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## FOUR-QUADRANT OPERATION OF SWITCHED RELUCTANCE MACHINE

**ABSTRACT** *Switched reluctance machines (SRM) topics in the range of motoring and generating mode of operation are presented in this paper. Torque direct control method has been discussed. Mathematical and simulation models developed in Matlab/Simulink system are also shown. The results of the simulated and experimental testing of the generating and motoring mode at steady and dynamic states are described here. Also conclusions are drawn in this paper.*

**Keywords:** *modelling and simulation, switched reluctance motors, control methods*

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## 1. INTRODUCTION

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There are many drive systems where there is a requirement for speed to be controlled in a wide range, as well as for good dynamic properties with an option for the energy to be returned to its supply. Switched reluctance machines (SRM) are, among the others, utilized as drive systems in electric vehicles. These machines are of simple design, high start-up torque and low inertia torque [1, 2]. Moreover have high maximum speed and are of high reliability and their speed can be controlled in a wide range of operation. The main disadvantage of the SRMs is high torque ripple inherent to the very design, as well to the pulse supply of the winding. It is thus necessary to apply some control to minimize torque ripple. In the electric car applications the drives have to have the ability to return a portion of energy into the supply. The correct operation of SRM is obtained by supplying the machine winding at the interval when the self-inductance phase in winding rises (motoring mode) or falls (generating mode). The SRMs can operate at two regions of speed control, i.e. constant torque and constant power regions.

It is the aim of this paper to show the results of the simulation and experimental research data, conducted for a three-phase SRM 6/4, operating in four quadrants.

The simulation and experimental research was performed on the model three-phase SRM machine of 6/4 configuration, developed by the authors. Simulation research is based on SRM models prepared for the Matlab/Simulink program environment. These models include non-linearity of the magnetic circuit. The non-linear flux linkage-current-angle and torque-current-angle characteristics have been obtained based on field calculations by finite elements method. The method of SRM control that minimizes torque ripple in the motor, developed by the authors is discussed in this paper. The control method described here has been realized by simulation in Matlab/Simulink system, as well as by simulation on the test stand fitted with dSpace DS 1104 card. The simulation and experimental data support the merits of the suggested method of the SRM control in the range of four-quadrant operation.

## 2. FOUR-QUADRANT OPERATION OF SRM

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The four-quadrant operation of the switched reluctance machine requires, depending on the operation mode (motoring or generating) as well as on the

direction of the rotor rotation, requires selecting of the correct method of winding supply. The SRMs can operate in any quadrant, similarly to other machine types, within the region of constant torque or constant power (Fig. 1.). The operation at constant torque is assured by using the current control or instantaneous torque control. The operation at constant power can be achieved by controlling the turn-on  $\theta_{on}$  and turn-off  $\theta_{off}$  angle above the base speed.

