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THE ANALYTICAL APPROACHES TO CALCULATION OF ELECTRIC MACHINES ON THE BASIS OF THE ANNULAR DOMAINS BOUNDARY PROBLEMS SOLUTION BY THE METHOD OF FOURIER VARIABLES SEPARATION

ABSTRACT *The air gap, with double-sided stairstep jaggies being two-downlink area, can be conformally imaged as circular ring. Traditionally this procedure executes approximately with the help of Carter factors. Cross sections of the stator and rotor cores also have the form of rings.*

The rotating magnetic field in the locations of conductors (in slots, windows between poles) can be presented by the sum of potential and some adding (easily calculated) fields. The localisation of an adding

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field in the location of conductors requires arrangement on a part of boundary of each slot (at the bottom or in the wedge part) infinitely thin current layers, the joint currents of them are equal to full currents of slots.

Usage of scalar magnetic potentials (SMP) of current layers allows to receive the analytical solution of Dirichlet problem for circular rings of three mentioned environments (air gap, magnetic stator core and rotor) assuming its magnetic permeabilities are constant.

For the sake of greater generality of the problem we assume that the core of the rotor is performed from massive steel. The selection of this material does the mathematical model more universal, as at transition to high enough values of magnetic permeability and resistivity the massive environment gains properties of laminated steel.

SMP of current layers of stator windings and rotor are submitted by Fourier series.

The parameters of an electromagnetic field in a solid rotor (radial and tangential component of flux density, current density and power loss) are obtained on the basis of the analytical solution of Bessel differential equation for a vector magnetic potential by a method of variables separation and are submitted by appropriate Bessel and Kelvin functions.

With reference to an asynchronous motor with a solid rotor the performance data are calculated and compared with the test results. The changes of magnetic permeability in steel of a rotor are taken into account by splitting it on concentric rings, in each of which the permeability is constant.

1. FORMULATION

The rotating magnetic field in the locations of conductors (in slots, windows between poles) can be presented by the sum of potential $\overline{H}_p$ and some adding $\overline{H}_0$ (easily calculated) magnetic fields. The localisation of adding field in the location of conductors requires arrangement on a part of each slot boundary (at the bottom or in the wedge part) infinitely thin current layers with a line density H_0, the joint currents of its are equal to full currents of slots.

The scalar magnetic potential (SMP) of a current layer j-ro of a slot will be defined by expression

$$\dot{U}^{j}(l) = \dot{I}^{j}_{\text{п}/2} - \int_{0}^{l} \dot{H}^{j}_{0\tau} dl$$

where

$I_{п}^{j}$ – full current of a slot,

l – variable of integration on slot boundary, read out from edges of a current layer,

$H_{o\tau}^{j}$ – projection of vector of a adding field to a direction of integration.

At summation SMP of current layers of all slots

$$U_{\Sigma}(l) = \sum_{j} U^{j}(l)$$

we will receive resultant relation, the geometrical site of which is dictated by convenience of calculation of magnetic field. For example, it is possible to suppose the field is fixed on a circle of an air gap passing on stator teeth edges.

In the beginning for simplification of a problem we neglect stairstep jaggies of stator and rotor cores (it is considered they are smooth; in subsequent, at refinement of the solution we shall come to conformal mapping of an air gap with bilateral stairstep jaggies on a circular ring)[1)] and we assume the length of the machine infinitely large (edge effects missed).

For the sake of greater generality we assume that the rotor of the machine is performed from massive steel. The selection of this material does a mathematical model by more universal, as at transition to high enough values of magnetic permeability and resistivity on an axis *z* the massive environment gains properties of laminated steel core.

The cross section of the machine is shown in Fig. 1. By digits 1, 2 and 3 are marked accordingly: the massive ferromagnetic rotor, air gap, stator core, magnetic permeability which in the beginning is considered as infinitely large.

Usage SMP of current layers allows receiving the analytical solution the Dirichlet problem for circular rings of three mentioned environments at constancy of its magnetic permeabilities.

