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VIBRATION SUPPRESSION IN TWO-MASS DRIVE SYSTEM USING PI SPEED CONTROLLER WITH DIFFERENT ADDITIONAL FEEDBACKS

ABSTRACT *In the paper an analysis of control structures for the electrical drive system with elastic joint was carried out. The synthesis of the control structure with PI controller supported by different additional feedbacks was presented. The pole placement method was applied. The analytical equations which allow for calculating the control structure parameters were given. In order to damp the torsional vibration effectively the application of the feedback from one selected state variable is necessary. In the literature a big number of possible feedbacks was reported. In the paper it was shown that all systems with one additional feedback can be divided into three different groups, according to their dynamical characteristics. Simulation results were confirmed experimentally in the laboratory set up.*

Keywords: *electric drives, two–mass drive system, suppression of vibration (oscillation)*

1. INTRODUCTION

The drive system is composed of a motor connected to a load machine through a shaft. In many cases the joint is assumed to be stiff, yet in a number

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of applications, such as rolling-mill drives, robot-arms, servo-systems, textile-drives, throttle systems, conveyor belts or deep space antenna drives, this assumption can lead to damaging oscillations [1]-[4]. The speed oscillations decrease the system characteristics and the product quality; the system can even loose the stability.

The control problem is especially difficult when not all system state variables are measurable, which happens very often in industrial applications. The most popular classical cascade structure with parameters of PI speed controller adjusted according to the symmetrical criterion can not damp the torsional vibrations effectively.

To improve the performances of the drive, the additional feedback loop from one selected state variable can be used. The additional feedback allows setting the desired value of the damping coefficient, yet the free value of resonant frequency cannot be achieved simultaneously. The additional feedbacks can be inserted to the electromagnetic torque control loop or to the speed control loop.

In [2] the additional feedback from derivative of the shaft torque inserted to the electromagnetic torque node was presented. The authors investigated the proposed method and applied it to the two- and three-mass system. Nevertheless, the proposed estimator of the shaft torque is quite sensitive to measurement noises, so suppression of high frequency vibrations is difficult and additionally the dynamics of the system decreases. In the paper [3] it was shown that in the case of the above mentioned structure, for the same value of the assumed damping coefficient, two different feedback gains can be designed, resulting in two different values of the resonant frequency of the system.

Another modification of the control structure results from inserting the additional feedback from the shaft torque. This type of feedback was utilised in [3]. The damping of the torsional vibration is reported to be successful. This structure is less sensitive to measurement noises than the former one, since in the analysed system the derivative of the shaft torque does not exist. In the paper [4] the feedback from the difference between motor and load speed was utilized. Although the oscillations were successfully suppressed the authors claim the loss of response dynamics and larger load impact effect. The additional feedback from the derivative of the load speed was proposed in [4], resulting in the same dynamical performance as for the previous control structure.

Another possible modification of the classical structure is based on the insertion of additional feedback to the speed control loop. In [3], [4] the feedback from the load speed was applied. The authors argued that this feedback can ensure good dynamical characteristics and is able to damp vibrations effectively. The same results can be obtained by applying the feedback from difference between motor and load speed.

The main goal of this work is a systematic analysis and the presentation of the design guidelines for the speed control structures of two-mass system with PI speed controller supported by different additional feedbacks as well as a comparison of dynamic properties of such structures. Besides the structures mentioned in the introduction, in the paper three additional structures were analyzed: two – with feedback inserted to the torque loop and one with feedback inserted to the speed loop. The theoretical investigation and the simulation results presented in the paper were confirmed by experimental tests in a laboratory set-up.

2. CONTROL STRUCTURE ANALYSIS

In the paper the commonly-used model of the drive system with the resilient coupling is considered. The system is described by the following state equation (in per unit system), with nonlinear friction neglected:

$$\frac{d}{dt}\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ \Gamma_s(t)\end{bmatrix}=\begin{bmatrix}0 & 0 & \frac{-1}{T_1}\\ 0 & 0 & \frac{1}{T_2}\\ \frac{1}{T_c} & \frac{-1}{T_c} & 0\end{bmatrix}\begin{bmatrix}\omega_1(t)\\ \omega_2(t)\\ \Gamma_s(t)\end{bmatrix}+\begin{bmatrix}\frac{1}{T_1}\\ 0\\ 0\end{bmatrix}[\Gamma_e]+\begin{bmatrix}0\\ \frac{-1}{T_2}\\ 0\end{bmatrix}[\Gamma_L] \tag{1}$$

where:

- ω_1 – motor speed,
- ω_2 – load speed,
- Γ_e – motor torque,
- Γ_s – shaft (torsional) torque,
- Γ_L – disturbance torque,
- T_1 – mechanical time constant of the motor,
- T_2 – mechanical time constant of the load machine,
- T_c – stiffness time constant.

