Jan IWASZKIEWICZ
Jacek PERZ

FOURIER SERIES AND WAVELET TRANSFORM APPLIED TO STEPPED WAVEFORM SYNTHESIS IN MULTILEVEL CONVERTORS

ABSTRACT *The paper is related to the problem of generating the high quality AC voltage waveforms using multilevel converters. This technique can be applied especially in the distributed power energy systems with renewable DC voltage sources like photo-voltaic farms and fuel cells. A novel approach to the synthesis of AC voltage waveforms, applying analytical methods like Fourier transform and Haar wavelets is presented. The analysis of the obtained waveforms is presented as well as examples of chosen parameter optimisation. The comparison of modulation methods and their impact on converter topology is discussed.*

Keywords: *multilevel converters, stepped waveform synthesis*

Jan IWASZKIEWICZ, D.Sc
e-mail: jan.iwaszkiewicz@iel.gda.pl

Jacek PERZ, D.Sc.
e-mail: jacek.perz@iel.gda.pl

The Gdansk Branch of The Electrotechnical Institute,
Narwicka 1, 80-557 Gdansk,
tel. +48 58 3430645,fax: +48 58 3431295,

1. INTRODUCTION

There are many applications i.e. uninterruptible power sources or distributed power generation systems, where the demand is to generate 50 Hz sinusoidal voltage waveforms of relatively high quality. The voltage obtained from DC sources like storage accumulators, photovoltaic farms or fuel cells are converted to AC using power electronics converters. The quality of generated waveforms, especially THD (*Total Harmonic Distortion*) factor, should comply with the appropriate standards. The commonly used solution for this purpose is two level VSI (*Voltage Source Inverter*), controlled using PWM (*Pulse Width Modulation*) method. This method has well known disadvantages, related to high frequency switching, like power losses in switching elements and necessity of using special filters for high frequency components in the output voltages. These disadvantages can be reduced using multilevel converters and amplitude modulation method. The mathematical approach to the control strategy of multilevel converters is presented in the paper. Two different mathematical methods for synthesis of the output waveforms applying Fourier transform and wavelet transform are discussed. Thanks to used mathematical methods it is possible to optimise the parameters of the obtained waveforms.

2. STANDARD WAVEFORMS SYNTHESIS BASED ON FOURIER SERIES

Given is a scaling function $\varphi(x)$:

$$\varphi(x) = \begin{cases} 1 & \text{for} \quad 0 \le x < \alpha, \\ 0 & \text{for} \quad \text{other} \quad x \end{cases} \qquad \alpha \neq 0 \tag{1}$$

The scaling function $\varphi_n(x)$ is defined as follows:

$$\varphi_n(x) = \varphi(x - n\alpha) \quad \text{for} \quad n = \ldots,-2,-1,0,1,2,\ldots \tag{2}$$

The equation defines a set of rectangular pulses of unitary amplitude and angle duration equal to α. The pulse position on x axis depends on parameter n. A few examples of the scaling functions are presented in Fig. 1.

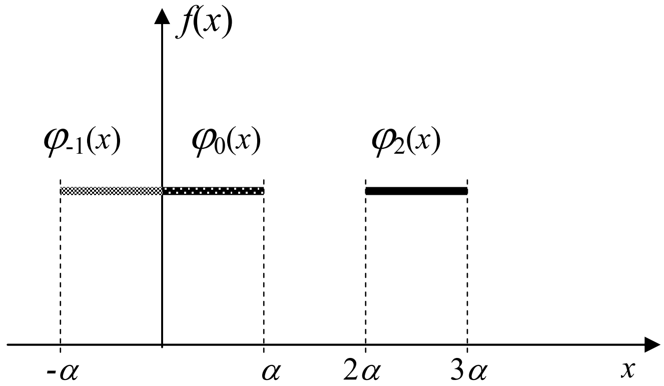

Fig. 1. Examples of the scaling functions:
$\varphi_0(x)=\varphi(x)$, $\varphi_{-1}(x)$, $\varphi_2(x)$

In an interval $x \in <a,b)$, which length is $k\alpha$ ($k \geq 1$), functions $\varphi_n(x)$ satisfy two conditions:

$$\|\varphi_n\|^2 = \int_a^b \varphi_n{}^2(x)\mathrm{d}x = \alpha \qquad \text{and} \quad \int_a^b \varphi_k(x)\varphi_m(x)\mathrm{d}x = 0 \quad \text{if} \quad k \neq m \tag{3}$$

therefore the notation (2) creates a set of orthogonal functions called an orthogonal base. Since all functions $\varphi_n(x)$ have their norms $\|\varphi_n\|^2$ equal to α, this base is called an orthonormal base.