[1)] As a first approximation for estimated value of a smooth air gap it is possible to accept value of a geometrical air gap δ (distance between circles passing on heads of fingers of a stator and a rotor), multiplied on air gap factor (Carter factor).

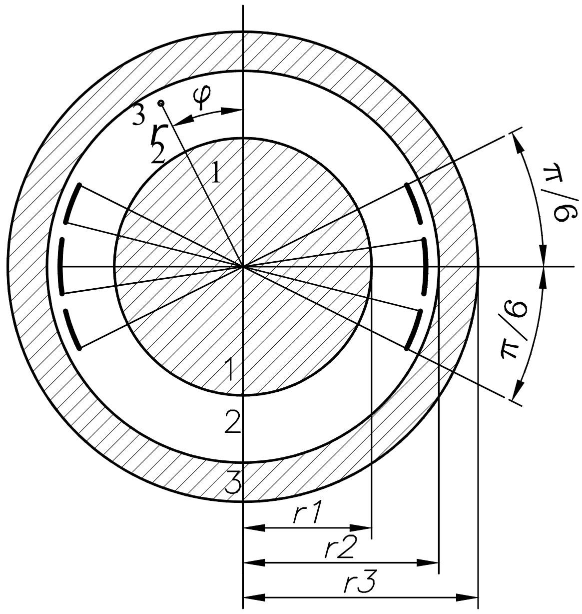


Fig. 1. Cross section of an electric machine with a solid rotor

2. CALCULATION OF A MAGNETIC FIELD IN AN AIR GAP

On the stator core surface with the bolded arcs are shown current layers of phase A slots. Their scalar magnetic potential (SMP) can be presented by a trigonometric series

$$U_{2A}(r_2, \varphi) = \sum_{n=1}^{\infty} a_n^{(2)} \cos\ n\,\varphi \tag{1}$$

where

$$a_n^{(2)} = F_n^{(2)} \cos \omega t \tag{2}$$

$$F_n^{(2)} = -\frac{2}{\pi} \frac{wk_{wn}}{n\,p} I_m \tag{3}$$

where

I_m – amplitude of a phase current of frequency ω;

w, k_{wn} – number of orbits and winding factor of a phase;

p – number of pairs of poles of a statoric winding,

φ – a ngular coordinate of a view point in a magnetic field ($\varphi = p\varphi_м$, here $\varphi_м$ – angular coordinate of the same view point, but in mechanical (geometrical) coordinate system).

Summarising (1) with SMP two other phases

$$U_{2B}(r_2, \varphi) = \sum_{n=1}^{\infty} F_n^{(2)} \cos\left(\omega t - 2\frac{\pi}{3}\right) \cos n\left(\varphi - 2\frac{\pi}{3}\right) \tag{4}$$

$$U_{2C}(r_2, \varphi) = \sum_{n=1}^{\infty} F_n^{(2)} \cos\left(\omega t + 2\frac{\pi}{3}\right) \cos n\left(\varphi + 2\frac{\pi}{3}\right) \tag{5}$$

we have

$$U_2(r_2, \varphi) = \frac{3}{2}\sum_{n=1}^{\infty} F_n^{(2)} \cos(\omega t \mp n\varphi) \Leftrightarrow \frac{3}{2}\sum_{n=1}^{\infty} F_n^{(2)} e^{\,j(\omega t \mp n\varphi)} \Leftrightarrow \frac{3}{2}\sum_{n=1}^{\infty} F_n^{(2)} e^{\mp jn\varphi} \tag{6}$$

where

the upper sign corresponding to harmonics numbers

$$n = 6k + 1 \quad (k = 0,\, 1,\, 2,\ldots,\, n = 1,\, 7,\, 13,\ldots)$$

the lower sign – $n = 6k - 1 \quad (k = 1, 2, 3,\ldots,\, n = 5, 11, 17,\ldots)$. [1)]

On other side of air area 2 (with radius r_1) current of phase A and the eddy currents of ferromagnetic rotor will induce SMP in the form

$$U_{2A}(r_1, \varphi) = \sum_{n=1}^{\infty} \dot{a}_n^{(1)} \cos n\varphi + \dot{b}_n^{(1)} \sin n\varphi \tag{7}$$

where

$\dot{a}_n^{(1)}, \dot{b}_n^{1}$ – unknown factors.