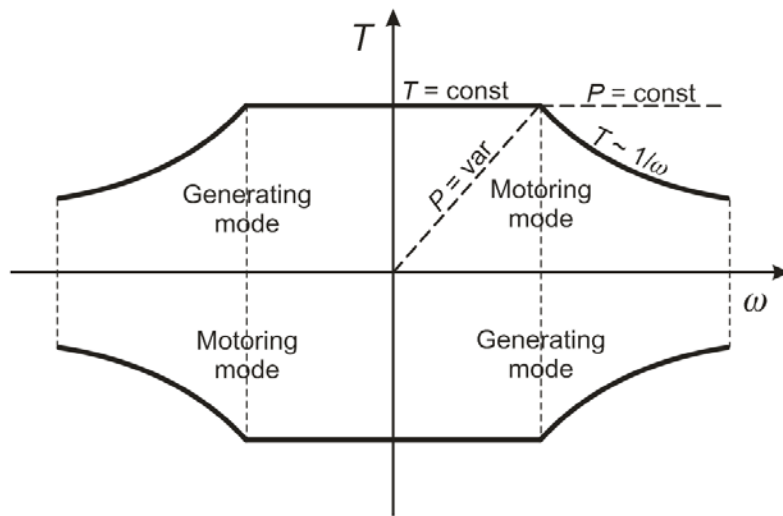


Fig. 1. Motoring and generating operation of the SRM

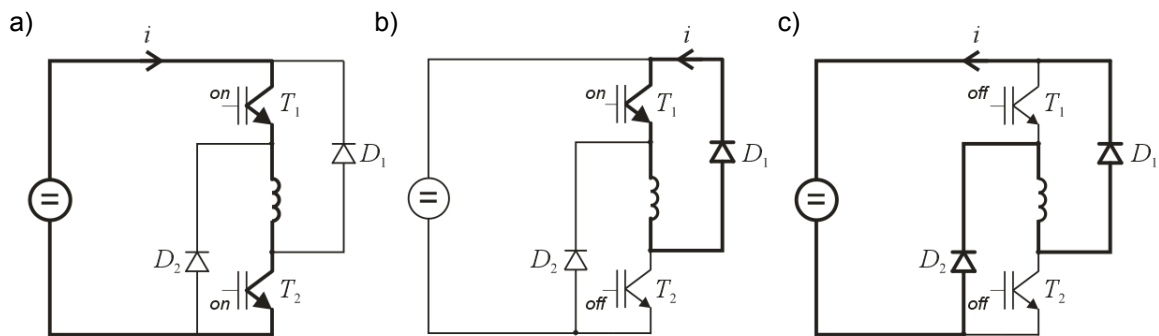
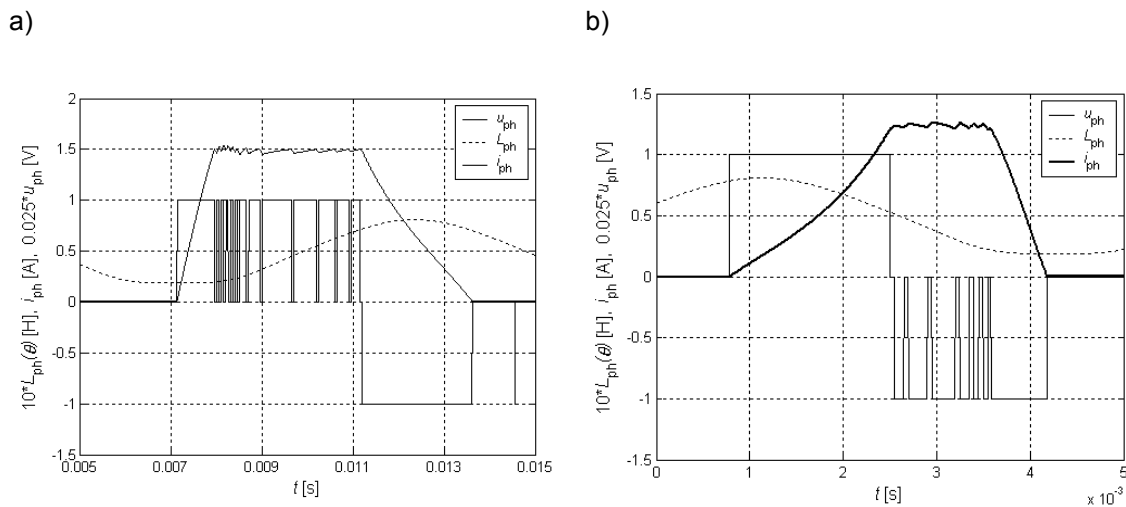


Fig. 2. Operation states of the SRM supply: supply (a), zero voltage (b) and energy return to the supply (c)

Figure 2 shows three basic states of the supply operation, i.e. supply (Fig. 2a), zero voltage (Fig. 2b) and energy return to the supply (Fig. 2c) states. The direction of the generated torque of the SRM does not depend on the direction

of the current flowing through the winding, it rather depends on the sign of the derivative of the self-inductance phase in winding along the angle of the rotor. In order to switch the SRM operation mode (from motoring to generating or vice versa), it is necessary to change the value of the angles controlling the supply ( $\theta_{on}$  and  $\theta_{off}$ ), so as conducting of the current occurs in the range of rising or falling inductance respectively. The Fig. 3 shows how typical current, voltage and the values of self-inductance phase in winding change in time at motoring (Fig. 3.a) and generating (Fig. 3b) operation.



**Fig. 3. Inductance, current and voltage trains for the:**  
motoring (a), and generating (b) operation

### **Motoring operation**

The switched reluctance machine generates the driving torque if only it is supplied in the region of rising inductance (Fig. 3a). The power supply has to occur with sufficient advance, so as the phase current achieved the required magnitude at the point when inductance starts to rise rapidly. Turn-on angle ( $\theta_{on}$ ) is a function of set current, speed and design properties of the machine [3]. The turning-off has to occur also at specified advance ( $\theta_{off}$ ), so as the current fades before the range of falling inductance. A correct selection of turn-on and turn-off angles makes it possible to optimize the operation of the machine to get minimum torque ripple, maximum efficiency and limited machine-generated noise. To control SRM at motoring operation, within the constant torque range, it will suffice to use two-level current or torque controller.

### **Generating operation**

Generating operation can occur, when the machine is controlled within the region of falling inductance (Fig. 3b). During the generating operation, in

contrast to the motoring operation, where the current rises at supply to the winding (there is positive voltage in winding – Fig. 2a), current rises as soon as the power converter operates at zero voltage (Fig. 2b). At low angular rotor speeds however, a dead region is found [4], where the machine is supplied both with mechanical and electric energy. It could be called a quasi-generating state of operation, as some braking torque is developed and energy supplied to the machine is entirely converted to losses. Return of energy to the supply is possible only after certain speed is exceeded (Fig. 2c). At zero-voltage condition current fails to rise due to too low rotation voltage. If the energy returned by the machine to the supply is greater than the energy drawn at excitation process, then the SRM operates at purely generating mode. In order to maintain the phase current value constant at generating operation, it is necessary to use three-level torque or current controllers. It is owed to the fact, that the rotation voltage value is a product of current, speed and  $\partial L / \partial \theta$  ratio related to rotor angle, which in this region of operation has a negative value. Thus, depending on required phase current value and actual angular rotor speed, it is necessary to apply voltage of  $U_{dc}$ , 0 or  $-U_{dc}$  to the winding.