Parameters of the analysed system are the following: T_1 = 203 ms, T_2 = 203 ms, T_c = 2.6 ms.

A typical electrical drive system is composed of a power converter-fed motor coupled to a mechanical system, a microprocessor-based speed and torque controllers, current, speed and/or positions sensors used for feedback signals. Usually, cascade control structure containing two major control loops is used. The inner control loop performs a motor torque regulation and consists of the power converter, electromagnetic part of the motor, current sensor and respective current or torque controller. Therefore, this control loop is designed to provide a sufficiently fast torque control, so it can be approximated by an equivalent first order term. In the case of the induction motor it could be field oriented or direct torque control method; in the system with DC motors it is usually a PI current controller tuned with the help of modulus criterion. If this control is ensured, the type of the driven machine makes no difference for the outer speed control loop. This outer control loop consists of the mechanical part of the drive, speed sensor, speed controller, and is cascaded to the inner torque control loop. It provides speed control according to its reference value.

As it was said before, the suitable oscillations damping of the two-mass system can be obtained using different additional feedbacks [3]-[8]. The block diagram of the drive system with a simplified inner loop and additional feedbacks is presented in Fig. 1.

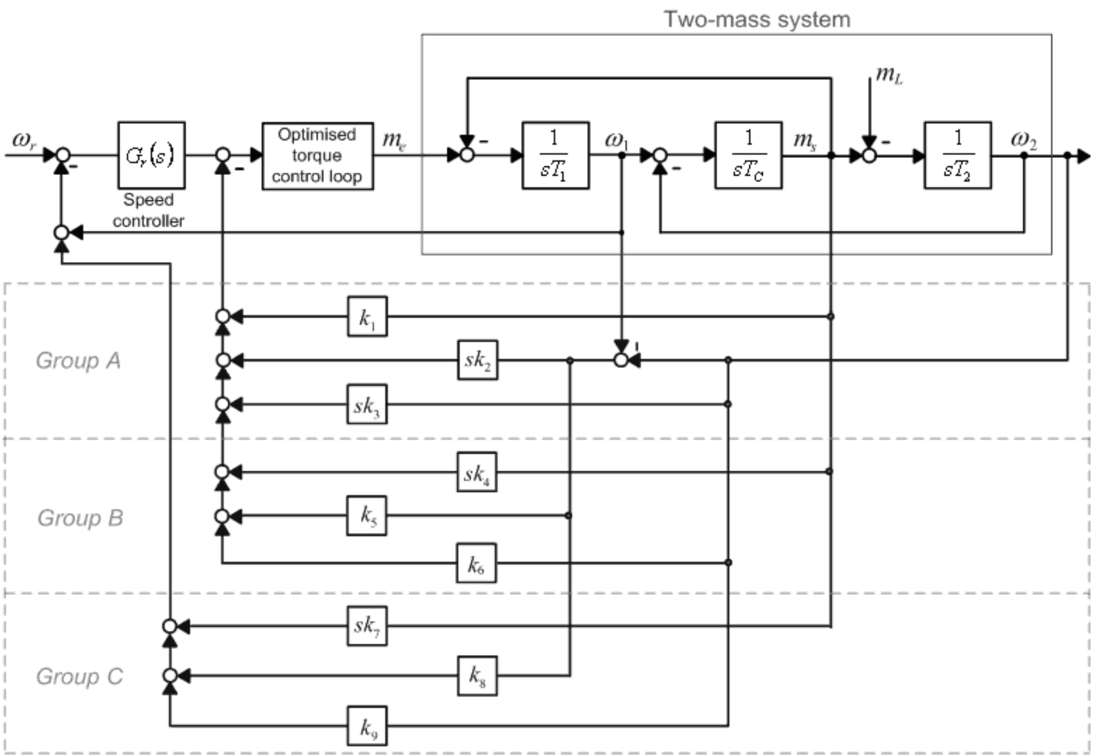

Fig. 1. The control structure with different additional feedbacks

In the literature every feedback has a specific name. It creates an impression of a large number of possibilities to shape the dynamical characteristics. However, the link between different feedbacks (in every group) can be found out from the Fig. 4. The relationship can be directly seen between feedbacks k_4 and k_5 in group B: the derivative of the shaft torque is simply the difference between motor and load speeds multiplied by the stiffness coefficient. The same relationship exist between the feedbacks k_7 and k_8 in group C. The last feedback k_9 is based on the motor and load speeds. The link between feedbacks k_1 and k_2 is not so clearly seen in group A. However if the electromagnetic and load torques are neglected, the derivative of the difference speeds is the shaft torque multiplied by the following coefficient: $d(\omega_1 - \omega_2)/dt = -m_s(1/T_1 + 1/T_2)$. So, despite of nine feedbacks, introduced as additional closed loops in the cascade control structure, in fact only three types of control structures exist, whose dynamical characteristics are different, as it was presented in Fig. 4. It will be proved in the following sections of this paper.