[1)] For the sake of simplicity of the analysis we assume that the stator winding has an integer of slots on a pole and phase. In this case miss even harmonics in the formulas (2),(5),(6).

SMP, assigned by the formulas (1) and (7), allows finding the solution the Dirichlet problem for an annular domain 2 [1]

$$U_{2A}(r,\varphi)=\sum_{n=1}^{\infty}(r^{n}A_{n}+r^{-n}C_{n})\cos n\varphi+\left(r^{n}B_{n}+r^{-n}D_{n}\right)\sin n\varphi \tag{8}$$

where the factors A_n, B_n, C_n, D_n are determined by the formulas

$$\left.\begin{aligned} A_n&=\left(r_2^n\dot{a}_n^{(2)}-r_1^n\dot{a}_n^{(1)}\right)/\left(r_2^{2n}-r_1^{2n}\right),\\ B_n&=\left(r_2^n\dot{b}_n^{(2)}-r_1^n\dot{b}_n^{(1)}\right)/\left(r_2^{2n}-r_1^{2n}\right),\\ C_n&=\left(r_1r_2\right)^n\left(r_2^n\dot{a}_n^{(1)}-r_1^n\dot{a}_n^{(2)}\right)/\left(r_2^{2n}-r_1^{2n}\right),\\ D_n&=\left(r_1r_2\right)^n\left(r_2^n\dot{b}_n^{(1)}-r_1^n\dot{b}_n^{(2)}\right)/\left(r_2^{2n}-r_1^{2n}\right).\end{aligned}\right\} \tag{9}$$

From expressions (8) and (9) follows the relation for radial component of magnetic field (MF) intensity on boundary of areas 1 and 2

$$H_{2rA}(r_1,\varphi)=-\left.\frac{\partial U_{2A}(r,\varphi)}{\partial r}\right|_{r=r_1}=\frac{1}{r_1}\sum_{n=1}^{\infty}n\left(\nu_{1n}\dot{a}_n^{(1)}+\nu_{2n}\dot{a}_n^{(2)}\right)\cos n\varphi+$$
$$+n\left(\nu_{1n}b_n^{(1)}+\nu_{2n}b_n^{(2)}\right)\sin n\varphi \tag{10}$$

where

$$\nu_{1n}=\left(r_1^{2n}+r_2^{2n}\right)/\left(r_2^{2n}-r_1^{2n}\right)$$

$$\nu_{2n}=-2\left(r_1r_2\right)^n/\left(r_2^{2n}-r_1^{2n}\right)$$

Taking into account operation of two other stator windings phases (*B* and *C*), the formulas (7) and (10) can be given in the form

$$U_2(r_1,\varphi)=\frac{3}{2}\sum_{n=1}^{\infty}\dot{F}_{n1}^{(1)}\cos(\omega t\mp n\varphi)-\dot{F}_{n2}^{(1)}\sin(\omega t\mp n\varphi)\Leftrightarrow\frac{3}{2}\sum_{n=1}^{\infty}\dot{F}_n^{(1)}e^{j(\omega t\mp n\varphi)}\Leftrightarrow$$
$$\Leftrightarrow\frac{3}{2}\sum_{n=1}^{\infty}\dot{F}_n^{(1)}e^{\mp jn\varphi} \tag{11}$$

$$H_{2r}(r_1,\varphi)=\frac{3}{2r_1}\sum_{n=1}^{\infty}n\left(\nu_{1n}\dot F_n^{(1)}+\nu_{2n}\dot F_n^{(2)}\right)e^{j(\omega t\mp n\varphi)}\Leftrightarrow$$
$$\Leftrightarrow\frac{3}{2r_1}\sum_{n=1}^{\infty}n\left(\nu_{1n}\dot F_n^{(1)}+\nu_{2n}\dot F_n^{(2)}\right)e^{\mp jn\varphi} \tag{12}$$

where

$$\left.\begin{aligned}&\dot a_n^{(1)}=\dot F_{n1}^{(1)}\cos\omega t,\quad b_n^{(1)}=\dot F_{n2}^{(1)}\sin\omega t,\\&\dot F_n^{(1)}=\dot F_{n1}^{(1)}+j\dot F_{n2}^{(1)}\end{aligned}\right\}$$