### 3. INSTANTANEOUS TORQUE CONTROL

One of the methods of minimizing torque ripple in SRM, is to control the instantaneous torque [4, 5]. In this method, winding voltage on the terminals of phase winding of the machine can be determined by:

$$u(i, \theta) = \begin{cases} U_{dc} & \text{if } [(\theta_{on} \leq \theta \leq \theta_{off}) \& (i \leq I_{min})] \\ U_{dc} \cdot f_{cT}(T_e, T_{ref}) & \text{if } [(\theta_{on} \leq \theta \leq \theta_{off}) \& (i > I_{min})] \\ -U_{dc} & \text{if } [\theta_{off} < \theta < \theta_{ex}] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where the following designations have been used:  $U_{dc}$  – supply voltage,  $\theta_{on}$  – turn-on angle,  $\theta_{off}$  – turn-off angle,  $\theta_{ex}$  – angle of zero phase current,  $I_{min}$  – minimum phase current, which maintains torque at required level [5]. The  $f_{cT}()$  function describes the operation of torque controller, and its value changes to motor torque value  $T_e$  as compared to the reference torque value  $T_{ref}$  and to the machine operation mode.

For the motoring operation, i.e. in the supply range ( $\theta_{on} \leq \theta \leq \theta_{off}$ ) it will usually suffice to utilize two-level controller. The operation of such controller one can describe with function  $f_{cT}()$ , defined as follows:

$$f_{cTm}(T_e, T_{ref}) = \begin{cases} 1 & \text{if } (T_e < T_{ref}) \\ 0 & \text{if } (T_e \geq T_{ref}) \end{cases} \quad (2)$$

One can notice considering the formula (1), that torque controller does its task on given machine phase only if the current in this phase exceeds the current value  $I_{min}$ .

In generating operation, a three-level controller has been suggested. Such controller can be realized using a classical delta controller with additional hysteresis loop inside which voltage over the controlled winding will be zero. The operation of the suggested torque controller for generating mode can be described by function  $f_{cTg}()$ , defined as:

$$f_{cTg}(T_e, T_{ref}) = \begin{cases} -1 & \text{if } (T_e \geq T_{ref} + \Delta T) \\ 0 & \text{if } (T_{ref} - \Delta T < T_e < T_{ref} + \Delta T) \\ 1 & \text{if } (T_e \leq T_{ref} - \Delta T) \end{cases} \quad (3)$$

where  $\Delta T$  defines the hysteresis span.

A block diagram for the suggested SRM control system is shown in Fig. 4. This unit keeps at a minimum the motor torque ripple in the regions of both motoring and generating operation.

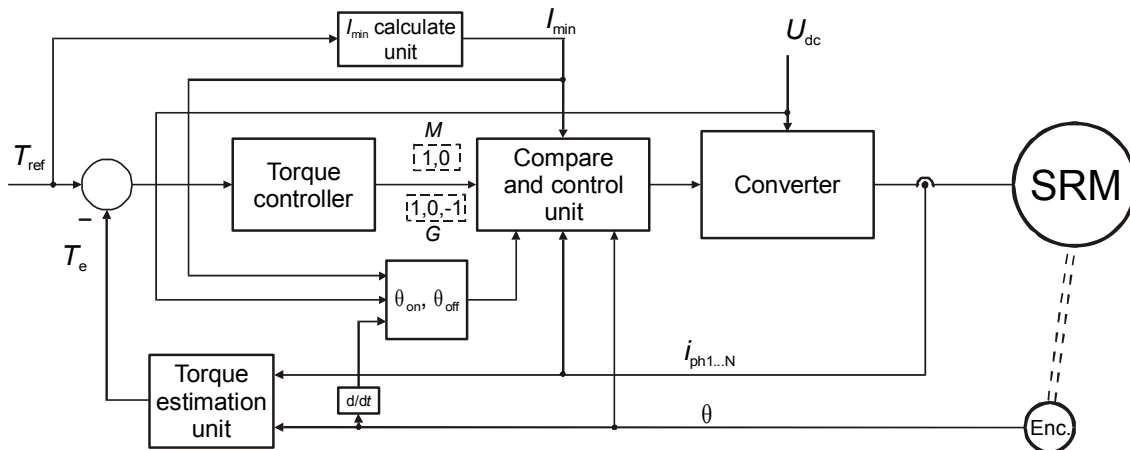


Fig. 4. A block diagram of SRM instantaneous torque control system

Performing such design solution needs application of some processor, which would be able to establish the values required for the motor-controlling values

instantly. In practice, to test the developed control algorithms, signal processing card was used.