The expansion of a function $f(x)$ in a generalized Fourier series related to the set of scaling functions (φ_n) is as follows:

$$f(x) = \sum_{n=0}^{\infty} c_n \varphi_n(x) \tag{4}$$

where

$$c_n = \frac{(f, \varphi_n)}{\|\varphi\|^2} = \frac{\int_a^b f(x)\varphi_n(x)\mathrm{d}x}{\alpha} \tag{5}$$

The expansion (4) is valid for any function $f(x) \in L^2{}_{<a,b>}$ The Fourier series contains an infinite number of elements and makes it possible to approximate a function $f(x)$ by use of an infinite set (a sum) of adequately scaled functions

$\varphi_n(x)$. Particularly it is possible to expand a function $f(x)=\sin(x)$ by summing an infinite set of rectangular pulses. It is in contradistinction to typical application of the Fourier series where any function $f(x)$ is expanded as a set of harmonics. According to (4) and (5) the expansion of $\sin(x)$ in the interval $x \in < a,b)$ is given as:

$$\sin(x)=\sum_{n=0}^{n=\infty}\left\{\frac{\int_a^b \sin(x)\varphi_n(x)\mathrm{d}x}{\alpha}\varphi_n(x)\right\} \quad x\in<a,b> \tag{6}$$

The expression (6) defines a series of consecutive rectangular pulses represented by functions $\varphi_n(x)$. Amplitudes of pulses are different and determined after calculation of the integral. This statement can be applied for composition of sine waveforms in power electronics. Rectangular pulses represent an essential shape of output voltage (current) of an inverter. Naturally the composition of stepped waveforms using rectangular pulses has been applied in many applications but it concerned a “vertical” addition of waveforms. The resulting phase voltage was synthesized by the addition of the voltages generated by different cells of a cascaded inverter. The presented proposal relates to the addition of pulses “along *x axis*” or in a time scale. Consecutive pulses form the resulting voltage or current of the converter.

Practically in power electronics applications, the approximation of a sine wave can be realized using a finite number N of the series members and natural aspiration of designers is to utilize the possibly lowest N number. The accuracy of approximation depends on it. Its numerical value can be measured in different ways. In mathematics the accuracy of approximation is defined as the average square error δ, a very useful criterion destined to that purpose. If the approximation of the function $f(x) = \sin(x)$ has been done in the interval $<a, b>$ by use of N elements, the average square error δ is determined by the following expression:

$$\sigma=\frac{1}{b-a}\int_a^b\left[\sin(x)-\sum_{n=0}^{N-1}c_n\varphi_n(x)\right]^2\mathrm{d}x \quad x\in<a,b>,\quad b>a \tag{7}$$

In power electronics the most important criterion of the accuracy or rather the quality of approximated waveforms is THD factor. The example of approximation has been presented in Fig. 2. The presented stepped waveform has been obtained after approximation based on the set defined according to (1) and (2).

The results of Fourier approximation for selected N are collected in Table 1. The results of average square errors, calculated according to the expression (7), are presented n the column denoted by δ_N. The symbol F_N denotes the number of steps in the scanned interval – herein it is the interval <0,2π>. The parameter $N_{|F_N|}$ denotes the number of demanded supply DC voltage or current sources. The correlation between these two parameters is described by the following: statement:

$$\text{if} \quad \left(\frac{F_N}{4}\right) \in W \quad \text{then} \quad N_{|F_N|} = \left(\frac{F_N}{4}\right) \quad \text{else} \quad N_{|F_N|} = E\left\{\frac{F_N}{4} + 1\right\} \tag{8}$$

where W denotes the set of whole numbers and E – function [x] known as function *Entier* $\{x\}$. The value $N_{|F_N|}$ is a very important parameter of multilevel converters.