From the formula (11) we can receive expression for tangential component of MF intensity

$$H_{2\varphi}(r_1,\varphi)=-\frac{\partial U_2(r_1,\varphi)}{r_1\partial\varphi}=\pm\frac{3j}{2r_1}\sum_{n=1}^{\infty}n\,\dot F_n^{(1)}e^{j(\omega t\mp n\varphi)}\Leftrightarrow\pm\frac{3j}{2r_1}\sum_{n=1}^{\infty}n\,\dot F_n^{(1)}e^{\mp jn\varphi} \tag{13}$$

3. CALCULATION OF A MAGNETIC FIELD IN A FERROMAGNETIC ROTOR

We draw attention now to the ferromagnetic rotor – environment 1, rotating with speed Ω (in a direction of a first harmonic MF). Assuming $\mu_1=const$[1)] and entering a vector magnetic potential

$$\mu_1\overline{H}_1=\mathrm{rot}\,\overline{A}_1$$

we have from the first Maxwell equation, written in fixed coordinate system

1) Below this condition will concern only to rather thin elementary concentric annular domains, into which will be divided the massive ferromagnetic cylinder of a rotor. On boundaries of these rings the magnetic permeability will change jumps. Inside rings the environment will be linear, i.e. its parameters μ and γ are saved invariable. The monotone change of these parameters will be watched at infinitely large number of elementary concentric rings.

$$\mathrm{rot}\ \ \mathrm{rot}\,\overline{A}_1 = \mu\,\overline{\delta}_1 \tag{14}$$

where

$$\overline{\delta}_1 = \gamma\,\overline{E}_1 = \gamma\left(-q\mathrm{rad}U_1 - \frac{\partial\,\overline{A}_1}{\partial t} + \left[\overline{v}_1\ \mathrm{rot}\,\overline{A}_1\right]\right) \tag{15}$$

is a current density in steel rotor (here $\overline{v}_1$ – running speed of a view point of the ferromagnetic environment).

From the vector analysis it is known

$$\mathrm{rot}\ \mathrm{rot}\,\overline{A}_1 = \mathrm{qrad}\,\mathrm{div}\overline{A}_1 - \nabla^2\overline{A}$$

Selecting for calibration of the vector potential the Culomb condition

$$\mathrm{div}\,\overline{A}_1 = 0$$

And neglecting strength of an electrical (Culomb) field inside the steel array

$$\mathrm{qrad}\,U_1 = 0$$

from an equation (14) we shall have

$$\nabla^2\overline{A}_1 + \mu\,\gamma\left(-\frac{\partial\,A_1}{\partial\,t} + \left[v_1\ \overline{B}_1\right]\right) = 0$$

At harmonically varying MF it is fair

$$\nabla^2\dot{\overline{A}} - j\,\omega\,\mu\,\gamma\,\dot{\overline{A}} - \mu\,\gamma\,\Omega\,r\,\dot{B}_{1r}\overline{e}_z = 0 \tag{16}$$

As considered MF is two-dimensional parallel, thus in cylindrical coordinate system

$$\mathrm{rot}_z\,\overline{A}_1 = B_{1z} = \frac{1}{r}\left(\frac{r\,\partial\,A_{1\varphi}}{\partial\,r} - \frac{\partial\,A_{1r}}{\partial\,\varphi}\right) = 0$$

