposition of the elements of the processes. The following considerations will prove that wavelets can be also useful in composition of the power electronics signals and structures.

For this purpose the Haar wavelets have been adopted. The fundamental Haar wavelet $\psi(t)=\psi_{00}(t)$ can be constructed by transforming the following scaling function $\varphi(t)$:

$$\varphi(t)=\begin{Bmatrix} 1 & \text{for} & 0\le t<1, \\ 0 & \text{for} & \text{other} \quad t \end{Bmatrix} \tag{6}$$

The composition of two consecutive scaling functions $\varphi(2t)$ and $\varphi(2t-1)$:

$$\varphi(2t)=\begin{Bmatrix} 1 & \text{for} & 0\le t<0{,}5), \\ 0 & \text{for} & \text{other} \quad t \end{Bmatrix} \qquad \varphi(2t-1)=\begin{Bmatrix} 1 & \text{for} & 0{,}5\le t<1), \\ 0 & \text{for} & \text{other} \quad t \end{Bmatrix}$$

creates the Haar wavelet: $\psi(t)=\varphi(2t)-\varphi(2t-1)$ which can be described as following:

$$\psi(t)=\begin{Bmatrix} 1 & \text{for} & 0\le t<\frac{1}{2}, \\ -1 & \text{for} & \frac{1}{2}\le t<1, \\ 0 & \text{for} & \text{other} \quad t \end{Bmatrix} \tag{7}$$

The scaling functions and the fundamental Haar wavelet are presented in Fig. 4.

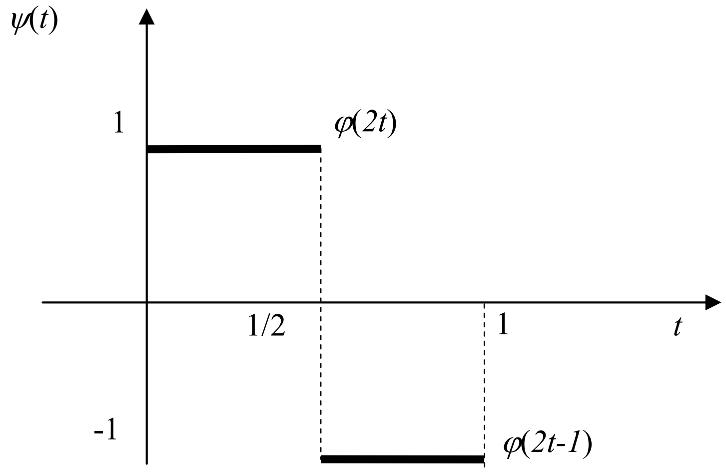

Fig. 4. The scaling functions $\varphi(2t)$ and $\varphi(2t-1)$ and the fundamental Haar wavelet $\psi(t)=\psi_{00}(t)$

By introducing two parameters: m – scale factor and n – displacement factor the generalised Haar transform is obtained:

$$\psi_{mn}(t)=\frac{1}{\sqrt{2^m}}\psi\left(2^{-m}t-n\right) \quad \text{for} \quad m,n=...,-2,-1,0,1,2,... \tag{8}$$

The scale factor m – settles the width and amplitude of the wavelet, and the displacement factor n – settles the wavelet position on time axis. The fundamental Haar wavelet corresponds to the factors: $m = 0$ and $n = 0$ and can be denoted as $\psi(t)=\psi_{00}(t)$.

The Haar wavelet form is similar to the form of the voltage or current pulse that can be obtained using simple one-phase inverter. The displacement and width of the wavelet can be freely controlled. Thanks to these properties it is possible to apply wavelets in power electronics i.e. to form the output waveforms of multilevel converters. For the one-phase power electronics purpose the scaling function $\varphi(x)$ is defined in interval $x \in <0, 2\pi)$:

$$\varphi(x)=\begin{Bmatrix} 1 & \text{for} & 0 \le x < 2\pi, \\ 0 & \text{for} & \text{other} \;\; x \end{Bmatrix} \tag{9}$$

The fundamental proposed wavelet is defined like the Haar one:

$$\psi(x) = \varphi(2x) - \varphi[2(x - \pi)]$$

and can be expressed as following:

$$\psi(x) = \begin{cases} 1 & \text{for} \quad 0 \le x < \pi, \\ -1 & \text{for} \quad \pi \le x < 2\pi, \\ 0 & \text{for} \quad \text{other} \quad x \end{cases} \tag{10}$$

The wavelet transform is defined as follows:

$$\psi_{mn}(x) = \psi\left[2^{-m}\left(x - n2^{m+1}\pi\right)\right] for\ m, n = ..., -2, -1, 0, 1, 2, ... \tag{11}$$

where $2^m 2\pi = 2^{m+1}\pi$ is a wavelet carrier.