The closed-loop transfer functions from reference speed to the motor and load speed respectively, for the control structure demonstrated in Fig. 4, are given by the following equations (with the assumption that the optimized transfer function of the electromagnetic torque control loop is equal 1):

$$G_{\omega_1}(s)=\frac{\omega_1(s)}{\omega_r(s)}=\frac{G_r(s)\left(s^2T_2T_c+1\right)}{s^3T_2T_c(T_1+k_2)+s^2T_2(G_r(s)T_c+G_r(s)k_7+G_r(s)T_ck_8)+s(T_1+T_2(1+k_1+sk_4+sT_ck_5)+k_3)+G_r(s)(1+k_9)+k_6} \tag{2}$$

$$G_{\omega_2}(s)=\frac{\omega_2(s)}{\omega_r(s)}=\frac{G_r(s)}{s^3T_2T_c(T_1+k_2)+s^2T_2(G_r(s)T_c+G_r(s)k_7+G_r(s)T_ck_8)+s(T_1+T_2(1+k_1+sk_4+sT_ck_5)+k_3)+G_r(s)(1+k_9)+k_6} \tag{3}$$

where:

$G_r(s) = K_P + K_I \frac{1}{s}$ is the transfer function of the PI controller.

Using the classical pole-placement method the following equations which allow to calculate the control structure parameters are grouped in Tab. 1.

TABLE 1
Mathematical formulas used for parameters setting of PI speed controllers of two-mass drive system

Control system without additional feedbacks

$$\xi=\frac{1}{2}\sqrt{\frac{T_2}{T_1}},\ \omega_0=\sqrt{\frac{1}{T_2T_c}},\ K_P=2\sqrt{\frac{T_1}{T_c}},\ K_I=\frac{T_1}{T_2T_c}$$

Feedback k_1

$$\xi^{k1}=\frac{1}{2}\sqrt{\frac{T_2(1+k_1)}{T_1}}\ \omega_0^{k1}=\sqrt{\frac{1}{T_2T_c}},\ k_1=\frac{4\xi_r^2T_1}{T_2}-1,\ K_P^{k1}=2\sqrt{\frac{T_1(1+k_1)}{T_c}},\ K_I^{k1}=\frac{T_1}{T_2T_c}.$$

Feedback k_2

$$\xi^{k2}=\frac{1}{2}\sqrt{\frac{T_2-k_2}{T_1+k_2}},\ \omega_0^{k2}=\sqrt{\frac{1}{T_2T_c}},\ K_I^{k_2}=\frac{T_1+k_2}{T_2T_c},\ k_2=\frac{T_2-4\xi_r^2T_1}{4\xi_r^2+1},\ K_P^{k2}=2\sqrt{\frac{(T_1+k_2)(T_2-k_2)}{T_2T_c}}$$

Feedback k_3

$$\xi^{k_3}=\frac{1}{2}\sqrt{\frac{T_2+k_3}{T_1}},\ \omega_0^{k_3}=\sqrt{\frac{1}{T_2T_c}},\ k_3=\xi_r^2 4T_1-T_2,\ K_P^{k_3}=2\sqrt{\frac{T_1(T_2+k_3)}{T_2T_c}},\ K_I^{k_3}=\frac{T_1}{T_2T_c}.$$

Feedback k_4

$$\xi^{k_4}=\sqrt{\frac{T_c+x}{4T_1T_c}(T_1+T_2)+\frac{T_c}{4(T_c+x)}-\frac{1}{2}},\ \omega_0^{k_4}=\sqrt{\frac{1}{T_2(T_c+x)}},\ k_4=xK_p^{k_4},\ K_P^{k_4}=\frac{4\xi_r\omega_0^{k_4}T_1T_c}{T_c+x},\ K_I^{k_4}=\left(\omega_0^{k_4}\right)^4T_1T_2T_c.$$

