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MATHEMATICAL MODEL OF A SINGLE-PHASE INVERTER'S WITH VOLTAGE REGULATION

Abstract: The paper deals with mathematical model of single-phase bridge inverters which outputs voltage is controlled by a pulse-width notching. The transistors in the inverter circuit are turned on and off so as to produce zero periods of equal length, for unipolar modulation, or negative voltage for bipolar modulation. The D.C. source is being a fixed level of voltage. Suppose the notching frequency to be constant for all frequency of the fundamental squarewaves and to be fixed on 500 Hz. The output inverter's voltage is expressed by an analytical formula based on Fourier complex series. On the base of output voltage, the waveforms of the output currents are analytically calculated for different inverter's loads.

1. Introduction

The inverter permits to product an alternating voltage of the variable or constant frequency from a continuous voltage source. The load of the inverter can be either passive, without the alternating voltage source, or active where the alternating voltage source is present.

Basically, the single-phase and three-phase voltage invertors are distinguishing. Generally the alternating output voltage is controlled not only in frequency, but even in amplitude.

The inverters are often used in security supply devices. In this application the inverter is supply from accumulator, which is constantly loaded from the main. In a case of a main disruption, the accumulator guarantees alimentation for shorter or longer segment of time.

The invertors are likewise very often set in the board mains of the airplanes, traction engine and train wagons. Here are their supplies by accumulators and converts DC voltage on AC voltage of constant frequency.

The power electronic has developed two type circuits of the invertors. There are the centre-tapped circuit and bridge circuit. The centre-tapped circuit requires for its operation transformer. This one presents some inconvenient, especially in the low frequency area. For this reasons is the centre-tapped circuit very seldom used.

Against that, the bridge circuit is very frequently used in single-phase or three-phase version.

Earlier the invertors were building using conventional thyristors as switching elements. In this case the circuits require extensive commutation networks using capacitors and inductors to bring about turn-off. The earliest inverters developed used the conventional thyristors because that was the only device available. More recent developments in device technology have a number of new devices appropriate for inverter application.

The transistor family of devices is now very widely used. On the Fig. 1 is illustrated the use of the insulated gate bipolar transistor (IGBT). This device is being increasingly used in the both the single-phase and three-phase inverters. The IGBT is a voltage-controlled device and hence is more attractive in its control feature than the thyristors family.

2. Single-phase bridge inverter

2.1. Condition for an idealized operation study

For inverter's operation study at steady state we consider following idealized conditions:

Power switch, that means the switch can handle unlimited current and blocks unlimited voltage.

The voltage drop across the switch and leakage current through switch are zero.

The switch is turned on and off with no rise and fall times.

This assumption helps us to analyze a power circuit and helps us to build a mathematical model for the inverter at steady state.

In a real case, ideal switches do not exist. During switching transitions, there are significant switching losses associated with growing of voltage and current. These phenomena depend on several issues such characteristics of power switch, control signals, gate drives, stray parameters and operating points of the system.

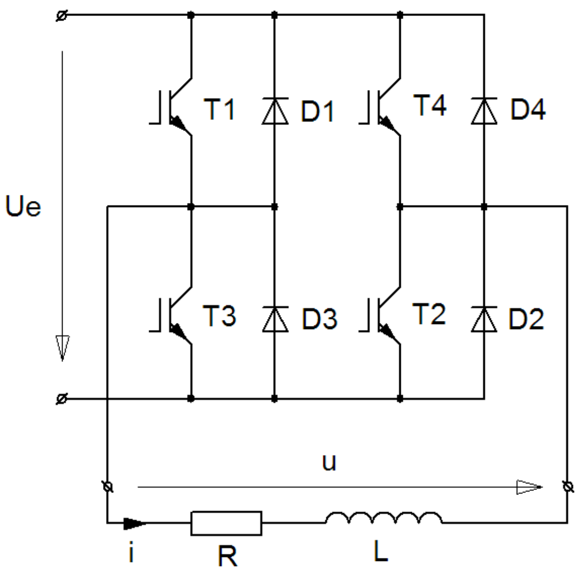

Fig. 1. Single-phase bridge inverter

2.2. Voltage control of inverter

Figure 1 shows a single-phase voltage source inverter using power transistors as the active elements. Since power transistors are self-commutating, no special commutation components are included in this circuit.