## 4. SRM MODELLING

### **Mathematical model of SRM**

The following equations describe mathematical model of SRM with  $N$  phases, when nonlinear magnetic circuit is assumed:

$$\mathbf{u} = \mathbf{R} \mathbf{i} + \frac{d}{dt} [\boldsymbol{\psi}(\mathbf{i}, \theta)] \quad (4)$$

$$J \frac{d\omega}{dt} + B\omega + T_L = T_e, \quad T_e = \sum_{j=1}^{N_{\text{ph}}} \frac{\partial}{\partial \theta} \left[ \int_0^{i_j} \psi_j(i_1, \dots, i_j, 0, \dots, 0, \theta) di_j' \right] \quad (5)$$

$$\frac{d\theta}{dt} = \omega \quad (6)$$

gdzie:

$$\mathbf{u} = \text{col}(u_1, \dots, u_N), \quad \mathbf{i} = \text{col}(i_1, \dots, i_N), \quad \mathbf{R} = \text{diag}(R_1, \dots, R_N) \quad (7)$$

$$\boldsymbol{\psi}(\mathbf{i}, \theta) = \text{col}(\psi_1(i_1, \dots, i_N, \theta), \dots, \psi_N(i_1, \dots, i_N, \theta)) \quad (8)$$

The following designations were introduced in equations (4) through (6):  $\theta$  – rotor position angle,  $J$  – rotor inertia torque,  $B$  – viscous friction coefficient,  $T_L$  – load torque,  $\omega$  – rotor angular speed. The  $T_e$  parameter found in equation (5) stands for electromagnetic torque developed with the motor, and is calculated as a derivative of coenergy of the magnetic field along the mechanical coordinate.

### **Torque estimation**

To perform torque estimation in SRMs, steady state torque-current-angle characteristics are most frequently used. These characteristics, do not however take into consideration any dynamic phenomena which occur during operation. The analytical methods of torque estimation are very complicated mathematically and difficult to implement in actual systems. In order to establish steady state characteristics, methods based on field calculations are generally used, as well as experimental methods, where torque, current and rotor angular position at

stopped rotor are measured. For the purpose of this paper, the field analysis was applied to establish torque-current-angle characteristics. To perform calculations, ANSYS package, which makes it possible to do professional field calculations based on finite elements method (FEM), was used. Figure 5 shows the flux distribution in the designed machine. The Figure 6 shows the torque-current-characteristics obtained as a result of field calculations.

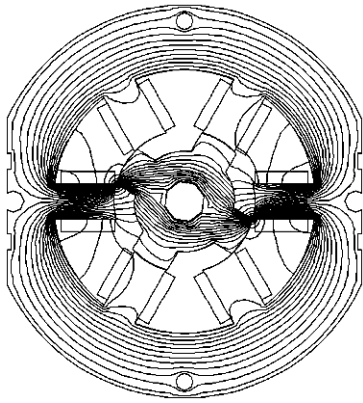


Fig. 5. Flux plot for SRM 6/4

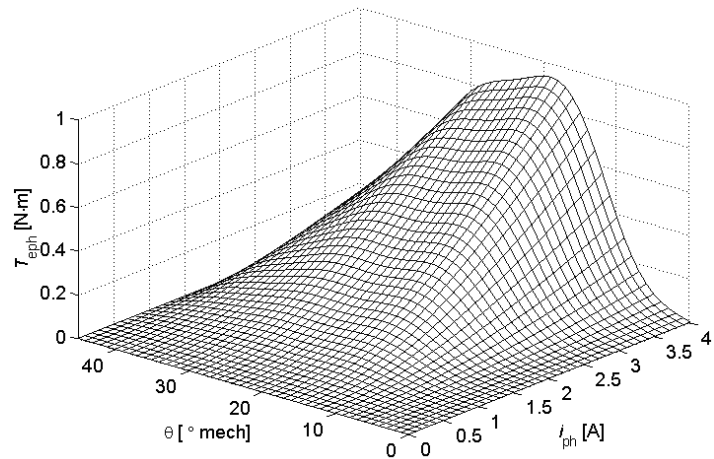


Fig. 6. Electromagnetic torque  $T_{eph}$  as a function of rotor position  $\theta$  and current  $I$

### Simulation model of SRM

Figure 7 shows developed in Matlab/Simulink system, flowchart model of the control system including the block of the SRM model based on the presented mathematical model equation (Fig. 8).