Feedback k_5

$$\xi^{k_5}=\sqrt{\frac{(T_1+T_2)(1+x)}{4T_1}+\frac{1}{4(1+x)^2}-\frac{1}{2}},\ \omega_0^{k_5}=\sqrt{\frac{1}{T_2T_c(1+x)}}.\ k_5=xK_p^{k_5},\ K_P^{k_5}=\frac{4\xi_r\omega_0^{k_5}T_1}{(1+x)},\ K_I^{k_5}=\left(\omega_0^{k_5}\right)^4T_1T_2T_c.$$

Feedback k_6

$$\xi^{k_6}=\sqrt{\frac{(1+x)^2T_1+T_1+T_2}{4T_1(1+x)}-\frac{1}{2}},\qquad \omega_0^{k_6}=\sqrt{\frac{(1+x)}{T_2T_c}}.\ k_6=xK_p^{k_6},\ K_P^{k_6}=4\xi_2\omega_0^{k_6}T_1,\ K_I^{k_6}=\left(\omega_0^{k_6}\right)^4T_1T_2T_c.$$

Feedback k_7

$$\xi^{k_7}=\frac{1}{2}\sqrt{\frac{(T_1+T_2)(T_c+k_7)}{T_1T_c}-1},\ \omega_0^{k_7}=\sqrt{\frac{1}{T_2(T_c+k_7)}},\ k_7=\frac{(4\xi_r^2+1)T_1T_c}{T_1+T_2}-T_c,\ K_P^{k_7}=4\xi_r\left(\omega_0^{k_7}\right)^3T_1T_2T_c,$$

$$K_I^{k_7}=\left(\omega_0^{k_7}\right)^4T_1T_2T_c.$$

Feedback k_8

$$\xi^{k_8}=\frac{1}{2}\sqrt{\frac{T_1k_8+T_2(1+k_8)}{T_1}},\ \omega_0^{k_8}=\sqrt{\frac{1}{(1+k_8)T_2T_c}},\ k_8=\frac{\xi_r^2 4T_1-T_2}{T_1+T_2},\ K_P^{k_8}=\frac{4\xi_r\omega_0^{k_8}T_1}{1+k_8},\ K_I^{k_8}=\frac{T_1}{(1+k_8)^2T_2T_c}.$$

Feedback k_9

$$\xi^{k_9}=\frac{1}{2}\sqrt{\frac{T_1+T_2}{T_1(1+k_9)}-1},\ \omega_0^{k_9}=\sqrt{\frac{1+k_9}{T_2T_c}},\ k_9=\frac{T_1+T_2}{T_1(4\xi_r^2+1)}-1,\ K_P^{k_9}=4\xi_r\omega_0^{k_9}T_1,\ K_I^{k_9}=\frac{T_1(1+k_9)}{T_2T_c}.$$

a) b)

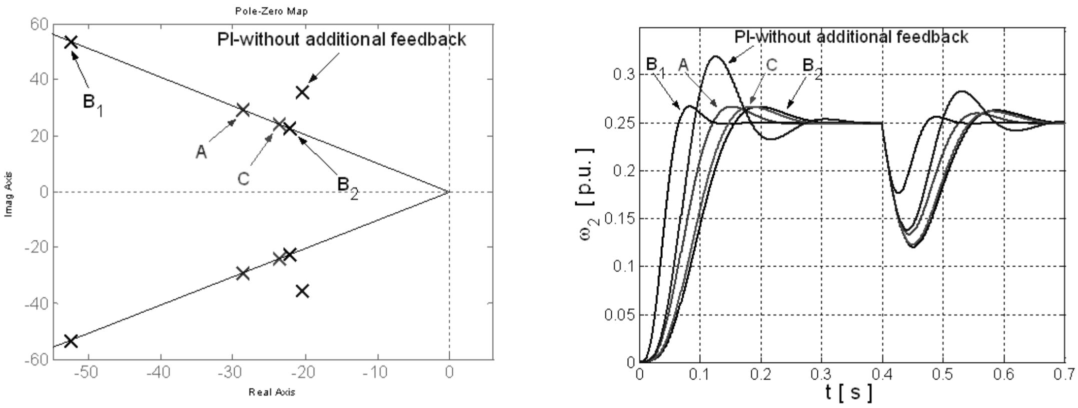

Fig. 2. The closed-loop poles location (a) and the load speed transient (b) of all considered system

In Fig. 2a the closed-loop poles loci of all considered control systems are presented. These systems are of the fourth order and the presented poles are double. The closed-loop poles location of the system without additional feedback are situated relatively close to the imaginary axis. The closed-loop poles location of the system with one additional feedback depends on the assumed damping coefficient, which in each case was set to ξ_r = 0.7. The closed-loop poles of the system from group B (in this case B_1) have the highest value of the resonant frequency, when the additional feedback coefficient (k_4, k_5 or k_6) has a negative value. The rising time of the speed response of the mentioned drive is approximately twice as short as that of the remaining systems. The next faster system is the control structure belonging to group A. The dynamical characteristic of the remaining structures (group C and group B_2) are quite similar. In Fig. 2b the load speed transients of all considered systems are presented.