If the load of inverter were pure resistance, then alternate switching of transistors $\mathrm{T1,T2}$ and $\mathrm{T3,T4}$ will place the D.C. source in alternate senses across the load, giving a squarewave. However, with an inductive load, the current waveform is delayed, although the voltage wave is still square.

Control of the output voltage can be obtained from a fixed D.C. source by a introducing zero periods or opposite voltage into the squarewave. We talk about unipolar or bipolar mo-dulation. In the case of unipolar modulation, one of the conducting transistor is periodically switched on and off, so to produce zero periods of equal length. In case of bipolar modulation, both conducting transistors are periodically switched on and off, so the supply D.C. voltage across the load is periodically change the polarity.

3. Mathematical model

The transistors of the inverters are controlled so that each of pairs of transistors conducts for 180^{o} of the output cycle. The squarewave of the output voltage waveform can be mathematically expressed by a Fourier complex series

$$u_1 = U_e \sum_{k=1}^{\infty} c_k \left(e^{jk\omega t} - e^{-jk\omega t} \right) \tag{1}$$

The fundamental circular frequency is given by a relation

$$\omega = 2\pi f$$

Fourier coefficient is given

$$c_k = \frac{1}{jk\pi}\left[1-(-1)^k\right]$$

Fig. 2 shows the plot of the squarewave voltage calculated on the base of equation (1) for 50 Hz frequency.

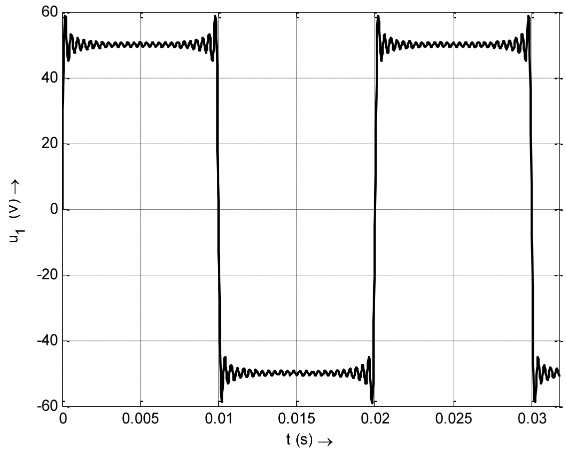

Fig. 2. Plot of the squarewave voltage

3.1. Unipolar modulation

In the case of unipolar modulation, the squarewave is modulated by a signal of the form

$$u_m = a_0 + \sum_{m=1}^{\infty} \left(b_m e^{jm\omega_1 t} - a_m e^{-jm\omega_1 t} \right) \tag{2}$$

Modulation circular frequency is given by a modulation frequency

$$\omega_1 = 2\pi f_1$$

The Fourier coefficients take a form

$$a_0 = 1 - \frac{\alpha}{2\pi}$$

$$b_m = \frac{j - j\cos m\alpha - \sin m\alpha}{2m\pi}$$

$$a_m = \frac{j - j\cos m\alpha + \sin m\alpha}{2m\pi}$$

While $\alpha \in \langle 0; 2\pi \rangle$ is a notching angle.

On the Fig. 3 is a plot of the modulation signal for unipolar modulation. Modulation frequency of 500 Hz is to be constant.