The x axis position is defined as n-times $2^{m+1}\pi$ displacement. The m factor scales not the wavelet amplitude but the carrier. For the wavelets of different m and equal n the integral:

$$\int_0^{2\pi} \psi_{m_k n}(x)\psi_{m_l n}(x)\mathrm{d}x = 0 \quad \text{for} \quad k \ne l \tag{12}$$

because each wavelet with smaller carrier is contained in interval, in which the wavelet of bigger carrier is constant. The expression (12) confirms that all the wavelets $\psi_{mn}(x)$ are orthogonal in the interval $x \in <0, 2\pi)$.

Let the approximation of function $f_\psi(x)$ in interval $x \in <0, 2\pi)$ be a combination of a few selected wavelets with m = -3, -2, -1, 0. It can be denoted as a sum:

$$f_\psi(x) = \sum_{m=-3}^{m=0} \sum_{n=0}^{2^{-m}-1} a_{mn}\psi_{mn}(x) = \sum_{m=-3}^{m=0} \sum_{n=0}^{2^{-m}-1} f_{mn}(x)$$

where the functions $f_{mn}(x)$ are component wavelets of amplitude and phase defined by the factors a_{mn}:

$$a_{mn} = \frac{1}{N_m} \int_0^{2\pi} \sin(x)\psi_{mn}(x)\mathrm{d}x$$

The approximating function $f_\psi(x)$ presents itself a stepped waveform which approximates in a limited way function $f(x) = \sin(x),\ x \in <0,2\pi)$. The limitation depends on the number of wavelets components. Collecting more wavelets it is possible to obtain more precise output waveforms of the inverter. The results of wavelet approximation for consecutive functions are collected in Table 2.

TABLE 2
Parameters of wavelet approximation for functions $f_{\Psi 0}$, $f_{\Psi 1}$, $f_{\Psi 2}$, $f_{\Psi 3}$ i $f_{\Psi 4}$

$f_{\Psi k}$	A	$N_{\lvert f_{\psi k} \rvert}$	$\delta_{\psi k}$	THD	THD$_B$
$f_{\Psi 0}$	π	1	0, 0947	48, 37 %	79,93 %
$f_{\Psi 1}$	π /4	2	0, 0252	23, 06 %	22, 63 %
$f_{\Psi 2}$	π /8	3	0, 0091	13, 70 %	13, 83 %
$f_{\Psi 3}$	π /8	4	0, 0065	11, 44 %	8, 19 %
$f_{\Psi 4}$	π /16	8	0, 0016	5, 73 %	2,96 %

The comparison between Table 1 and Table 2 shows that the for the same number of independent voltage sources the THD factor varies depending on the used approximation method.

As an example a function $f(x) = \sin(x)$ approximated by use of wavelets $f_{00}(x)$, $f_{-2n}(x)$ and $f_{-3n}(x)$ is presented in Fig. 5.