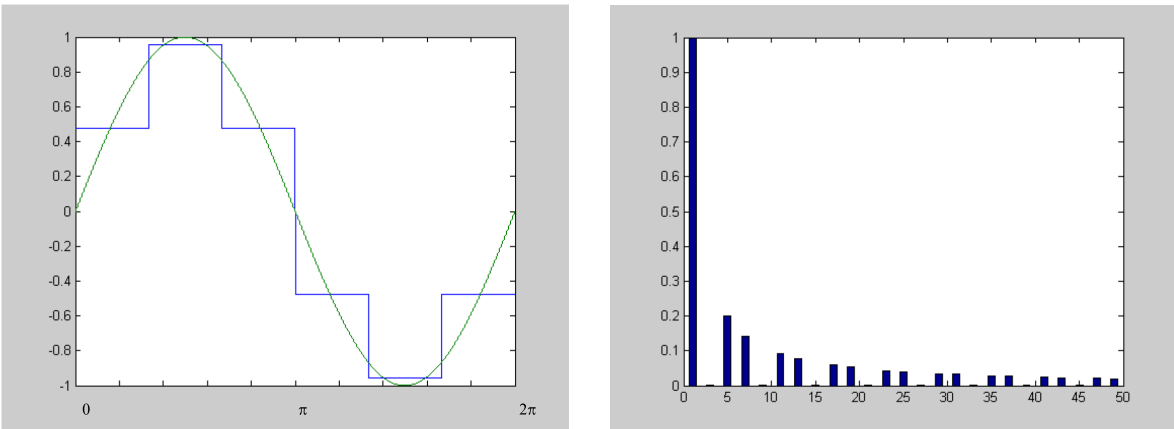

Fig. 2. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 6 (α = π/3) and the spectrum analysis of the waveform

Figure 2 presents the stepped waveform obtained using a very low approximation level. The ratio of the measures of steps is equal to 2 and it is the same as in a three phase inverter with connected load and controlled by an adequate set of rectangular waves. In order to produce such a waveform in one phase applications the converter needs only two DC sources easy to get e. g. by dividing one DC supply voltage.

TABLE 1
The parameters of standard Fourier approximation for different N.

F_N	α	$N_{\lvert F_N \rvert}$	σ_N	THD
$F_{N=2}$	π	1	0,0947	48,37 %
$F_{N=6}$	π/3	2	0,0440	31,09 %
$F_{N=12}$	π/6	3	0,0113	15,23 %
$F_{N=16}$	π/8	4	0,0064	11,41 %
$F_{N=24}$	π/12	6	0,0028	7,63 %

3. FOURIER APPROXIMATION WITH OPTIMISATION

The proportion of step measures presented in Fig. 2 does not assure the minimal harmonic content of the waveform available in a converter equipped with only two DC sources. It will be shown further that changing the ratio of the supply voltages and matching the duration of pulses it is possible to decrease the THD factor of the output waveform.

Generally the application of the Fourier series in the domain of approximation does not demand the use of an orthonormal base of scaling functions. The necessary condition is the orthogonality of the base. Therefore another set of orthogonal functions can be applied for approximation. It may be a set consisting of rectangular pulses of different length α_k on the x axis:

$$\varphi_k(x) = \begin{Bmatrix} 1 & \text{for} & 0 \le x < \alpha_k, \\ 0 & \text{for} & \text{other} \quad x \end{Bmatrix} \qquad \alpha_k > 0, \quad k = 0,1,2,... \tag{9}$$

The scaling function $\varphi_n(x)$ is defined as follows:

$$\varphi_n(x) = \varphi\left(x - \sum_{k=0}^{k=n-1} \alpha_k \right) \quad \text{for} \quad n = 0,1,2,... \tag{10}$$

An example of approximation using two scaling functions: $\varphi_0(x), \varphi_1(x)$ has been presented in Fig. 3. Relevant parameters are: α_0 = π/6 and α_1 = 2π/3. In this case the harmonic spectrum is not regular and contains lower order harmonics including the third one. The total harmonic content is less than in case presented in Fig. 2: THD = 21,62 % in comparison to 31,09 % in case of standard approximation. The approximation accuracy reaches the value δ = 0,0222.

In case of the waveform presented in Fig. 2 all even harmonic components are equal to zero and odd components b_k are given as:

$$b_k = \frac{4}{k\pi}\left[V_1 + \left(V_2 - V_1\right)\cos\alpha\right] \quad k \in \mathrm{N} \tag{11}$$

where V_1 and V_2 denote the measures of steps and *α* the angle of the first step. It was assumed here that in every half cycle the waveform was symmetrical with respect to the straight line $\alpha = \frac{\pi}{2} + n\pi \quad n = ..,-2,-1,0,1,2,...$.