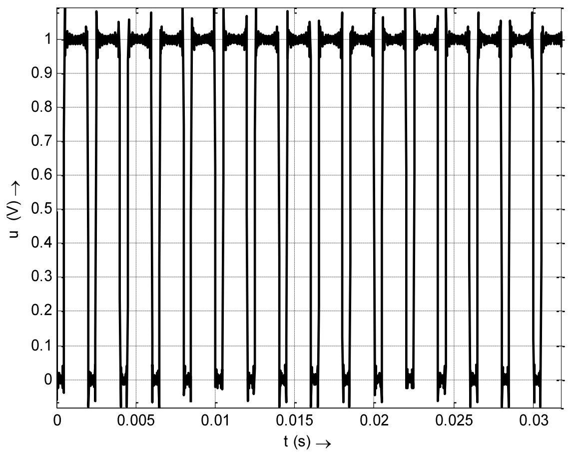

Fig. 3. Plot of the modulation signal for unipolar modulation

The inverter's output voltage is given as a product of a squarewave and modulation voltage and takes a form

$$u = U_e \left(\sum_{k=1}^{\infty} a_0 c_k \left[e^{jk\omega t} - e^{-jk\omega t} \right] + \right.$$
$$\sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \left\{ c_k b_m \left[e^{j(k\omega + m\omega_1)t} - e^{-j(k\omega - m\omega_1)t} \right] - \right. \tag{3}$$
$$\left. \left. c_k a_m \left[e^{j(k\omega - m\omega_1)t} - e^{-j(k\omega + m\omega_1)t} \right] \right\} \right)$$

Fig. 4 shows the calculated output waveform for unipolar modulation.

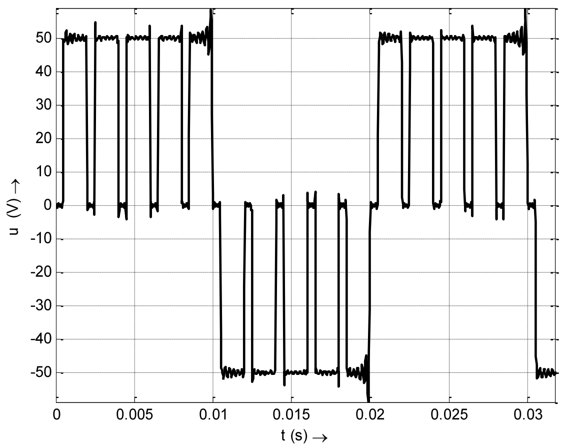

Fig. 4. Plot of the inverter's output voltage for unipolar modulation

3.2. Bipolar modulation

For bipolar modulation the voltage across the load periodically changes the polarity. The conducting transistors are periodically switched on and off. The modulation signal can by mathematically expressed by a formula (2), but the Fourier coefficients for bipolar modulation take a form

$$a_0 = 1 - \frac{\alpha}{\pi}$$
$$a_m = \frac{j - j\cos m\alpha - \sin m\alpha}{m\pi}$$
$$b_m = \frac{j - j\cos m\alpha + \sin m\alpha}{m\pi}$$

On the Fig. 5 is shown the plot of the modulation signal for bipolar modulation.

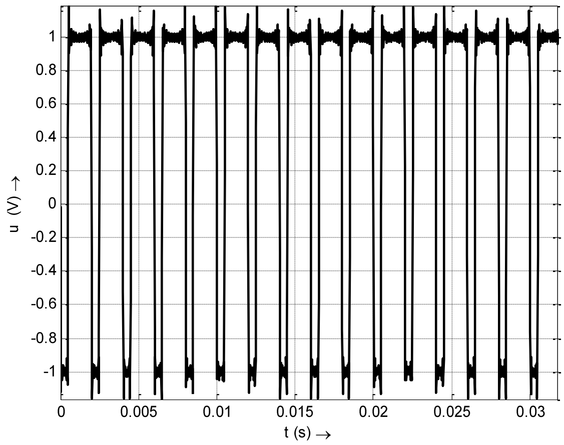

Fig. 5. Plot of the modulation signal for bipolar modulation

The inverter's output voltage is again expressed by a formula (3). The plot of inverter's output voltage calculated on the base of (3) is given on the Fig. 6.