From here follows

$$A_{1r} = A_{1\varphi} = 0, \quad \overline{A}_1 = A_z \overline{e}_z$$

$$\left.\begin{aligned} \operatorname{rot}_r \overline{A}_1 = B_{1r} &= \frac{1}{r}\left(\frac{\partial A_{1z}}{\partial \varphi} - \frac{r\partial A_{1\varphi}}{\partial z}\right) = \frac{1}{r}\frac{\partial A_{1z}}{\partial \varphi}, \\ \operatorname{rot}_\varphi \overline{A}_1 = B_{1\varphi} &= -\left(\frac{\partial A_{1z}}{\partial r} - \frac{\partial A_{1r}}{\partial z}\right) = -\frac{\partial A_{1z}}{\partial r} \end{aligned}\right\} \tag{17}$$

Therefore equation (16) can be written

$$\frac{\partial^2 \dot{A}_{1z}}{\partial r^2} + \frac{1}{r}\frac{\partial \dot{A}_{1z}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \dot{A}_{1z}}{\partial \varphi^2} - \mu\gamma\Omega\frac{\partial \dot{A}_{1z}}{\partial \varphi} - j\omega\mu\gamma\dot{A}_{1z} = 0 \tag{18}$$

We shall solve it by a method of variables separation, supposing

$$\dot{A}_{1z} = R(r)\Phi(n\varphi) \tag{19}$$

And, if to select

$$\Phi(n\varphi) = e^{\mp jn\varphi}$$

then after the substitution (19) in (18) we receive an ordinary differential equation

$$r^2\frac{d^2R(r)}{dr^2} + r\frac{dR(r)}{dr} + \left(-n^2 + k^2 S_n r^2\right)R(r) = 0$$

where

$$k^2 = -j\omega\mu\gamma$$

$$S_n = \frac{\omega \mp p\Omega n}{\omega} = 1 \mp (1 - S_1)n$$

where

$S_1 = 1 - {p\Omega}/{\omega}$ – slip of a rotor for first harmonic of a magnetic field;

The upper sign (-) corresponds to values $n = 6k + 1$ $(k = 0,1,...; n = 1,7,13,...)$;

The lower sign (+) – to $n = 6k - 1$ $(k = 1,2,3,...; n = 5,11,17,...)$.

Its solution is known [2]

$$R(r)=\sum_{n=1}^{\infty} C_{1n} J_n\left(k\sqrt{S_n}\, r\right)+C_{2n} Y_n\left(k\sqrt{S_n}\, r\right) \tag{20}$$

where

$J_n\left(k\sqrt{S_n}\, r\right)$, $Y\left(k\sqrt{S_n}\, r\right)$ – cylindrical Bessel functions of *n* order, accordingly of the first and second kind;

C_{1n}, C_{2n} – arbitrary constants.

Taking into account, that $Y_n(0)=\infty$, it is necessary to accept $C_{2n}=0$. In outcome we shall have

$$\dot{A}_{1z}=\sum_{n=1}^{\infty} C_{1n} J_n\left(k\sqrt{S_n}\, r\right) e^{\mp j n \varphi} \tag{21}$$

where the constants C_{1n} will be retrieved from boundary conditions of a problem.

Using (17) we have

$$\dot{B}_{1r}=\mp \frac{j}{r} \sum_{n=1}^{\infty} n\; C_{1n} J_n\left(k\sqrt{S_n}\, r\right) e^{\mp j n \varphi} \tag{22}$$

$$\dot{B}_{1\varphi}=-k \sum_{n=1}^{\infty} \sqrt{S_n}\; C_{1n}\, J_n'\left(k\sqrt{S_n}\, r\right) e^{\mp j n \varphi} \tag{23}$$

where

$J_n'(x)=\partial J_n / \partial x$.