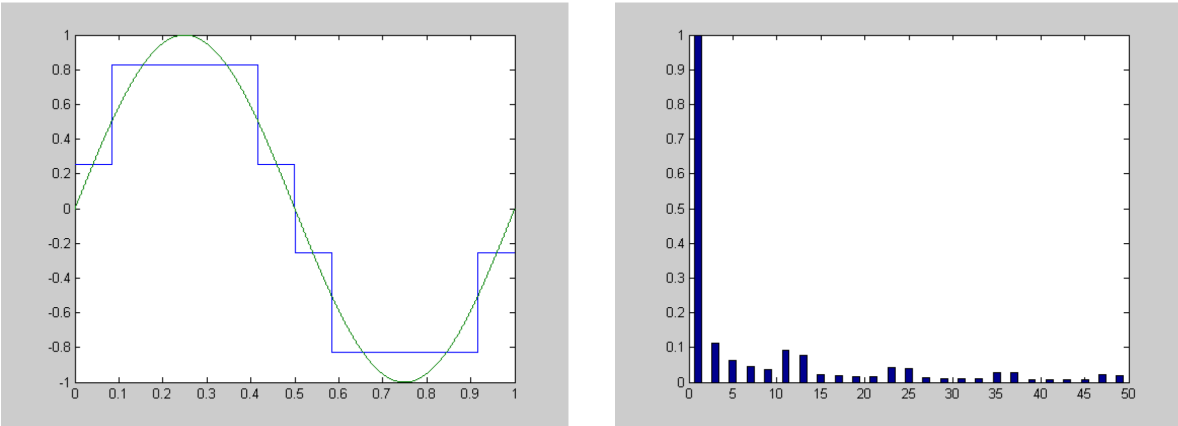

Fig. 3. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 2 ($\alpha_0 = \pi/6$, $\alpha_1 = 2\pi/3$) and the spectrum analysis of the waveform

The relationship (11) can be utilized to select the step levels in order to cancel selected harmonic components. The psroblem of the selective harmonic elimination is well known as a fundamental of the state of the art. The example discussed here shows only the results available in the three level converter. Assuming canceling of third and fifth harmonic components a set of equation can be written:

$$\begin{cases} 4\left[V_1 + \left(V_2 - V_1\right)\cos\alpha\right] = \pi \\ V_1 + \left(V_2 - V_1\right)\cos(3\alpha) = 0 \\ V_1 + \left(V_2 - V_1\right)\cos(5\alpha) = 0 \end{cases} \tag{12}$$

Solving the set of equations one can receive:

$$V_1 = 0{,}3927, \quad V_2 = 0{,}9481, \quad \alpha = \frac{\pi}{4}, \quad b_1 = 1, \quad b_3 = b_5 = 0$$

The results of calculations are presented in Fig. 4. The THD ratio is 23,1 %.

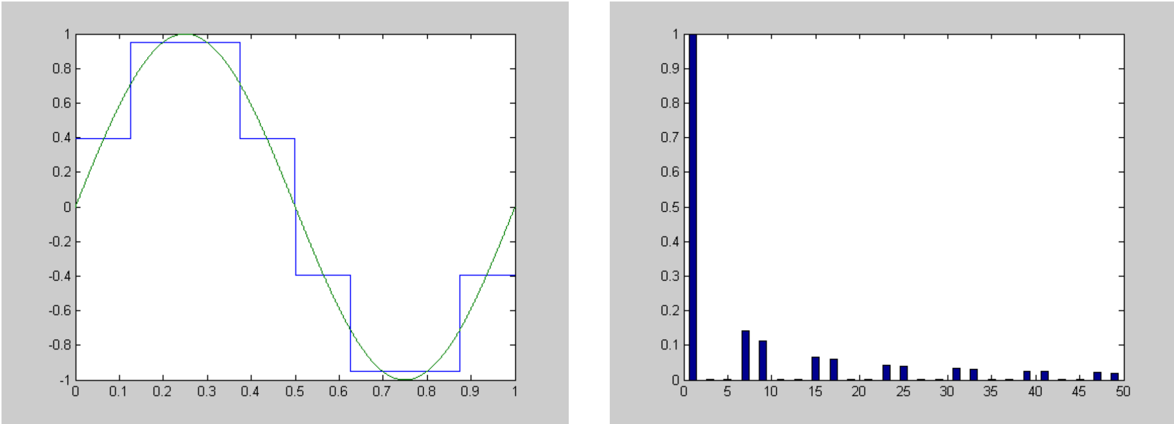

Fig. 4. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 2 ($\alpha_0 = \pi/4$, $\alpha_1 = \pi/2$) with third and fifth harmonics cancelled and the spectrum analysis of the waveform

