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REPRESENTATION OF PERMANENT MAGNETS IN THE 3-D FINITE ELEMENT DESCRIPTION OF ELECTRICAL MACHINES

ABSTRACT *Finite element methods for 3-D magnetic field calculation in permanent magnet motors are discussed. An edge element method using a magnetic vector potential and the nodal element method using a scalar magnetic potential are considered. For both formulations the methods of field source description are presented. Attention is paid to sources in the permanent magnet regions. The methods have been successfully applied in the analysis of motors with inhomogeneously magnetized permanent magnets.*

Keywords: *permanent magnets, representation method, finite element method (FEM)*

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1. INTRODUCTION

The FE methods for the calculation of 3-D magnetic field are considered. Two FE approaches are discussed: (a) a scalar potential formulation in the nodal element space and (b) a vector potential formulation in the edge element space. The FE equations are expressed using the notion of equivalent magnetic and electric networks.

The description of the magnetic field that is based on the scalar potential and nodal elements may be considered as an equivalent to the nodal analysis of the permeance network (PN) [2]. The formulation involving edge elements and vector potential is equivalent to loop analysis of reluctance network (RN) [1, 2]. Nodes of permeance network correspond to the nodes of elements. Nodes of reluctance network are positioned in the centre of elements and centres of faces. In both formulations the sources of the magnetic field can be defined by the edge value of the current vector potential $\boldsymbol{T}$ [1,4] and the edge value of magnetizing vector $\boldsymbol{T}_m$.

The paper presents the methods of source description in the regions with permanent magnets. It has been assumed that the vector $\boldsymbol{H}$ of magnetic field intensity can be described as follows:

$$\boldsymbol{H} = \nu_w \boldsymbol{B} - \boldsymbol{H}_m \qquad (1)$$

where ν_w is the reluctivity tensor for magnet region, $\boldsymbol{H}_m$ is the coercive force (rigid magnetization) [3]. The vector $\boldsymbol{H}_m$ may be considered as the magnetizing vector or electric vector potential $\boldsymbol{T}_m$, $\boldsymbol{T}_m = \boldsymbol{H}_m$. It has been assumed that (1) applies to the local co-ordinate system and in element vector $\boldsymbol{H}_m$ has only one component H_m in the direction that is defined in magnetizing process.

In order to find the coercive force H_m it is advantage to approximate the demagnetization curve of magnet by tangent segments – see Fig. 1. For each segment the intensity H_m and reluctivity ν_w are constant, independent of flux density. The values of H_m and ν_w can be found in iteration process. In the case of rare-earth permanent magnets the demagnetization characteristic may be assumed to be linear. Thus, the vector $\boldsymbol{H}_m$ and reluctivity ν_w are independent of flux density and are known in advance.