3. RESULTS OF EXPERIMENTAL TESTS

All theoretical considerations were confirmed experimentally. The laboratory set-up, presented in Fig. 3, was composed of a DC motor driven by a four-quadrant chopper. The motor was coupled to a load machine by an elastic shaft (a steel shaft of 5 mm diameter and 600 mm length). The motors had the nominal power of 500 W each. The control and estimation algorithms were implemented by a digital signal processor using the dSPACE software. To avoid the limitation of the electromagnetic torque, the reference value was set to 25 % of the nominal speed.

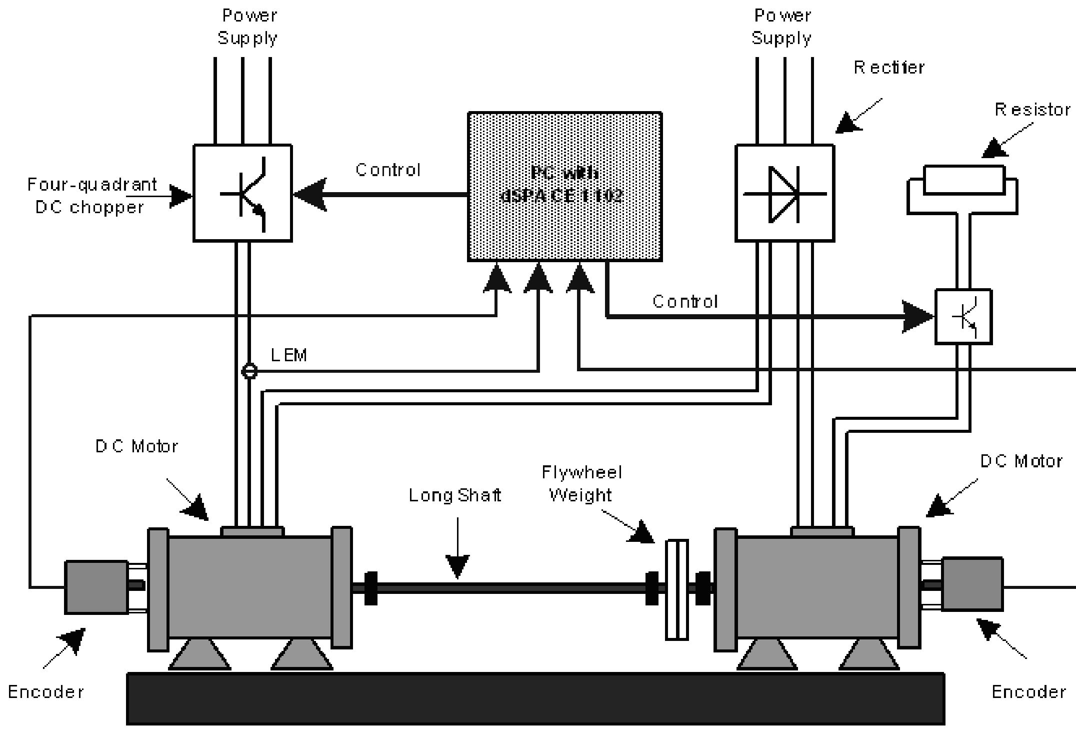

Fig. 3. Schematic diagram of experimental set-up

a) b)