Taking as a criterion the minimal value of the THD ratio it is possible to find an optimal ratio of step parameters as a minimal value of the expression:

$$\mathrm{THD} = \frac{1}{b_1}\sqrt{\sum_{k=1}^{k=\infty} b_{2k+1}^2} \tag{13}$$

Considering (11)

$$\mathrm{THD} = \frac{4V_1}{\pi b_1}\sqrt{f(\alpha,\theta)} \quad k = 1,2,... \tag{14}$$

and the minimal value of THD is related to the minimum of the function:

$$f(\alpha,\theta) = \sum_{k=1}^{k=\infty}\left\{\frac{1}{2k+1}\left[\theta + (1-\theta)\cos\left[(2k+1)\alpha\right]\right]\right\}^2 \tag{15}$$

The three dimensional picture of the analyzed function $f(\alpha,\theta)$ is presented in the Fig. 5.

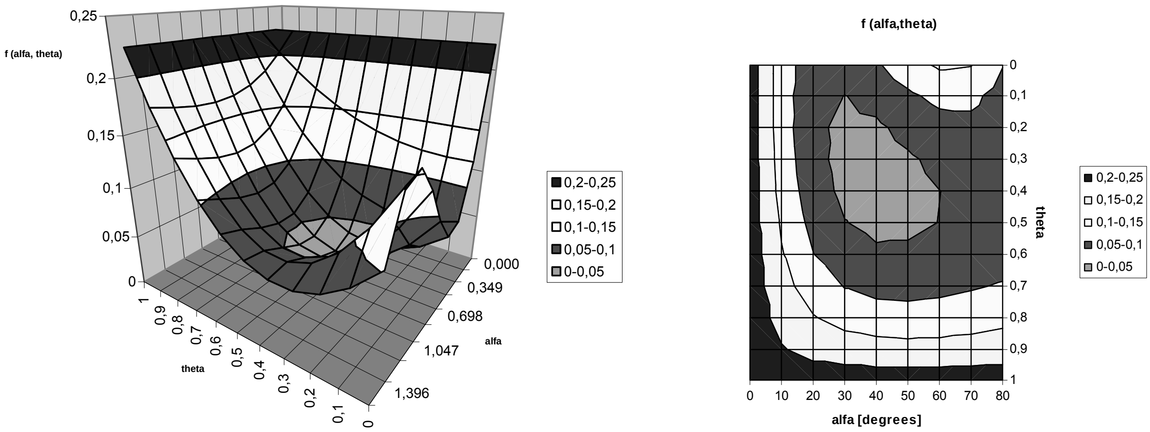

Fig. 5. Three dimensional picture of the function *f*(α, θ), and its projection on (α, θ) plane showing the optimal area

The parameters of the obtained optimal approximation for the minimum THD value are as following: α = 40˚ (2π/9), θ = 0,4, b_1 = 1, V_0 = 0,3655, V_1 = 0,9136. The THD factor for this waveform is 20,40 %. The optimal waveform and its harmonic spectrum is presented in the Fig. 6.

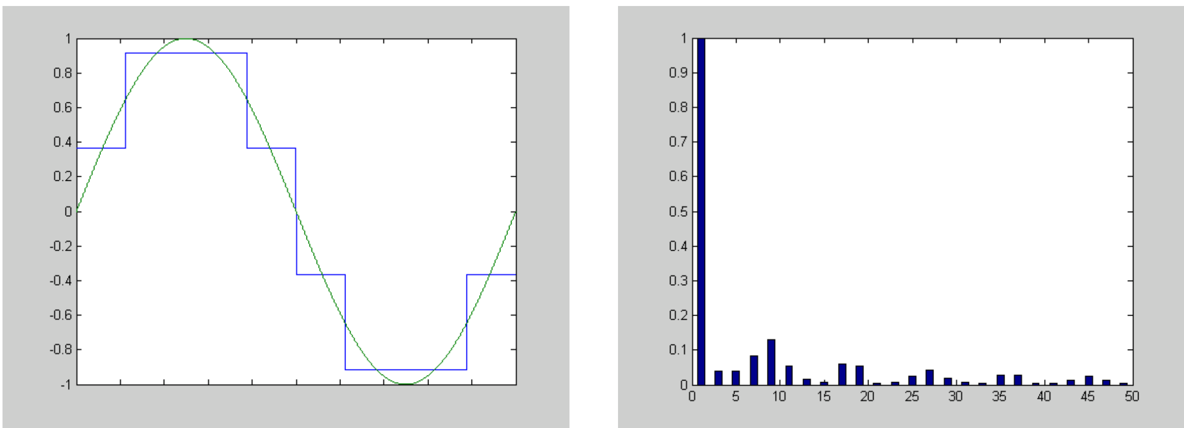

Fig. 6. The approximation of the function *f*(*x*) = sin(*x*) for *N* = 2 and minimal THD factor with optimal parameters (α_0 = 2π/9, α_1 = 5π/9, THD = 20,40)