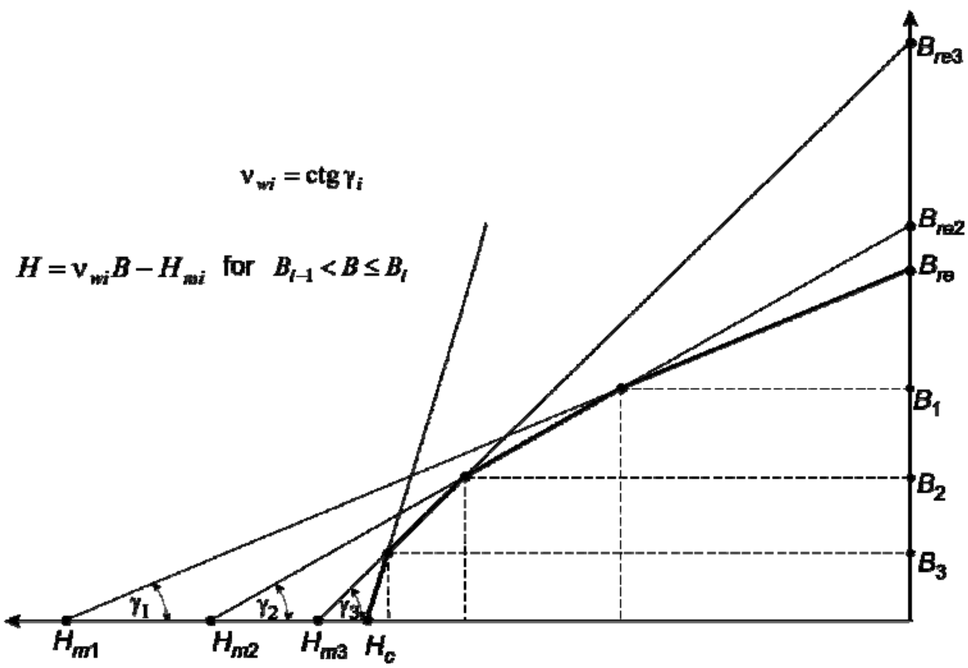

Fig. 1. Demagnetization characteristic of permanent magnet

2. PERMANENT MAGNETS IN THE EDGE ELEMENT SPACE

The nodal FE formulation using the scalar magnetic potential Ω has been considered. The FE equations represent the nodal equation of permeance network and can be written in the following matrix form:

$$\boldsymbol{k}_w^T \Lambda_g \boldsymbol{k}_w \Omega = -\Phi \tag{2}$$

Here Λ_g is the matrix of branch permeances associated with element edges and Ω is the vector of nodal values of Ω [2]. The matrix $\boldsymbol{k}_w$ transforms the nodal values of Ω into the magnetic voltages across the edges. The source vector Φ is expressed as

$$\Phi = \boldsymbol{k}_w^T \Lambda_g \Theta \tag{3}$$

where Θ is the vector of the magnetomotive forces (*mmfs*) in the branches of the equivalent PN [2]. In general, the vector Θ represents the *mmfs* of conducting currents Θ_u (in region with winding) and the *mmfs* of magnetizing current Θ_m (in regions with permanent magnets). Thus

$$\Theta = \Theta_m + \Theta_u \tag{4}$$

The branch *mmfs* Θ_m represent the edge values of magnetizing vector $\boldsymbol{T}_m$ and can be considered as a loop magnetization currents $\boldsymbol{i}_{om}$ in loops around element edges. These currents are calculated for a given distribution of magnetizing vector $\boldsymbol{T}_m$. Because of discontinuity of vector $\boldsymbol{T}_m$ in external surface of magnet the calculations of branch *mmfs* should be performed for the disjoint set of elements. In the calculations the following relationships are taken into consideration

$$\Theta_{md} = \boldsymbol{k}_e \Theta_m \tag{5}$$

and

$$\Lambda_g \Theta_m = \boldsymbol{k}_e^T \Lambda_{gd} \Theta_{md} \tag{6}$$

Here, Θ_{md} is the vector of branch *mmfs* and Λ_{gd} is the matrix of branch permeances for the disjoint set of elements, the matrix $\boldsymbol{k}_e$ relates the edge values of the disjoint set of elements to the edge values of the conjoint set of elements. In (6) product $\Lambda_{gd}\Theta_{md}$ represents the vector of flux sources ϕ_{md} for disjoint set of elements.

In order to explain the method a simple example of permeance magnet model is presented. A part of the permeance network in the magnet region has been considered. The part includes six loops related to the element facets - see Fig. 2...6.

It has been assumed that vector $\boldsymbol{T}_m$ has only component in the direction of axis *x* (Fig. 2). In the permance model branch *mmfs* are equal to magnetization currents in the loops around edges and represent the edge value of $\boldsymbol{T}_m$. Thus the mmf $\Theta_{mN_{i,j}}$ in branch $N_{i,j}$ of nodes P_iP_j is

$$\Theta_{mN_{i,j}} = i_{omN_{i,j}} = \int_{P_i}^{P_j} \boldsymbol{T}_m \, \mathrm{d}\boldsymbol{l} \tag{7}$$

In the presented example the vector $\boldsymbol{T}_m$ is parallel to the axis x. Therefore the loop currents in loops around edges parallel to the axis x are only different from zero. Thus the *mmfs* occur only in the branches associated with the edges parallel to the axis x, see Fig. 3.

The permanent magnet is homogenous magnetized. Therefore, the branch *mmfs* are identical,

$$\Theta_m = \Delta x T_{mx} \tag{8}$$

where Δx and T_{mx} are the symbols shown in Fig. 2.