On boundary of areas 1 and 2 $(r=r_1)$ we have

$$\dot{B}_{1r}=\dot{B}_{2r}, \quad \dot{H}_{1\varphi}=\dot{H}_{2\varphi} \tag{24}$$

To draw attention to the equations (12) and (13) for H_{2r} and $H_{2\varphi}$, from equations (24) we shall receive a set of equations for finding of unknowns C_{1n} and $\dot{F}_n^{(1)}$

$$\left.\begin{aligned} &\mp j C_{1n} J_n\left(k\sqrt{S_n}\, r_1\right)=\frac{3\mu_0}{2}\left(\nu_{1n}\dot{F}_n^{(1)}+\nu_{2n}\dot{F}_n^{(2)}\right), \\ &-\frac{k}{\mu_1}\sqrt{S_n}\, C_{1n}\, J_n'\left(k\sqrt{S_n}\, r_1\right)=\pm\frac{3j}{2r_1} n\, \dot{F}_n^{(1)} \end{aligned}\right\}$$

Its solution will be

$$C_{1n} = \pm \frac{3j\,\mu_1\mu_0\,\nu_{2n}\,n\,\dot{F}_n^{(2)}}{2\,\mu_1 n\,J_n\left(k\sqrt{S_n}\,r_1\right) + 2\,\mu_0\,\nu_{1n}\,k\sqrt{S_n}\,r_1\,J_n\left(k\sqrt{S_n}\,r_1\right)} \tag{25}$$

$$\dot{F}_n^{(1)} = -\frac{\dot{F}_n^{(2)}}{\dfrac{\mu_1}{\mu_0}\,\dfrac{n}{r_1}\,\dfrac{J_n\left(k\sqrt{S_n}\,r_1\right)}{k\sqrt{S_n}\,r_1 J_n'\left(k\sqrt{S_n}\,r_1\right)} + \dfrac{\nu_{1n}}{\nu_{2n}}} \tag{26}$$

Considering known equalities

$$\left.\begin{aligned} 2J_n'(x) &= J_{n-1}(x) - J_{n+1}(x), \\ 2n\,J_n(x) &= x\left[J_{n-1}(x) + J_{n+1}(x)\right] \end{aligned}\right\} \tag{27}$$

it is possible to give the formulas (25), (26) in form

$$C_{1n} = \pm \frac{3j\,\mu_1\nu_{2n}\,n\,\dot{F}_n^{(2)}}{k\sqrt{S_n}\,r_1\,\nu_{1n}\left[a_{1n}J_{n-1}\left(k\sqrt{S_n}\,r_1\right) + a_{2n}J_{n+1}\left(k\sqrt{S_n}\,r_1\right)\right]} \tag{28}$$

$$\dot{F}_n^{(1)} = -\frac{\dot{F}_n^{(2)}}{\dfrac{\mu_1}{\mu_0\,\nu_{2n}}\,\dfrac{J_{n-1}\left(k\sqrt{S_n}\,r_1\right) + J_{n+1}\left(k\sqrt{S_n}\,r_1\right)}{J_{n-1}\left(k\sqrt{S_n}\,r_1\right) - J_{n+1}\left(k\sqrt{S_n}\,r_1\right)} + \dfrac{\nu_{1n}}{\nu_{2n}}} \tag{29}$$

where

$$a_{1n} = 1 + \frac{\mu_1}{\mu_0\,\nu_{1n}}, \quad a_{2n} = -1 + \frac{\mu_1}{\mu_0\,\nu_{1n}}$$

After a substitution of expression (28) for a factor C_{1n} in the formulas (22), (23) for radial and tangential components of a magnetic flux density in the steel cylinder we shall have

$$\dot{B}_{1r} = \frac{3}{2}\,\frac{\mu_1}{r_1}\sum_{n=1}^{\infty}\frac{\nu_{2n}}{\nu_{1n}}\,\frac{n\,\dot{F}_n^{(2)}\left[J_{n-1}\left(k\sqrt{S_n}\,r\right) + J_{n+1}\left(k\sqrt{S_n}\,r\right)\right]e^{\mp jn\varphi}}{a_{1n}\,J_{n-1}\left(k\sqrt{S_n}\,r_1\right) + a_{2n}\,J_{n+1}\left(k\sqrt{S_n}\,r_1\right)} \tag{30}$$

$$\dot{B}_{1\varphi} = \mp \frac{3}{2} \frac{j\mu_1}{r_1} \sum_{n=1}^{\infty} \frac{\nu_{2n}}{\nu_{1n}} \frac{n\dot{F}_n^{(2)} \left[J_{n-1}\left(k\sqrt{S_n}\, r\right) - J_{n+1}\left(k\sqrt{S_n}\, r\right)\right] e^{\mp jn\varphi}}{a_{1n} J_{n-1}\left(k\sqrt{S_n}\, r_1\right) + a_{2n} J_{n+1}\left(k\sqrt{S_n}\right)} \qquad (31)$$