3. WAVEFORMS SYNTHESIS BASED ON WAVELET WAVEFORMS

The wavelets is a term for mathematical functions, which allow the analysis of signals in different time scale and with different resolution. Thanks to this adjustable „scope of the view" the wavelets can be used to distinguish and analyse small and big details of the investigated process as well. Especially they are useful in analyse of discontinuous processes or processes with step level changing. The wavelets application are in many not directly related areas like seismology, video analysis, quantum mechanics or electronic. The term „wawelets" is the direct translation of french term *„ondelettes"* or *„petites ondes"*, which means „little waves".

Till now the wavelets have been used mainly for analysis of processes or signals based on decomposition of the elements of the processes. The following considerations will prove that wavelets can be also useful in composition of the power electronics signals and structures.

For this purpose the Haar wavelets have been adopted. The fundamental Haar wavelet $\psi(t)=\psi_{00}(t)$ can be constructed transforming the following scaling function $\varphi(t)$:

$$\varphi(t)=\begin{cases}1 & \text{for} \quad 0\le t<1,\\ 0 & \text{for} \quad \text{other} \quad t\end{cases} \tag{16}$$

The composition of two consecutive scaling functions $\varphi(2t)$ and $\varphi(2t-1)$:

$$\varphi(2t)=\begin{cases}1 & \text{for} \quad 0\le t<0{,}5),\\ 0 & \text{for} \quad \text{other} \quad t\end{cases}$$

$$\varphi(2t-1)=\begin{cases}1 & \text{for} \quad 0{,}5\le t<1),\\ 0 & \text{for} \quad \text{other} \quad t\end{cases}$$

creates the Haar wavelet: $\psi(t)=\varphi(2t)-\varphi(2t-1)$ which can be described as following:

$$\psi(t)=\begin{cases}1 & \text{for} \quad 0\le t<\frac{1}{2},\\ -1 & \text{for} \quad \frac{1}{2}\le t<1,\\ 0 & \text{for} \quad \text{other} \quad t\end{cases} \tag{17}$$

The scaling functions and the fundamental Haar wavelet are presented in Fig. 7.

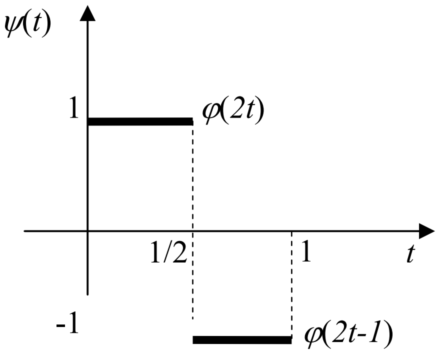

Fig. 7. The scaling functions $\varphi(2t)$ and $\varphi(2t-1)$ and the fundamental Haar wavelet $\psi(t)=\psi_{00}(t)$

By introducing two parameters: m – scale factor and n – displacement factor the generalised Haar transform is obtained:

$$\psi_{mn}(t)=\frac{1}{\sqrt{2^m}}\psi\left(2^{-m}t-n\right) \quad \text{for} \quad m,n=...,-2,-1,0,1,2,... \tag{18}$$

The scale factor m – settles the width and amplitude of the wavelet, and the displacement factor n – settles the wavelet position on time axis. The fundamental Haar wavelet corresponds to the factors: m = 0 and n = 0 and can be denoted as $\psi(t)=\psi_{00}(t)$.

The Haar wavelet form is similar to the form of the voltage or current pulse that can be obtained using simple one-phase inverter. The displacement and width of the wavelet can be freely controlled. Thanks to these properties it is possible to apply wavelets in power electronics i.e. to form the output waveforms of multilevel converters. For the one-phase power electronics purpose the scaling function $\varphi(x)$ is defined in interval $x \in <0, 2\pi)$:

$$\varphi(x)=\begin{Bmatrix} 1 & \text{for} & 0 \le x < 2\pi, \\ 0 & \text{for} & \text{other} \quad x \end{Bmatrix} \tag{19}$$

The fundamental proposed wavelet is defined like the Haar one: $\psi(x)=\varphi(2x)-\varphi[2(x-\pi)]$ and can be expressed as following:

$$\psi(x) = \begin{Bmatrix} 1 & \text{for} & 0 \le x < \pi, \\ -1 & \text{for} & \pi \le x < 2\pi, \\ 0 & \text{for} & \text{other} \quad x \end{Bmatrix} \tag{20}$$

The wavelet transform is defined as follows:

$$\psi_{mn}(x) = \psi\left[2^{-m}\left(x - n2^{m+1}\pi\right)\right] for\ m, n = ..., -2, -1, 0, 1, 2, ... \tag{21}$$

where $2^m 2\pi = 2^{m+1}\pi$ is a wavelet carrier.