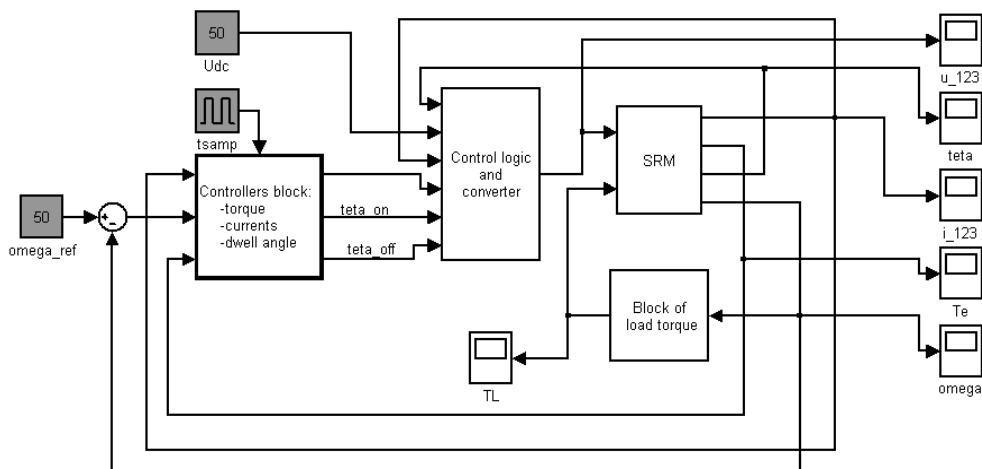


Fig. 7. SRM control system flowchart



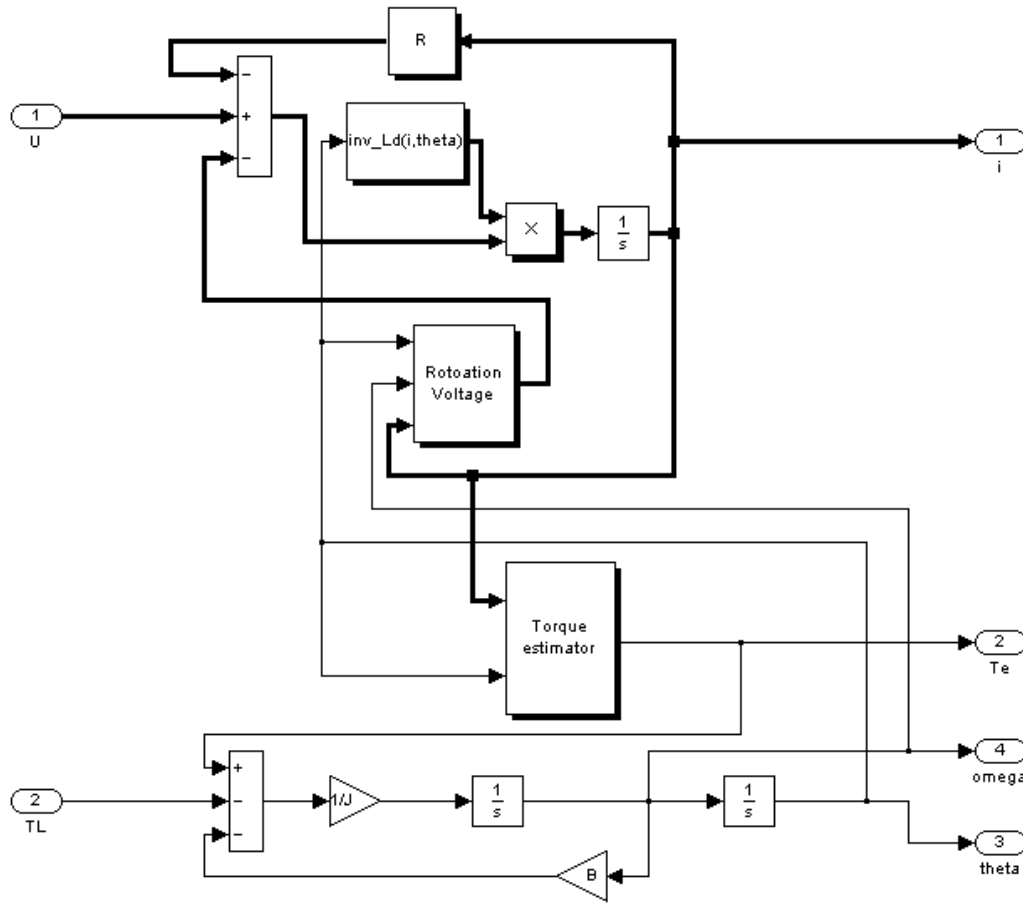


Fig. 8. SRM block model flowchart

## 5. SIMULATION AND LABORATORY TESTING RESULTS

A three phase SRM developed by authors of this paper, rated at:  $U_N = 50$  V,  $N_{ph} = 3$ ,  $N_s = 6$ ,  $N_r = 4$ ,  $R_s = 5,2 \Omega$  (at  $20^\circ\text{C}$ ),  $L_u = 19$  mH (self-inductance phase of the winding at an unaligned position),  $L_a = 82$  mH (self-inductance phase of the winding at an aligned position taken at currentless state) was subject to the testing. Testing of the operation of the machine at reverse direction setup, was done by simulation as well as on the actual arrangement. In order to perform the simulation testing, a drive system model was constructed with a switched reluctance machine in Matlab/Simulink package. In actual arrangement, the motor was supplied by classical half-bridge complete with two transistors and two diodes per motor phase. Such unit was controlled by two-processor dSpace DS1104 card. The sampling interval, when the discussed here, controlling

algorithms are performed, was  $35 \mu\text{s}$ . The SRM under testing was coupled mechanically with a classical DC machine supplied with a converter unit. The tests were performed both at steady and dynamic states of drive operation.