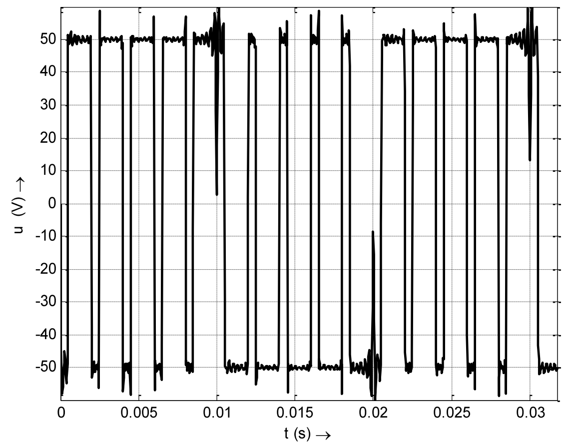

Fig. 6. Plot of the inverter's output voltage for bipolar modulation

4. The current waveform calculation

Suppose that the inverter is loaded by a resistive and inductive passives load. The voltage equation is valid

$$u = Ri + L\frac{di}{dt} \tag{4}$$

The supply voltage is expressed by an equation (3). This voltage differential equation has an analytical general solution of the form

$$\begin{aligned} i = {} & a_0\sum_{k=1}^{\infty} c_k\left(\frac{e^{jk\omega t}}{R+jk\omega L}-\frac{e^{-jk\omega t}}{R-jk\omega L}\right)+ \\ & +\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\Bigg\{b_m c_k\left[\frac{e^{j(k\omega+m\omega_1)t}}{R+j(k\omega+m\omega_1)L}-\right. \\ & \left.-\frac{e^{-j(k\omega-m\omega_1)t}}{R-j(k\omega-m\omega_1)L}\right]- \\ & -a_m c_k\left[\frac{e^{j(k\omega-m\omega_1)t}}{R+j(k\omega-m\omega_1)L}-\right. \\ & \left.\left.-\frac{e^{-j(k\omega+m\omega_1)t}}{R-j(k\omega+m\omega_1)L}\right]\right\}+Ce^{-\frac{R}{L}t} \end{aligned} \tag{5}$$

For steady state

$$t\to\infty;\ \text{thus}\ Ce^{-\frac{R}{L}t}\to 0$$

On the Fig. 7 is a plot of output voltage and current for unipolar modulation. The currant waveform is calculated for load composed of resistance R = 5 Ω and inductance L = 10 mH.

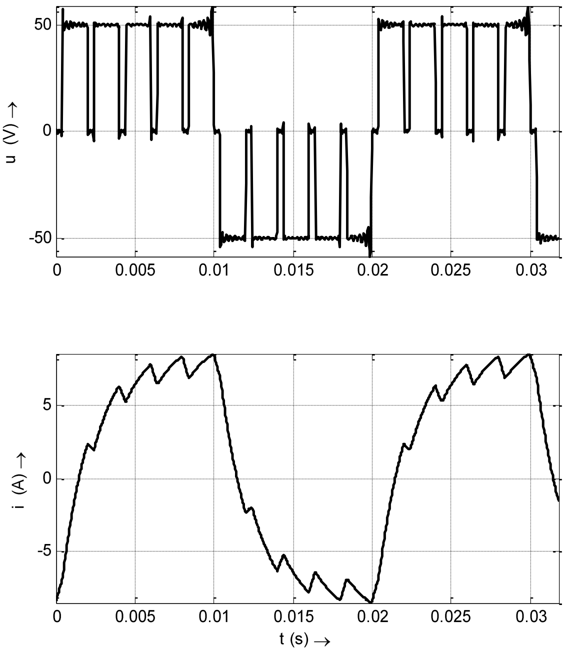

Fig. 7. Output voltages and current for unipolar modulation

The Fig. 8 shows the calculated waveforms of the inverter's output quantities for bipolar modulation.