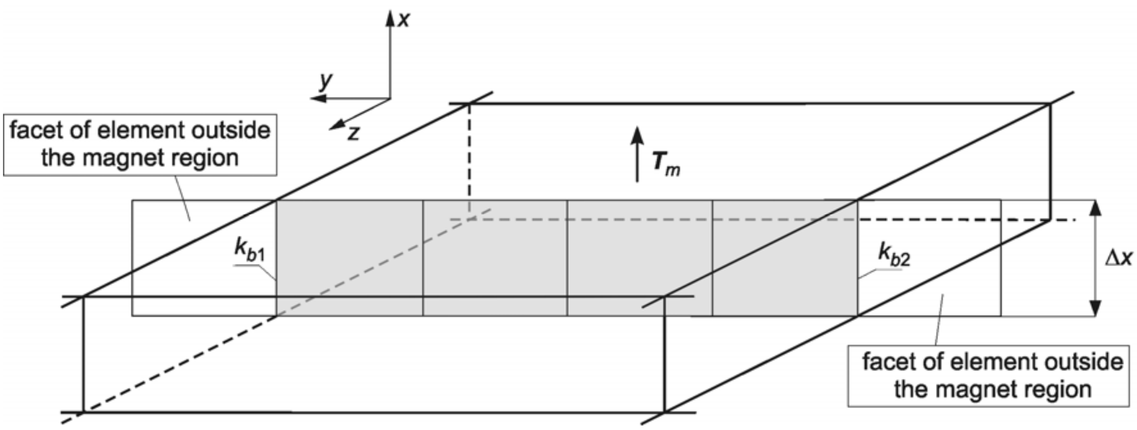

Fig. 2. Part of the network in the magnet region (k_{b1}, k_{b2} are the edges on the boundary of permanent magnet)

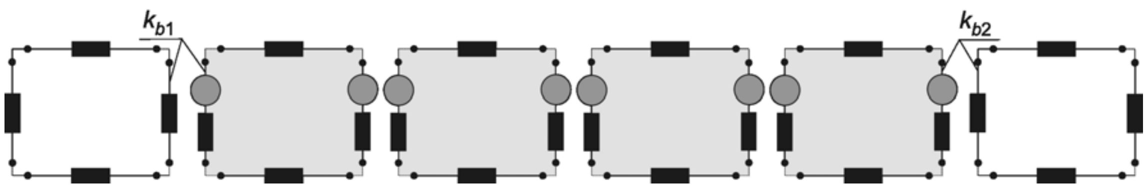

Fig. 3. Loops of permeance network related to facets of disjoin set of elements in Fig. 2

In the nodal analysis of permeance network the branch *mmfs* are replace by flux sources. As a result we obtain the system in Fig. 4.

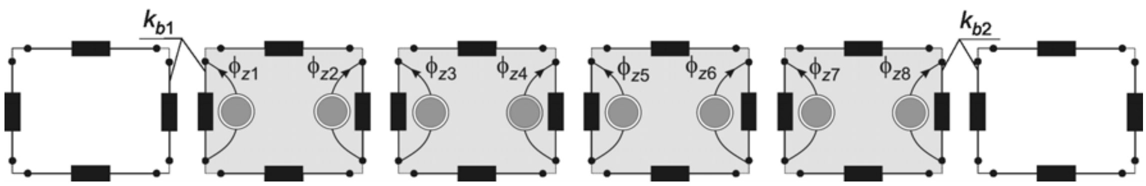

Fig. 4. Loops of permeance network in Fig. 3 with flux sources that replace branch *mmfs*

In the model with regular elements all flux sources are identical and

$$\phi_{z1} = \phi_{z2} = \cdots = \phi_{z8} = \frac{1}{4}\mu\frac{V_e}{(\Delta x)^2}\Theta_m = \frac{1}{4}\mu\frac{V_e}{\Delta x}T_{mx} \tag{9}$$

where V_e is the element volume.