#### Steady state operation

Figures 9 and 10 show simulated current (a) phase voltage (c) and electromagnetic torque  $T_e$  (b) including with their FFT analysis (d), at generating operation (Fig. 9) and motoring operation (Fig.10). The machine was operated at set torque of 0.2 N·m and speed of 80 rad/s.

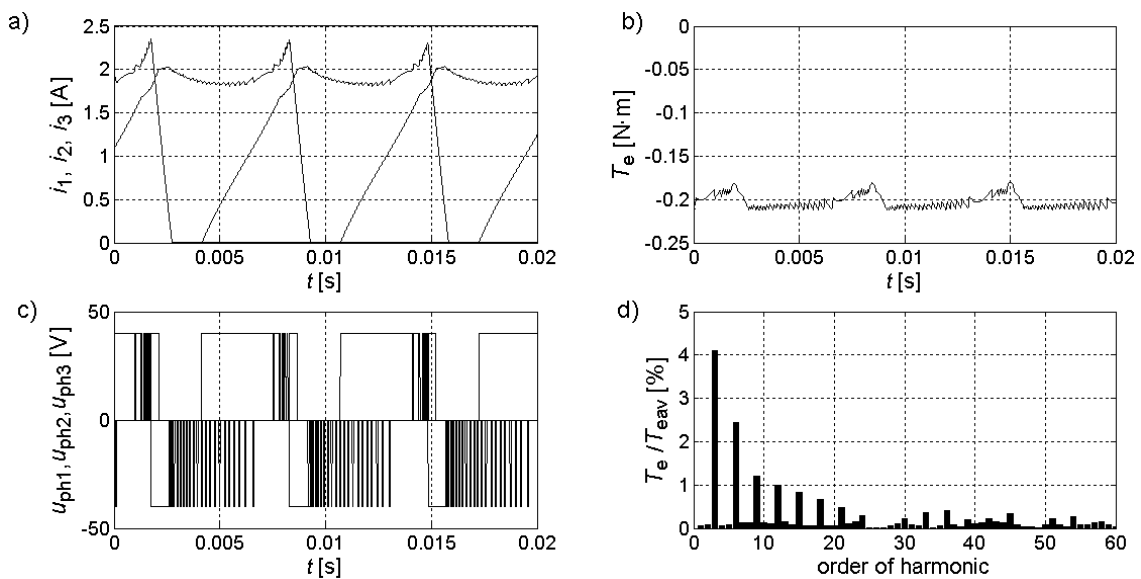


Fig. 9. Generating operation

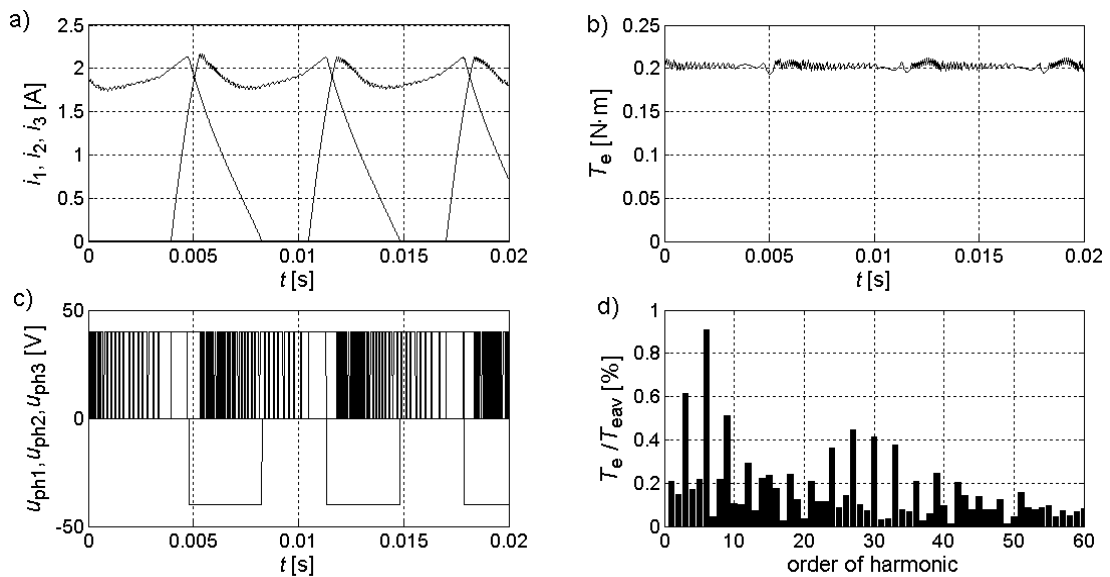
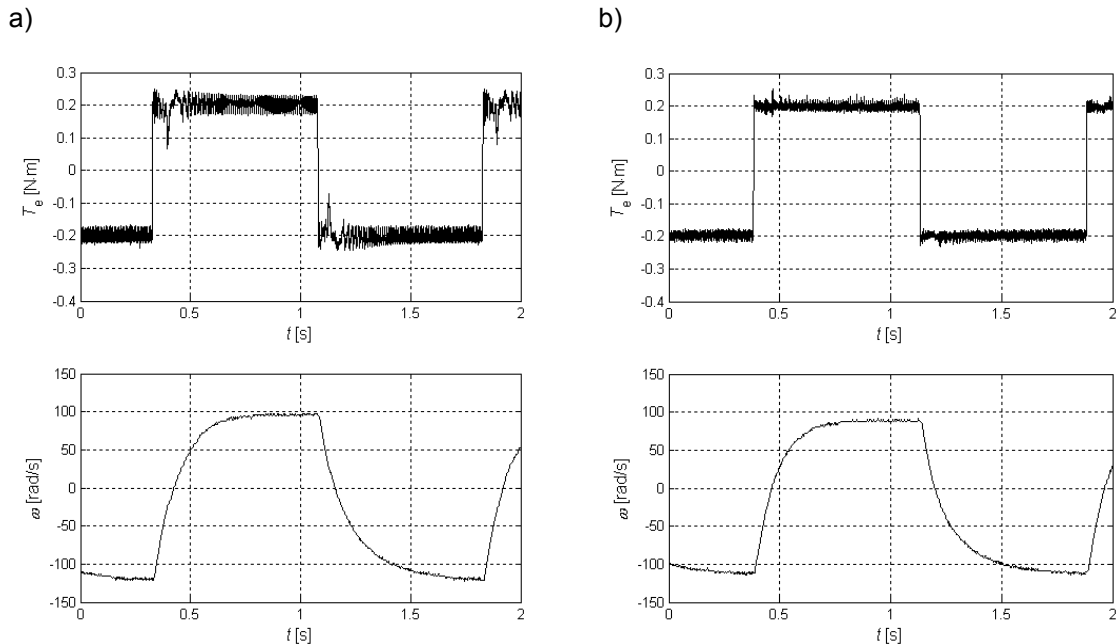


Fig. 10. Motoring operation

In phase voltage train, as shown in Fig. 9c, one can notice the operation of the three-level torque controller. Harmonic amplitudes in generating operation found in the torque train of the machine are much greater than in motoring operation, as it is necessary to excite the machine when it is still in the region of rising inductance.

### Operation of the machine at dynamic states

In order to verify the behaviour of the drive with switched reluctance machine during the operation at dynamic states, electromagnetic torque and speed have been recorded (Fig. 11) as well as current and phase voltage (Fig. 12) during reverse direction. Figure 11 shows experimental trains