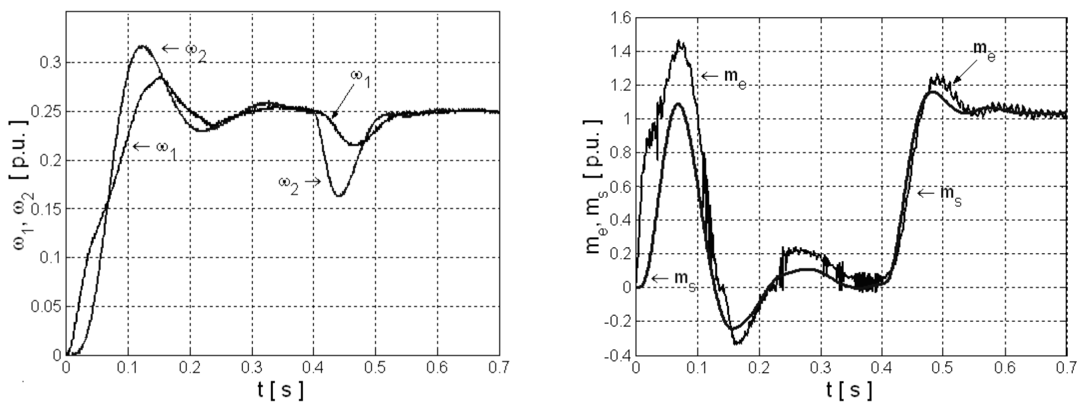

Fig. 4. Experimental transients of the system without additional feedback

The first considered control structure was the system with a PI controller without additional feedback. As shown in Fig. 4a, the load speed of the system has a large overshoot and quite a big settling time. The maximum value of electromagnetic torque is about 1.4 value of the nominal torque (Fig. 4b).

Next the system with additional feedback from the derivative of the difference between motor and load speeds (k_2) was investigated (the group A). The damping coefficient of the control structure was set to $\xi_r = 0.7$ (Fig. 5).

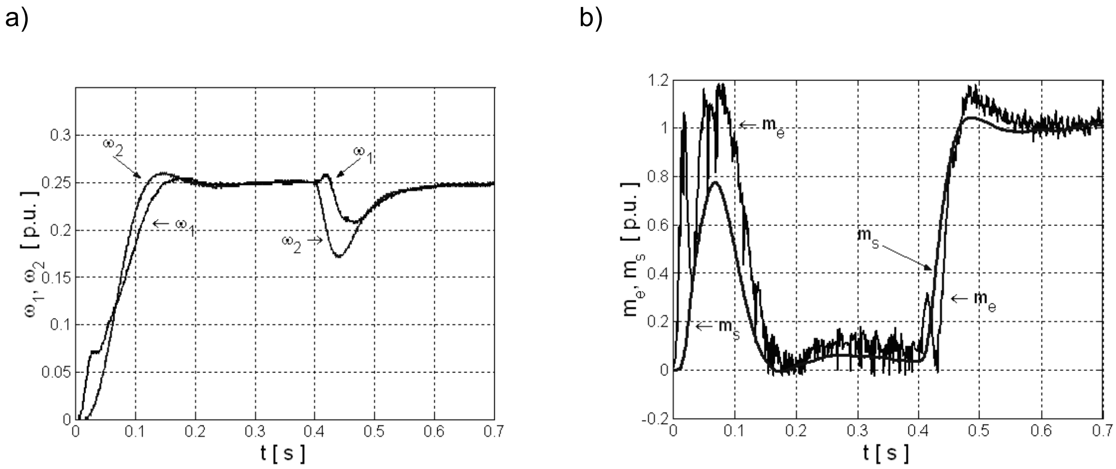

Fig. 5. Experimental transients of the system with additional feedback form the derivative of the difference between speeds (k_2) – group A; $\xi_r = 0.7$

Then the control structure with additional feedback from the difference between speeds (k_5) was examined (group B). This system has two sets of parameters which allow setting the desired value of the damping coefficient. First the drive system with a bigger value of resonant frequency was investigated. In Fig. 6a,b the system transients are presented. The responses to the speed reference change as well as the load torque change are very fast, what is clearly seen in the motor speed transient. But the maximum value of electromagnetic torque during start-up was four times bigger than the nominal one. Then the drive system with a smaller value of resonant frequency was tested. The system transients are shown in Fig. 6c,d.

The next examined system was the structure with additional feedback from the difference between the motor and the load speeds inserted in the speed node (k_8 - group C). In Fig. 7 transients of the analyzed system are presented. The load speed overshoot has a small value, resulting from the assumed value of the system damping coefficient.