The sources ϕ_{zi} of the disjoint set of elements create the vector ϕ_{md}. This vector has been described as follows

$$\phi_{md} = \Lambda_{gd}\Theta_{md} \tag{10}$$

The flux sources of the conjoint parallel branches associated with the common edges of elements (see Fig. 5) are defined as follows

$$\phi_m = \boldsymbol{k}_e^T\phi_{md} \tag{11}$$

In the obtained model the flux sources related to boundary edges k_{b1}, k_{b2} are half as big as flux sources associated with edges in magnet interior. In the nodal equations the flux sources are represented by nodal flux injections Φ_m, see Fig. 6. The vector nodal flux injections is

$$\boldsymbol{\Phi}_m = \boldsymbol{k}_w^T\phi_m \tag{12}$$

Using this formula and (10), (11) we obtain the following description of flux injections

$$\boldsymbol{\Phi}_m = \boldsymbol{k}_w^T\boldsymbol{k}_e^T\Lambda_{gd}\Theta_{md} \tag{13}$$

In the presented method of magnet representation the sum of flux injections is equal to zero. For example in Fig. 6, the sum of fluxes Φ_{mi} is equal to zero because $\Phi_{m,2j-1} = -\Phi_{m,2j}$ for $j = 1,2,\ldots,5$. This is the most important property of the proposed method of field source computation. Thanks to this property the FE method using single scalar potential can be successfully applied for the analysis of magnetic field in the region with permanent magnet.

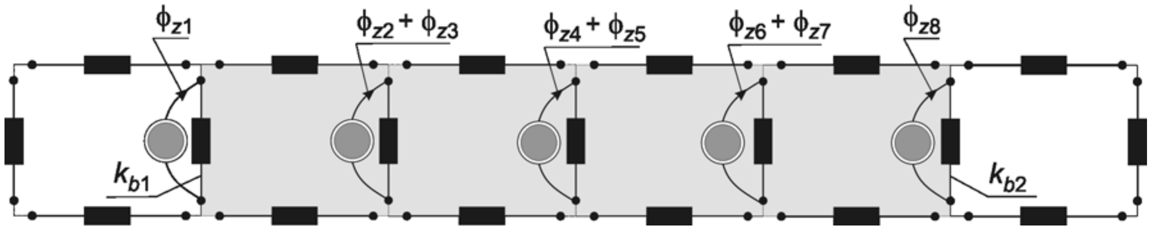

Fig. 5. The system of the conjoint branches associated with the common edges of elements

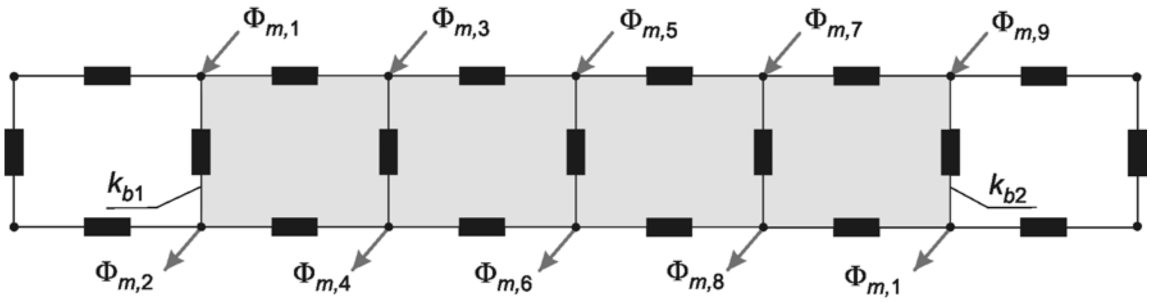

Fig. 6. Permeance network model of the conjoin set of elements with nodal flux injections

3. DESCRIPTION OF PERMANENT MAGNETS IN THE FACET ELEMENT SPACE

The edge element method using the vector magnetic potential $\boldsymbol{A}$ is considered. The edge element equations represent the loop equation of reluctance network (RN) [1, 2]. The branches of RN connect the centres of the elements. The vector of edge values of $\boldsymbol{A}$ represents the loop fluxes φ in the loops around edges. The equations that describe fluxes φ can be written as

$$\boldsymbol{k}_s^T \boldsymbol{k}_{sd}^T \boldsymbol{R}_{\mu gd} \boldsymbol{k}_{sd} \boldsymbol{k}_s \varphi = \theta_o \tag{14}$$