The x axis position is defined as n-times $2^{m+1}\pi$ displacement. The *m* factor scales not the wavelet amplitude but the carrier. All the wavelets $\psi_{mn}(x)$ are orthogonal in interval $x \in <0, 2\pi)$. For the wavelets of different *m* and equal *n* the integral:

$$\int_0^{2\pi} \psi_{m_k n}(x)\psi_{m_l n}(x)\mathrm{d}x = 0 \quad \text{for} \quad k \ne l$$

because each wavelet with smaller carrier is contained in interval, in which the wavelet of bigger carrier is constant.

Let the approximation function $f_\psi(x)$ in interval $x \in <0, 2\pi)$ be a combination of wavelets with m = -3, -2, -1, 0. It can be denoted as a sum:

$$f_\psi(x) = \sum_{m=-3}^{m=0} \sum_{n=0}^{2^{-m}-1} a_{mn}\psi_{mn}(x) = \sum_{m=-3}^{m=0} \sum_{n=0}^{2^{-m}-1} f_{mn}(x)$$

where the functions $f_{mn}(x)$ are component wavelets of amplitude and phase defined by the factors a_{mn}:

$$a_{mn} = \frac{1}{N_m} \int_0^{2\pi} \sin(x)\psi_{mn}(x)\mathrm{d}x$$

The function $f(x)=\sin(x)$ approximated using wavelets $f_{00}(x)$, $f_{-20}(x)$, $f_{-21}(x)$, $f_{-22}(x)$ and $f_{-23}(x)$ is presented in Fig. 8.

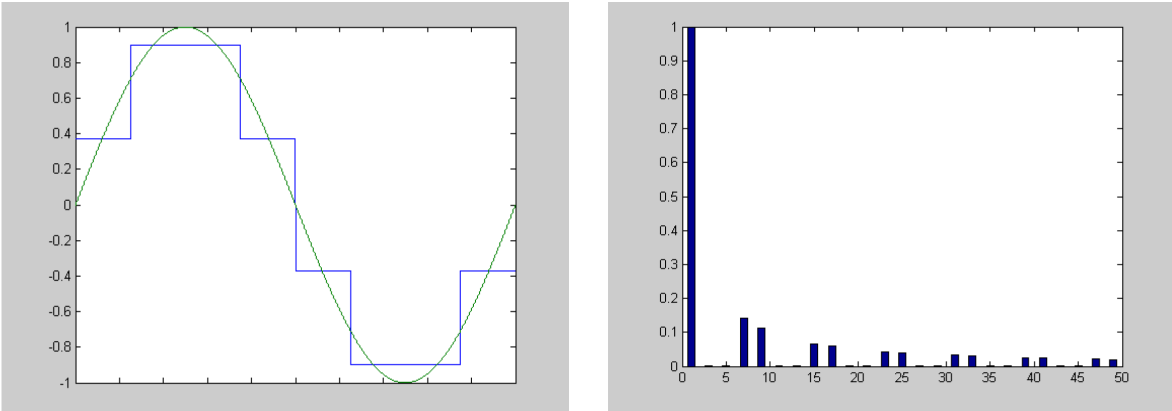

Fig. 8. The approximation of *f*(*x*)=sin(*x*) using wavelets $f_{00}(x)$, $f_{-20}(x)$, $f_{-21}(x)$, $f_{-22}(x)$ and $f_{-23}(x)$

The results of wavelet approximation for consequent N are collected in Table.2.

TABLE 2
Wavelet approximation for different N.

f_{ψ_k}	α	$N_{\lvert f_{\psi_k}\rvert}$	THD
f_{ψ_0}	π	1	48,37 %
f_{ψ_1}	π/4	2	23,60 %
f_{ψ_2}	π/6	3	13,70 %
f_{ψ_3}	π/8	4	11,44 %

The comparison between Table 1 and Table 2 shows that the for the same number of independent voltage sources the THD factor varies depending on the used transforming method.