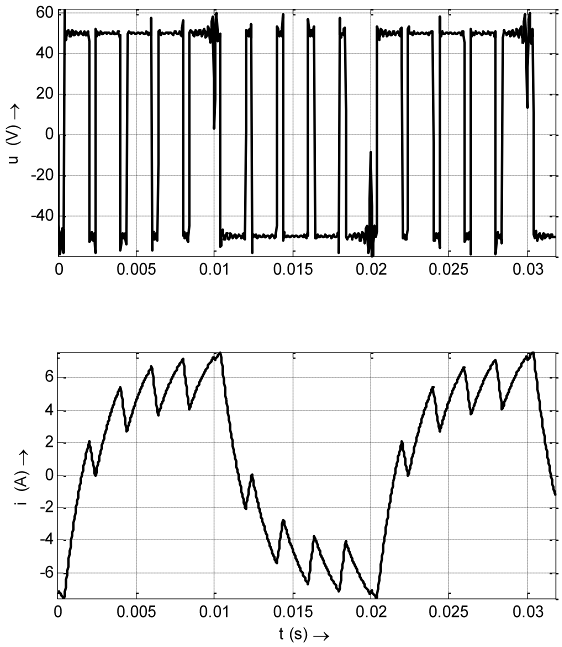

Fig. 8. Output voltages and current for bipolar modulation

On the Fig. 9 is a plot of the voltage and current of the load of resistance R = 5 Ω and inductance L = 30 mH. The frequency of the squarewave is f = 30 Hz. On the Fig. 10 is shown the plot of the outputs quantities for the bipolar modulation and for the same load parameters.

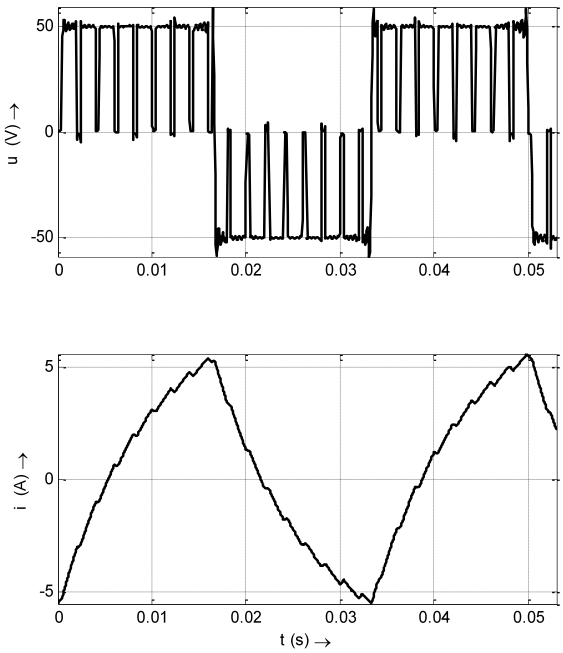

Fig. 9. Output voltages and current for unipolar modulation

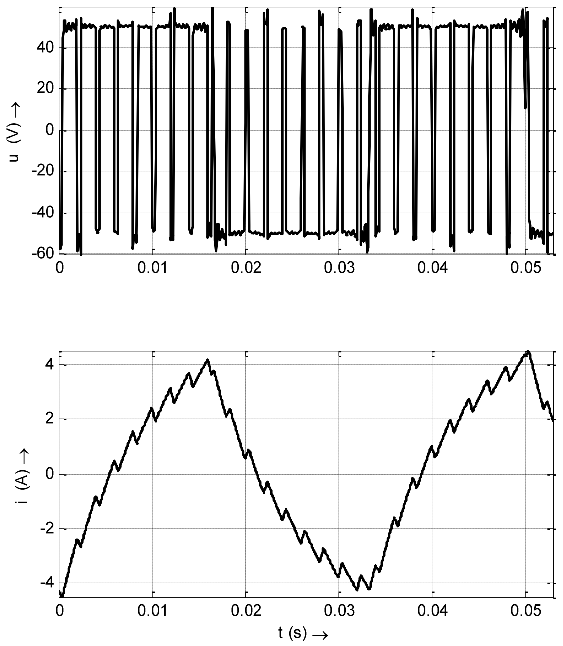

Fig. 10. Output voltages and current for bipolar modulation

5. Conclusion

The presented paper deals with mathematical method, which permits analytically to calculate the waveforms of the current and torque of a single-phase inverter. The method is based on the complex Fourier series. Accuracy of the calculated quantities depends on the quantities of the Fourier series, which are takes to consideration.

Acknowledgement

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7. Bibliography

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