Here $\boldsymbol{R}_{\mu gd}$ is the matrix of branch reluctances for the disjoint set of elements, matrix $\boldsymbol{k}_s$ transforms the edge values of $\boldsymbol{A}$ into the facet values of flux density $\boldsymbol{B}$, matrix $\boldsymbol{k}_{sd}$ relates the facet values of the disjoint set of elements to the facet values of the conjoint set of elements, θ_o is the vector field sources in the facet element space. The component of vector θ_o represents loop *mmfs* in the RN that models the permanent magnet region. These *mmfs* are calculated for a given distribution of magnetizing current density $\boldsymbol{J}_m$. The magnetizing current density $\boldsymbol{J}_m$ is expressed by curl $\boldsymbol{T}_m$. The edge values of magnetizing vector $\boldsymbol{T}_m$ represent the loop magnetization currents $\boldsymbol{i}_{om}$ in loops around edges. Therefore in the calculation of θ_o we can apply the currents $\boldsymbol{i}_{om}$ and *mmfs* Θ_{md} that are defined in Section 2. This approach gives

$$\theta_{om} = \boldsymbol{k}_s^T \boldsymbol{k}_{sd}^T \boldsymbol{N}_{ed} \Theta_{md} \tag{15}$$

Here matrix $\boldsymbol{N}_{ed}$ transforms the loop currents $\boldsymbol{i}_{omd}$ in the loops around edges into the currents $\boldsymbol{i}_{osd}$ in the loops associated with element facets [2]. The currents $\boldsymbol{i}_{osd}$ represent the branch *mmfs* θ_{gmd} in the branches of reluctance network. Thus

$$\theta_{gmd} = \boldsymbol{i}_{osd} = \boldsymbol{N}_{ed} \Theta_{md} \tag{16}$$

In order to explain the method a simple example of magnet model has been considered. The permanent magnet has been divided into four curved rectangular parallelepipeds (Fig. 7). It has been assumed that magnet is homogeneous magnetizing in direction of *r*-axis in a cylindrical coordinate system r, z, ψ, i.e. $\boldsymbol{T}_m = \mathbf{1}_r\, T_m$. The edge values of magnetizing vector $\boldsymbol{T}_m$, i.e. loop magnetization currents are non-zero only for the edges parallel to the *r*-axis. Fig. 7 shows the loop magnetizing currents which flows around edges and the currents $\boldsymbol{i}_{osd}$ defined by (16). The currents $\boldsymbol{i}_{osd}$ are equal to the branch *mmfs* in RN. Fig. 8 shows the reluctance model of permanent magnet with branch *mmfs*. If magnet is homogenous magnetizing then the sum of branch *mmfs* in the loop of RN inside the magnet are equal to zero. Only in the loops around the edges F_i lying in the flank of magnet the loop *mmfs* are nonzero. Thus, the homogenous magnetizing magnet can be represented by infinite thin coil that sticks to the magnet flank, see Fig. 9.

Usually, in the 3D edge element models the coils with current are described by the facet values of current density, $\boldsymbol{i}_{sd}$, and loop *mmfs* in RN are defined as follows

$$\theta_{om} = \boldsymbol{N}_{ed}^T \boldsymbol{k}_{sd} \boldsymbol{i}_{sd} \tag{17}$$

However, in the case of infinite thin coil that models permanent magnet sources this description should not be applied