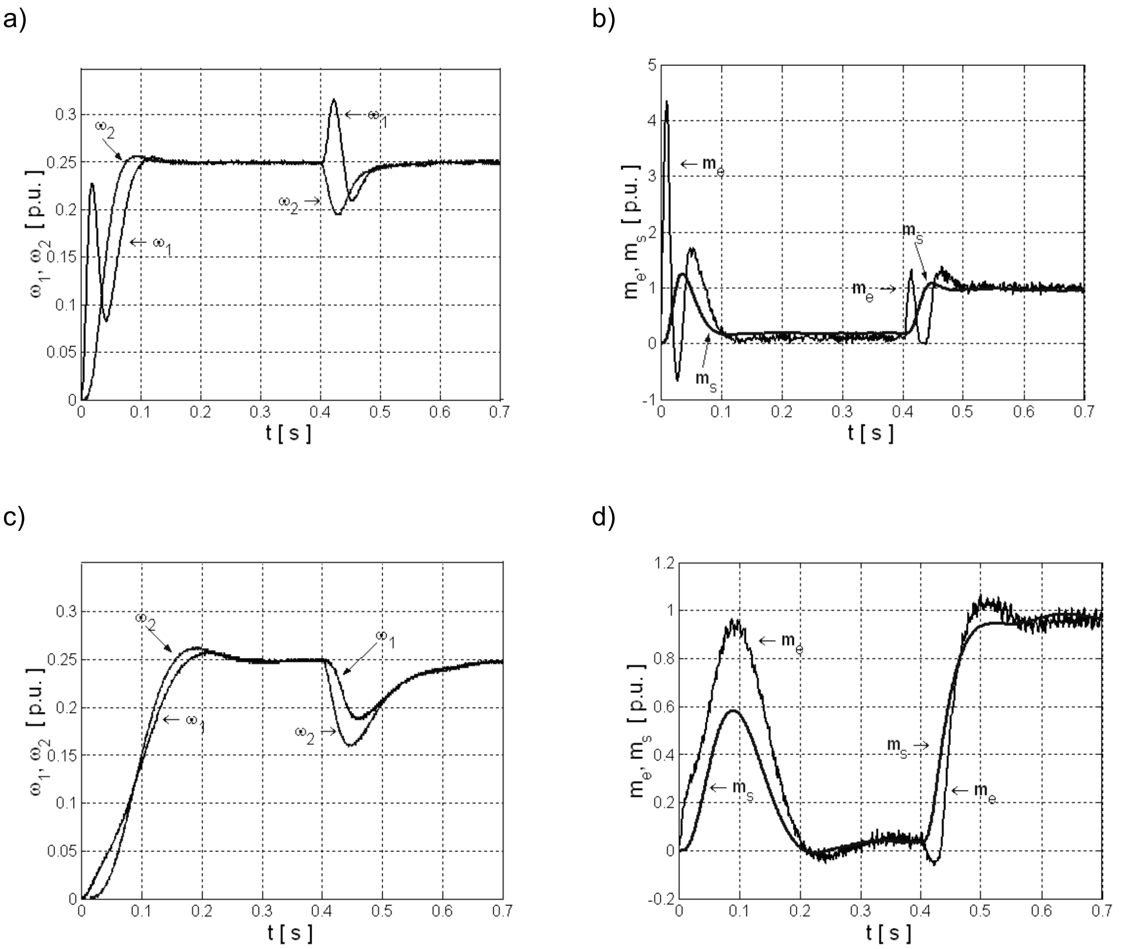

Fig. 6. Experimental transients of the system with additional feedback form the difference between speeds (k_5) – the group B_1(a, b) and B_2(c, d); $\xi_r = 0.7$

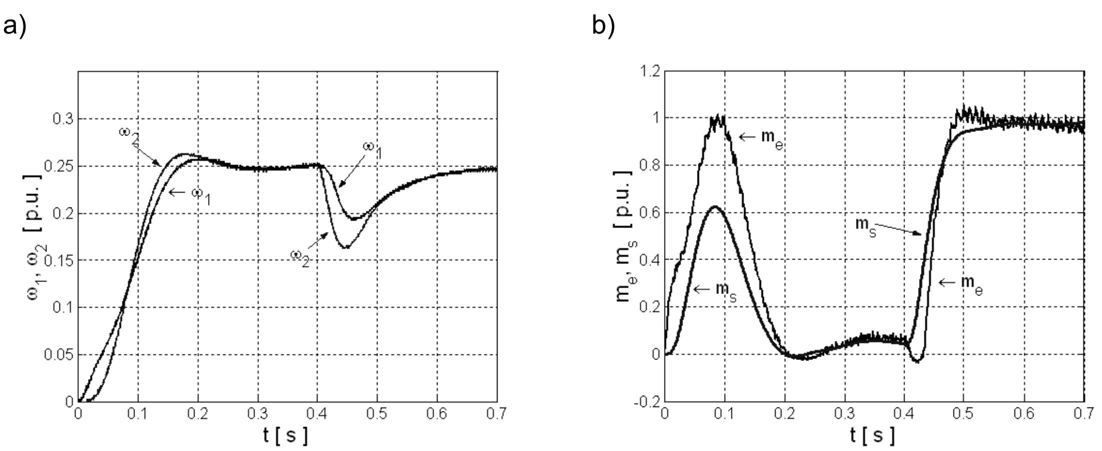

Fig. 7. Experimental transients of the system with additional feedback form the difference between speeds (k_8) – the group C; $\xi_r = 0.7$

The presented results confirmed the analytical investigations and simulation tests. The slight difference between the real and simulation transients comes from the fact that in the real system nonlinearities such as friction, non-linear characteristic of the shaft exist, which have been neglected in simulation.