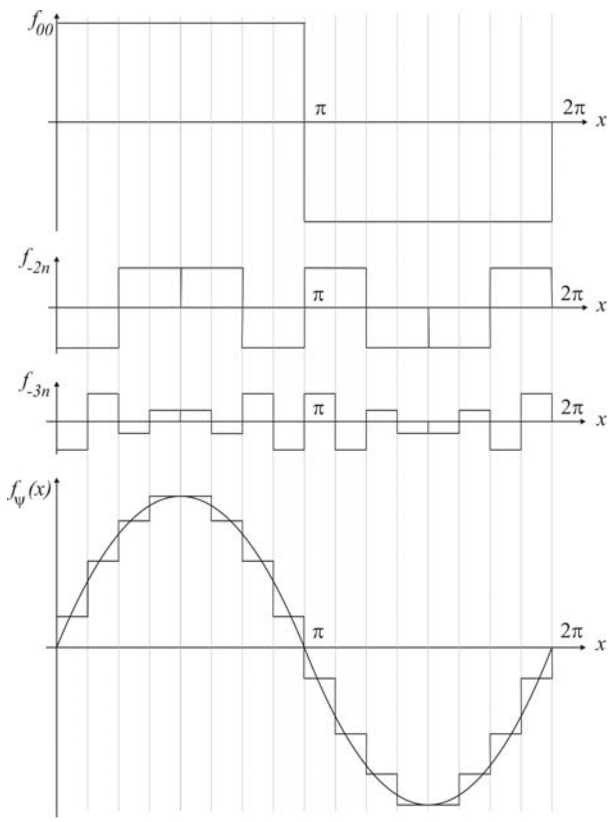

Fig. 5. The approximation of *f*(*x*) = sin(*x*) using wavelets $f_{00}(x)$, $f_{-2n}(x)$ and f_{-3n}

4. APPLICATIONS OF THEORETICAL MODELS IN AC DRIVES

The mathematical models presented in chapter 2 and 3 are very suitable in designing of multilevel inverters. This implies novel possibilities in designing of structures and controlof multilevel inverters. The Fourier model as well as the wavelet one are suitable in designing of cascade inverters. This idea is presented in Fig. 6. The converter consists of two one phase bridge inverters: $\boldsymbol{F1}\ (\boldsymbol{T1}, \boldsymbol{T2}, \boldsymbol{T3}, \boldsymbol{T4})$ i $\boldsymbol{F2}\ (\boldsymbol{T5}, \boldsymbol{T6}, \boldsymbol{T7}, \boldsymbol{T8})$ and a summing block – Σ.

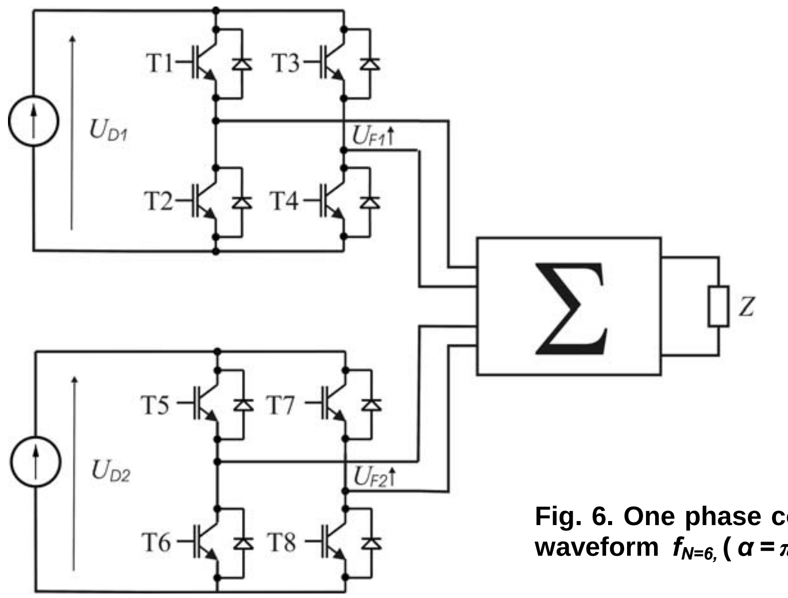

Fig. 6. One phase converter generating output waveform $f_{N=6,}$ ($\alpha = \pi/3$)

A simplified circuit realization of the converter has been presented in Fig. 7.