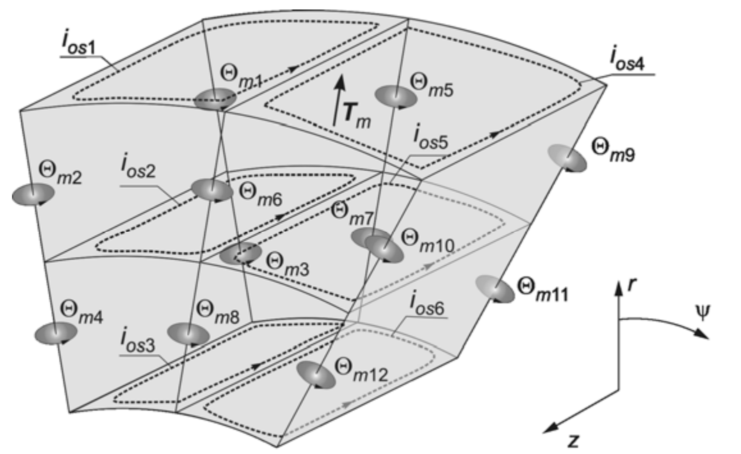

Fig. 7. The permanent magnet divided into 4 curved rectangular parallelepipeds

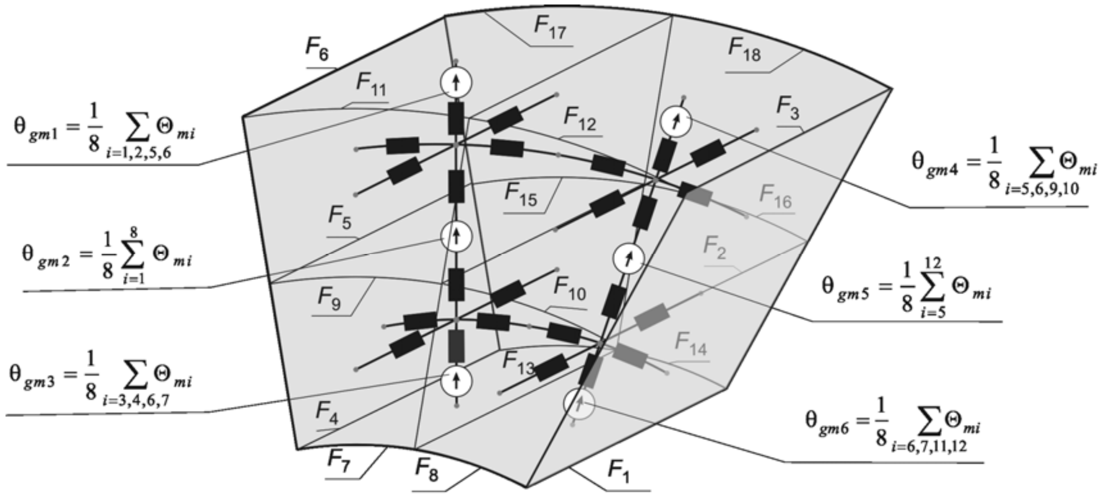

Fig. 8. The reluctance model of permanent magnet with branch *mmfs* calculated using loop currents in Fig. 7

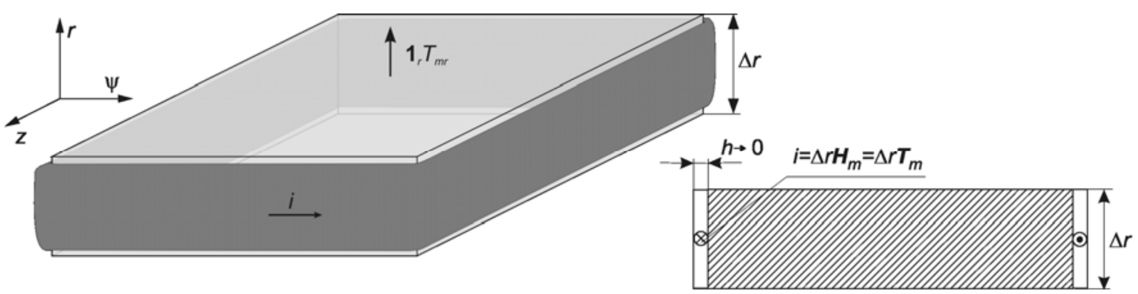

Fig. 9. The equivalent model of homogeneous permanent magnet and his cross section

The presented above method has been used in the calculations of permanent magnet motor (PMM). The 3D model has been applied. Motors with radial and inhomogeneously magnetized magnets are analyzed [5]. Permanent magnets are composed of segments (sectors). In the case of inhomogeneous magnets the magnetising vector $\boldsymbol{T}_m$ has different direction that dependents on the position of segment (*z*-component of $\boldsymbol{T}_m$ is equal to zero) – see Fig. 10. The components T_{mr}, $T_{m\psi}$ of vector $\boldsymbol{T}_m$ are sinusoidal function of angle β that describes the segment position. Systems with different angle λ that defines the direction of $\boldsymbol{T}_m$ in the terminal segment have been analysed (Fig. 10 b).