of the electromagnetic torque  $T_e$  and speed  $\omega$  at surges of set torque  $T_{ref}$  between -0.2 and 0.2 Nm. The trains have been recorded with the classical current control (Fig. 11a) as well as with proposed method (Fig. 11b). From the moment of changing the sign of the set torque until the rotor speed reaches zero, the generating (braking) operation occurs. Considering the obtained results, one can notice, that there is a marked reduction in torque ripple, when using the method of control as proposed in this paper.



**Fig. 11. Torque (up) and speed trains (bottom) with current control (a) and with the suggested method (b)**

Figure 12 shows the oscillograph record with phase current and voltage recorded in the situation when rotor rotating direction changes, when SRM is

controlled by the suggested method. One can notice, that starting from the moment of stopping the rotor, braking occurs (current and voltage trains are typical to generating operation), and then motoring operation is initiated.

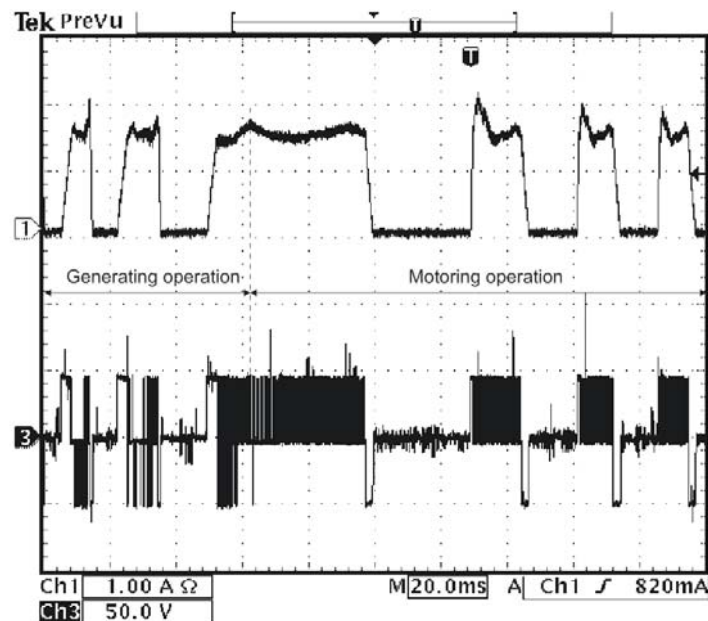


Fig. 12. Oscillograph record of current and voltage trains at generating-to-motoring transition