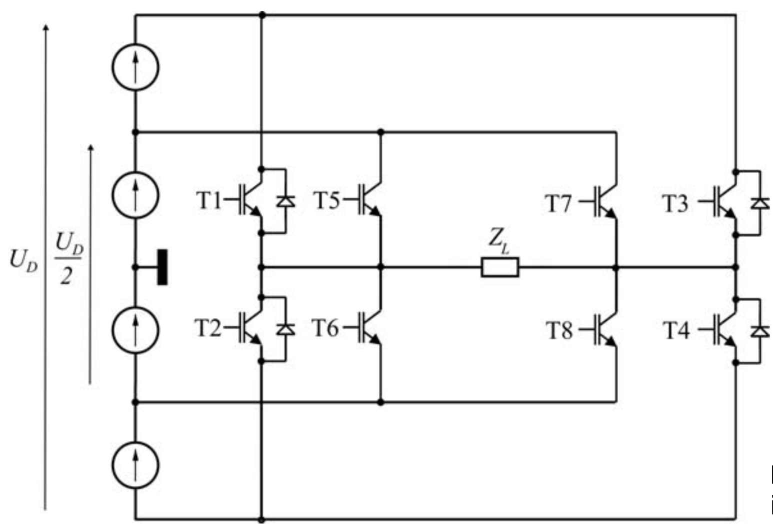

Fig. 7. Simplified three level inverter built on transistors

The control rule results in described above Fourier model idea permitting to construct an output waveform $f_{N=6}$. Another circuit solution is based on the wavelet model and is presented in Fig. 8. The converter is able to generate a four level output waveform $f_{\Psi 1}$. It consists of three one phase bridge inverters built respectively on transistors: $T1 \div T4$, $T5 \div T8$ i $T9 \div T12$. All inverters are connected in a cascade and are supplied from separate voltage sources. Supplying voltage levels are calculated according to the wavelet model description.

More detailed discussion of presented models is available in [6].