4. CONCLUSIONS

In the paper different cascade control structures with additional feedbacks for the electrical drive system with flexible connection were investigated. The performances of the control structure without additional feedback depend on the mechanical parameters of the considered drive and are rather poor. It results from the fact that the system is of the fourth order and there are only two controller parameters (K_I, K_P), which cannot form the desired damping coefficient and resonant frequency simultaneously. In order to damp the torsional vibrations effectively the application of one additional feedback is necessary. As results from the review of the literature, the application of different feedbacks is possible. Despite of the large number of existing structures, the systems with one additional feedback can be divided into three different groups, according to their dynamical characteristic. It was proved in the paper that all structures within a certain group have the same pole placement and thus the same transient responses.

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OGRANICZANIE DRGAŃ DWUMASOWEGO UKŁADU STEROWANIA Z DODATKOWYMI SPRZĘŻENIAMI ZWROTNYMI

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STRESZCZENIE *W układach napędowych, elementy mechaniczne przenoszenia momentu posiadają skończoną sztywność. Podlegają one w czasie pracy odkształceniom sprężystym i plastycznym. W większości przypadków, w napędzie elektrycznym pomija się elastyczność połączenia wału z silnikiem. Istnieje jednak grupa napędów, w której nieuwzględnienie skończonej sztywności wału może prowadzić do powstania drgań mechanicznych, a w konsekwencji do znaczących oscylacji zmiennych elektromagnetycznych. Może to spowodować pogorszenie przebiegu procesu technologicznego, skrócenie żywotności napędu itp. Przykładową grupą są napędy walcarek, taśmociągów, napędy robotów przemysłowych [1]-[4].*

Aby poprawić właściwości dynamiczne takich napędów i ograniczyć oscylacje prędkości i momentu silnika, powszechnie stosuje się zmodyfikowane struktury sterowania. Wykorzystują one dodatkowe sprzężenia zwrotne od wybranych zmiennych stanu. Tłumienie oscylacji prędkości obciążenia możliwe jest poprzez wprowadzenie dodatkowego sprzężenia zwrotnego od: momentu skrętnego, różnicy prędkości silnika i obciążenia, prędkości obciążenia lub ich pochodnych wprowadzonych do węzła momentu elektromagnetycznego lub prędkości.

W referacie przedstawiono analizę układu składającego się silnika napędowego połączonego z maszyną obciążającą za pośrednictwem elastycznego sprzęgła. Przeprowadzono syntezę sterowania układu dwumasowego z regulatorem prędkości PI z różnymi dodatkowymi sprzężeniami zwrotnymi (rys. 1). Przedstawiono wzory umożliwiające dobór nastaw regulatorów i uzyskanie dowolnego współczynnika tłumienia układu regulacji (Tab.1). Wskazano na ograniczenia w kształtowaniu charakterystyk dynamicznych w zależności od stopni swobody układu regulacji. Referacie pokazano, że mimo dużej liczby możliwych sprzężeń zwrotnych przedstawianych w literaturze, wszystkie struktury sterowania z jednym dodatkowym sprzężeniem zwrotnym mogą być podzielone na trzy grupy o identycznych właściwościach dynamicznych (rys. 1).

Badania symulacyjne zawarte w pracy zostały zweryfikowane na stanowisku laboratoryjnym którego schemat przedstawiono na rys. 3. Na rys. 4 przedstawiono przebiegi zmiennych układu bez dodatkowych sprzężeń zwrotnych. Podobnie jak w badaniach symulacyjnych w prędkości obciążenia występują duże słabo tłumione oscylacje. Kolejno przebadano układy z dodatkowymi sprzężeniami zwrotnymi. Zgodnie z wzorami przedstawionymi w Tab. 1 nastawy układu regulacji dobrano w sposób zapewniający uzyskanie współczynnika tłumienia drgań $\xi_r = 0.7$. Otrzymane wyniki potwierdzały przeprowadzone badania symulacyjne. Układy sterowania z jednym dodatkowym sprzężeniem zwrotnym efektywnie tłumiły drgania skrętne układu (rys. 5-7).