## 6. CONCLUSIONS

Problems associated with the control of switched reluctance machines (SRM) in the range of motoring and generating operation, have been discussed in this paper. A method of controlling instantaneous torque to minimize torque ripple at four-quadrant operation has been presented. Current and torque trains obtained as a result of simulation and experimental tests when operating the drive at steady and dynamic states, has been compared.

Based on the performed simulation and laboratory tests, the following conclusions can be drawn:

- Suggested method of controlling SRMs allows for great reduction of torque ripple as compared to the classical current control method.
- Application of three-level torque controller makes it possible to minimize effectively torque ripple within the region of generating operation of the SRMs.

- The merit of the suggested method is the capability of four-quadrant operation with minimizing torque ripple.

The suggested method of control makes it possible to extend the range of application of switched reluctance machines in the scope of direct drives, i.e. servo and traction drives.

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*Manuscript submitted 14.11.2006*

*Reviewed by Paolo DiBarba*

## PRACA CZTEROKWADRANTOWA MASZYNY RELUKTANCYJNEJ PRZEŁĄCZALNEJ

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**STRESZCZENIE**      *W pracy przedstawiono problematykę sterowania maszyn reluktancyjnych przełączalnych (ang. Switched Reluctance Machines – SRM) w zakresie pracy czterokwadrantowej. Taki rodzaj pracy wymagany jest między innymi w napędach pojazdów samochodowych, gdzie niezbędna jest zmiana kierunku wirowania wirnika oraz praca z możliwością zwrotu energii do źródła*

podczas hamowania. Zasadniczą zaletą maszyn reluktancyjnych przełączalnych jest między innymi prosta konstrukcja, szeroki zakres regulacji prędkości, mała bezwładność wirnika, natomiast istotną wadą są duże pulsacje momentu.

Celem niniejszej pracy jest prezentacja wyników badań symulacyjnych i eksperymentalnych, przeprowadzonych dla trójfazowej maszyny SRM 6/4, pracującej w zakresie pracy czterokwadrantowej.

W pracy omówiono zagadnienia sterowania maszyny reluktancyjnej przełączalnej zarówno w zakresie pracy silnikowej jak i prądnicowej (rys. 1, 2 i 3). Szczególną uwagę zwrócono na problem minimalizacji pulsacji momentu. Przedstawiono opracowaną przez autorów metodę sterowania wartością chwilową momentu zapewniającą minimalizację pulsacji momentu w zakresie pracy silnikowej i prądnicowej. Za pomocą zależności (1), (2) i (3) opisano pracę regulatora wartości chwilowej momentu. Schemat blokowy proponowanego układu sterowania SRM przedstawiono na rys. 4. Przedstawiono ogólną postać modelu matematycznego maszyny reluktancyjnej przełączalnej (wzory (4) – (8)), na podstawie którego opracowano model symulacyjny. Model symulacyjny wykonano dla trójfazowej maszyny SRM o konfiguracji 6/4 w systemie Matlab/Simulink. Strukturę blokową tego modelu przedstawiono na rys. 7 i 8. Parametry modelu oraz charakterystyki statyczne maszyny wyznaczano na podstawie obliczeń połowych za pomocą metody elementów skończonych. Na rysunkach 9 i 10 przedstawiono przebiegi czasowe prądów i napięć pasmowych oraz momentu elektromagnetycznego wraz z jego analizą harmoniczną, dla pracy prądnicowej i silnikowej. Przebiegi te uzyskano z zastosowaniem prezentowanej metody minimalizacji pulsacji momentu. Badania eksperymentalne maszyny przeprowadzono na stanowisku laboratoryjnym wyposażonym w kartę DS1104, na którym badano stany dynamiczne pracy maszyny. Na rysunku 11 zamieszczono przebiegi momentów: zadanego i elektromagnetycznego maszyny dla klasycznego sterowania prądowego i sterowania wartością chwilową momentu według metody zaproponowanej przez autorów. Na rysunku 12 zamieszczono oscylogram prezentujący przebiegi prądu i napięcia maszyny podczas nawrotu. Przedstawiona metoda sterowania pozwala na rozszerzenie zakresu zastosowań maszyn reluktancyjnych przełączalnych w napędach bezpośrednich, tj. serwonapędach oraz napędach trakcyjnych. W zakończeniu zaprezentowano wnioski.