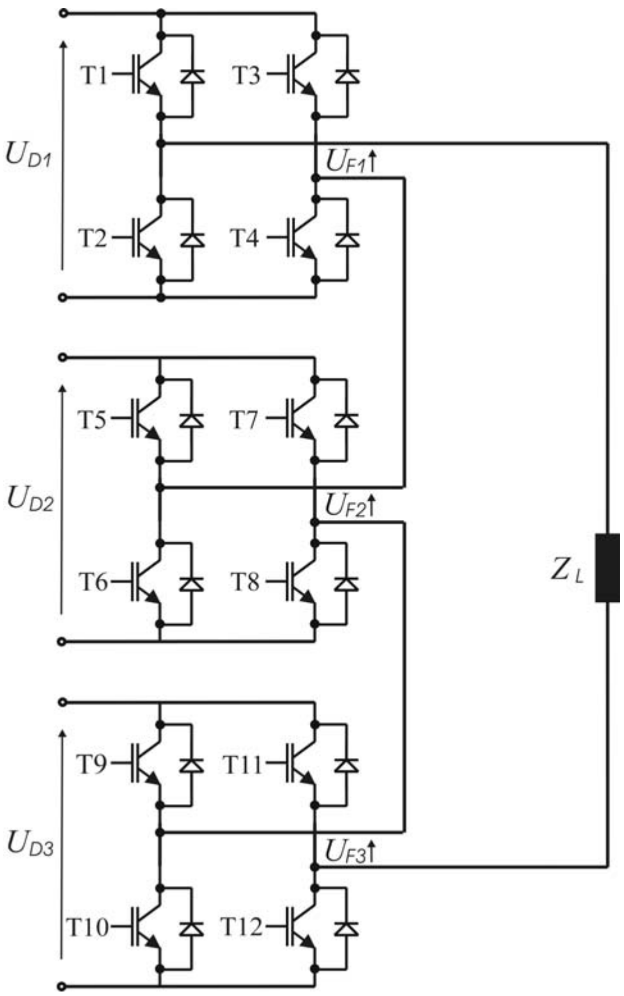

Fig.8. One phase four level inverter according to the idea of wavelet model

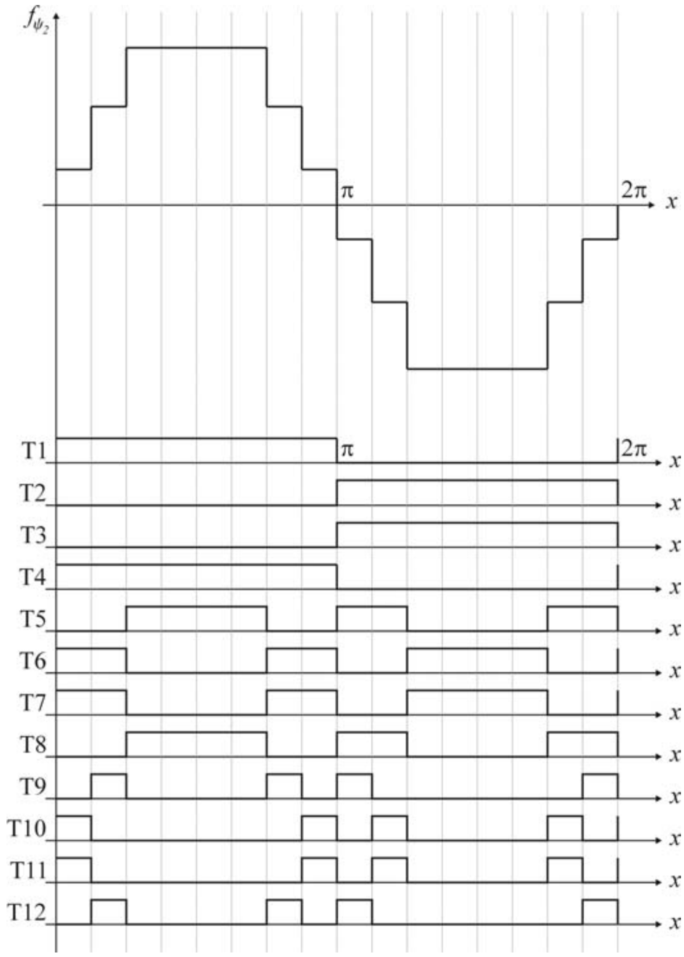

Fig. 9. The output waveform and control diagram of consecutive inverters

5. CONCLUSIONS

The objective of the paper was to describe a novel proposal of the AC stepped waveforms synthesis. The discussion proved the usefulness of such mathematical tools as alike Fourier series and wavelet transform. The Fourier series has been used to make a composition of the stepped waveform built from consecutive rectangular pulses. This proposal relates to the addition of pulses "along x axis" or in the time scale. It is in contradiction to the typical use of vertical addition of pulses applied largely in multilevel converters. It could be proved [6] that even in case of two DC sources it was possible to reduce significantly the harmonic content thanks to optimization of shape parameters. It can be useful in industrial applications.

The second proposal is based on wavelet transform as a handy mathematical tool to design multilevel converters (topology and control). The resulting coefficients of wavelets are proportional to DC sources needed in cascaded converter. The comparison between two methods of synthesis indicated the advantages of wavelet transform when there was a low number of DC sources.

Generally the possible application area belongs mainly to higher power converters especially in distributed power generation systems with DC sources like photovoltaic or fuel cells with discrete voltage levels. In the sources of this kind there is an easy way to group cells to obtain the desired voltage levels for the multilevel converter. This new solution becomes an attractive alternative to a traditional output filtering technique utilizing passive components. But in power electronics applications it is not useful to use a large number of component inverters because of economic and technical limitations. In low and very low power applications this number is not so much restricted. In micro power AC drives it is easy to compose a circuit built of increased number of component inverters. Such a multilevel inverter together with a control circuit can be realized in one integrated chip. Such a solution is extremely suitable for micro drives and it assures an excellent performance – large range of output frequency and very low THD factor. This technique can be applied particularly in small devices, servomechanisms and robot supply and drive component units.

Acknowledgements

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WIELOPOZIOMOWY PRZEKSZTAŁTNIK
DO ZASTOSOWAŃ W UKŁADACH NAPĘDOWYCH
MALEJ MOCY

J. IWASZKIEWICZ

STRESZCZENIE *W artykule opisano metodę generowania przebiegów napięcia przemiennego wysokiej jakości do układów napędowych małej mocy prądu przemiennego (AC). Przebiegi napięcia są generowane z wykorzystaniem wielopoziomowych falowników. Ta technika może być zastosowana szczególnie w małych urządzeniach, serwomechanizmach i robotyce. Przedstawiono metody syntezy przebiegów napięcia przemiennego z wykorzystaniem metod analitycznych, takich jak przekształcenie Fouriera i przekształcenie Falkowe.*

W artykule przedstawiono również analizę uzyskanych przebiegów oraz propozycje odpowiednich schematów przekształtników.