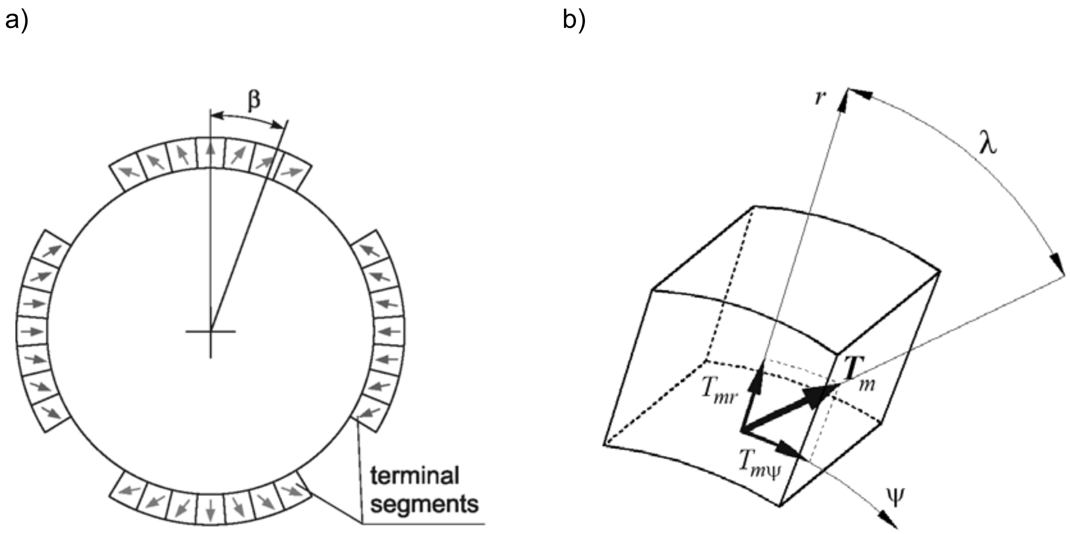

Fig. 10. Permanent magnets divided into segments of different magnetization (a) and terminal segment of magnet (b)

In the paper the results of cogging torque calculation are given. The calculations have been performed for different values of the angle λ. For λ = 0° we obtain magnets with radial magnetization. The calculated torque- angle characteristics are shown in Fig. 11. It is interesting to see that the application of inhomogeneously magnetized magnets can lead to the significant reduction of cogging torque.

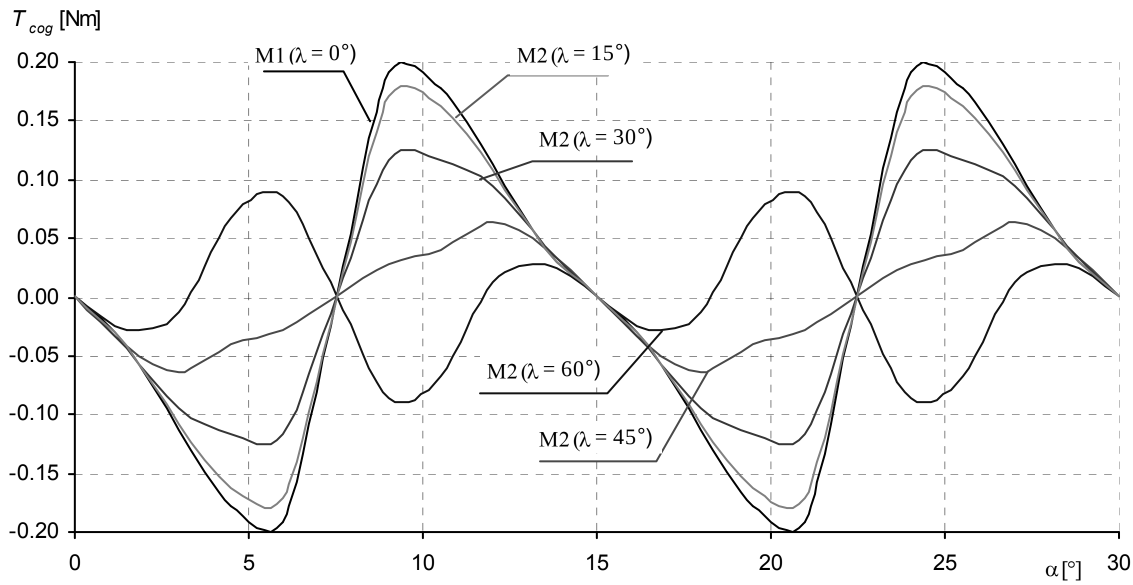

Fig. 11. Calculated cogging torque-angle characteristic for PMM with radial (M1) and inhomogeneously magnetized (M2) permanent magnets

4. CONCLUSION

The presented methods of permanent magnet description are based on the calculation of edge values of magnetizing vector $\boldsymbol{T}_m$. The methods are universal and can be successfully applied in the FE analysis of permanent magnet machines using nodal and edge elements. The methods enable the analysis of system with inhomogeneously magnetized permanent magnets.