4. CONCLUSIONS

The objective of the paper was to describe a novel proposal of the AC stepped waveforms synthesis. The discussion proved the usefulness of such mathematical tools like Fourier and wavelet transform. The Fourier transform has been used to make a composition of the stepped waveform built from consecutive rectangular pulses. This proposal relates to the addition of pulses “along x axis” or in the time scale. It is in contradiction to the typical use of vertical addition of pulses applied largely in multilevel converters. It was presented that even in case of two DC sources it was possible to reduce significantly the harmonic content by optimization of shape parameters. It can be useful in industrial applications.

The second proposal is based on wavelet transform as a handy mathematical tool to design multilevel converters (topology and control). The resulting coefficients of wavelets are proportional to DC sources needed in cascaded converter. The comparison between two methods of synthesis indicated the advantages of wavelet transform when there was a low number of DC sources.

The possible application area belongs mainly to higher power converters especially in distributed power generation systems with DC sources like photovoltaic or fuel cells with discrete voltage levels. In the sources of this kind there is an easy way to group cells to obtain the desired voltage levels for the multilevel converter. This new solution becomes an attractive alternative to a traditional output filtering technique utilizing passive components.

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SZEREG FOURIERA I TRANSFORMATA FALKOWA ZASTOSOWANE DO SYNTEZY SCHODKOWYCH PRZEBIEGÓW WYJŚCIOWYCH PRZEKSZTAŁTNIKÓW WIELOPOZIOMOWYCH

J. IWASZKIEWICZ, J. PERZ

STRESZCZENIE *W wielu zastosowaniach, takich jak systemy energetyki rozproszonej lub urządzenia zasilania bezprzerwowego, podstawowym wymaganiem jest dostarczanie do odbiorników sinusoidalnego napięcia o częstotliwości 50 Hz, charakteryzującego się niską zawartością harmonicznych. Napięcie dostarczane ze źródeł prądu stałego takich jak akumulatory, systemy fotowoltaiczne czy ogniwa paliwowe jest przetwarzane na napięcie przemienne za pomocą przekształtników energoelektronicznych. Jakość przebiegów generowanych przez te urządzenia, w szczególności zaś współczynnik zawartości wyższych harmonicznych* THD *(Total Harmonic Distortion), powinny odpowiadać odpowiednim standardom. Typowym rozwiązaniem problemu przekształcania energii elektrycznej jest dwu poziomowy przekształtnik energoelektroniczny (VSI - Voltage Source Inverter), sterowany metodą modulacji szerokości impulsów (PWM - Pulse Width Modulation). Ta metoda kształtowania przebiegów ma dobrze znane niedogodności związane z wysoką częstotliwością przełączeń łączników, takie jak straty mocy w łącznikach półprzewodnikowych oraz potrzeba stosowania dodatkowych filtrów w celu eliminacji z przebiegów wyjściowych składowych wysokiej częstotliwości. Niedogodności te mogą być znacznie zmniejszone przy zastosowaniu przekształtników wielopoziomowych sterowanych metodą modulacji amplitudy. Referat prezentuje nowe metody formowania przebiegów schodkowych falowników wielopoziomowych. Rozdział 1 przedstawia propozycję rozwiązania zagadnienia aproksymacyjnego polegającego na aproksymacji funkcji* $f(x)=sin(x)$ *za pomocą ciągu funkcji* $g_n(x)$ *opisujących impulsy prostokątne. Parametry tego ciągu określono wykorzystując współczynniki Fouriera szeregu ortogonalnego funkcji* $g_n(x)$. *W rozdziale 2 przeanalizowano spektra harmonicznych przebiegów napięcia schodkowego generowanego w ten sposób i poszukano optymalnego, w sensie minimalnej zawartości wyższych harmonicznych (*THD*), kształtu takiego przebiegu dla falownika trójpoziomowego. Wykazano, że stosując proponowane metody opty-*

malizacyjne można uzyskać znaczące zmniejszenie współczynnika THD *(z 31,09 % do 20,40 %). Rozdział 3 poświęcony został wykorzystaniu nowego narzędzia matematycznego, jakim jest transformata falkowa, do syntezy przebiegów schodkowych. Opisano pierwsze przekształcenie falkowe Haara, a następnie przeanalizowano możliwości wykorzystania dyskretnej odwrotnej transformaty falkowej do syntezy przebiegów przemiennych przekształtników wielopoziomowych.*