The proposed formulations give the field sources that exactly satisfy the current continuity condition for the FE models. Therefore the methods provide a high accuracy of FE method using single scalar potential for nodal elements and guarantee a good convergence of iterative procedure of solving edge element equations for ungaged formulation.

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ODWZOROWANIE MAGNESÓW TRWAŁYCH W DYSKRETNYCH TRÓJWYMIAROWYCH MODELACH MASZYN ELEKTRYCZNYCH

A. DEMENKO, D. STACHOWIAK

STRESZCZENIE *W pracy przedstawiono metody formułowania wektora wymuszeń dla obszarów z magnesami trwałymi. Rozpatrzono dwie metody opisu pola magnetycznego: metodę potencjału skalarnego oraz potencjału wektorowego. Do rozwiązywania równań pola zastosowano metodę elementów skończonych (MES). Rozpatrzono nowe ujęcie MES, w którym wielkości wektorowe opisuje się za pomocą funkcji interpolacyjnych elementów krawędziowych i funkcji interpolacyjnych elementów ściankowych.*

W metodzie skalarnego potencjału magnetycznego wykorzystuje się funkcje interpolacyjne elementu krawędziowego i wielkości krawędziowe. Wielkościami krawędziowymi są napięcia magnetyczne. Poszukiwane wartości krawędziowe wyraża się za pomocą wartości węzłowych potencjału skalarnego i rozwiązuje się równania opisujące wartości węzłowe.

W metodzie potencjału wektorowego wykorzystuje się funkcje interpolacyjne elementu ściankowego i wielkości ściankowe. Wielkościami ściankowymi są strumienie przenikające przez ścianki elementów. Posłużywszy się językiem teorii obwodów strumienie przenikające przez ścianki można nazwać strumieniami gałęziowymi. W algorytmach obliczeniowych strumienie gałęziowe wyraża się za pomocą strumieni oczkowych. Reprezentantami tych strumieni są wielkości krawędziowe wektorowego potencjału magnetycznego $\mathbf{A}$ *tj. zorientowane całki liniowe z* $\mathbf{A}$ *wzdłuż krawędzi elementów.*

W pracy przedstawiono metody opisu źródeł od prądów magnetyzacji w przestrzeni elementów krawędziowych i ściankowych.

Przyjęto, że w obrębie magnesu wektor $\boldsymbol{H}$ *natężenia pola opisany jest wyrażeniem:*

$$\boldsymbol{H} = \nu_w \boldsymbol{B} - \boldsymbol{H}_m$$

przy czym ν_w *jest tensorem reluktywności „wewnętrznej" magnesu, a* $\boldsymbol{H}_m$ *zastępczym natężeniem powściągającym reprezentowanym przez wektor namagnesowania* $\boldsymbol{T}_m$, $\boldsymbol{T}_m = \boldsymbol{H}_m$. *Przy zapisywaniu powyższej relacji przyjęto, że dotyczy ona lokalnego układu współrzędnych, w którym wektor* $\boldsymbol{H}_m$ *ma tylko jedną składową w kierunku namagnesowania.*

W obszarze z magnesami trwałymi źródłami pola są prądy magnetyzacji o gęstości $\boldsymbol{J}_m$. *Przy opisie źródeł w przestrzeni elementów krawędziowych i ściankowych posługiwano się krawędziowymi wartościami wektora magnesowania* $\boldsymbol{T}_m$, *uwzględniano, że* $\boldsymbol{J}_m = rot\,\boldsymbol{T}_m$. *Krawędziowe wartości wektora* $\boldsymbol{T}_m$ *odpowiadają oczkowym prądom magnetyzacji* $\boldsymbol{i}_{om}$ *w oczkach wokół krawędzi elementów. Na podstawie tych prądów można wyznaczyć iniekcje strumieni źródłowych w metodzie potencjału skalarnego oraz wymuszenia reprezentujące oczkowe siły magnetomotoryczne w metodzie potencjału wektorowego. Przedstawiono przykład zastosowania opracowanych metod. Analizowano moment zaczepowy w maszynie o magnesach złożonych z segmentów niejednorodnie namagnesowanych.*