On other side of an air gap (in the environment 2) – on a circle $r = r_2$ – – we have according to the formulas (8), (11), (12), (29)

$$\dot{B}_{2r}(r_2,\varphi) = \frac{3\mu_0}{2r_2} \sum_{n=1}^{\infty} n\nu_{1n} \dot{F}_n^{(2)} \frac{\widetilde{a}_{1n} J_{n-1}(j^{3/2} q_n r_1) - \widetilde{a}_{2n} J_{n+1}(j^{3/2} q_n r_1)}{a_{1n} J_{n-1}(j^{3/2} q_n r_1) + a_{2n} J_{n+1}(j^{3/2} q_n r_1)} e^{\mp jn\varphi} \qquad (32)$$

$$\dot{B}_{2\varphi}(r_2,\varphi) = \pm \frac{3j\mu_0}{2r_2} \sum_{n=1}^{\infty} n\dot{F}_n^{(2)} e^{\mp jn\varphi} \qquad (33)$$

where

$$\widetilde{a}_{1n} = \left(\frac{\nu_{2n}}{\nu_{1n}}\right)^2 - a_{1n}, \quad \widetilde{a}_{2n} = \left(\frac{\nu_{2n}}{\nu_{1n}}\right)^2 + a_{2n}.$$

The obtained formulas were used with reference to an asynchronous motor (AM) with a solid rotor executed on the basis mass-produced three-phase AM type 4A200M 2У3, with the following data:
$P_{2н} = 37$ kW; $n_н = 2943$ rpm; $U_{нф} = 220$ V; $I_н = 70$ A; $M_н = 120{,}06$ Nm; $\cos\varphi = 0{,}89$; $\eta = 0{,}90$; $z_1 = 36$; $q = 6$; $W = 60$; $R = 0{,}0652\,\Omega$ (0,0207 rel.u.); $\omega = 314$ rad/s; $\gamma = 0{,}5 \cdot 10^7$ $(\Omega{\cdot}\mathrm{m})^{-1}$; $r_1 = 97$ mm; $r_2 = 97{,}9$ mm; $\delta = 0{,}9$ mm; $l = 130$ mm; material of rotor – steel 3.

In Figs. 2-3 the peak values of the first and seventh harmonics of a magnetic flux density on a surface of a solid rotor in a function of slip for the given motor, calculated on the formulas (30), (31)[1)], are shown at $\mu = \mu_e = 100\mu_0$ and $\mu = \mu_e = \mathrm{var}$ (here μ_e - magnetic permeability of steel on a rotor surface).

[1)] Cylindrical, Bessel functions of the first kinds with complex arguments:

$$k\sqrt{S_n}\, r = \sqrt{-j\omega\mu\gamma S_n}\, r = q_n r e^{j3\pi/4} = x e^{j3\pi/4}, \; S_n > 0$$

$$k\sqrt{S_n}\, r = \sqrt{j\omega\mu\gamma |S_n|}\, r = q_n r e^{j\pi/4} = x e^{j\pi/4}, \; S_n < 0$$

where

$$q_n = \sqrt{\omega\mu\gamma |S_n|}, \; x = q_n r$$

expresses through Kelvin functions:

$$J_n\left(x e^{j3\pi/4}\right) = \left(\mathrm{ber}_n x + j\mathrm{bei}_n x\right)$$

$$J_n\left(x e^{j\pi/4}\right) = \left(\mathrm{ber}_n x - j\mathrm{bei}_n x\right)\cos\pi n$$

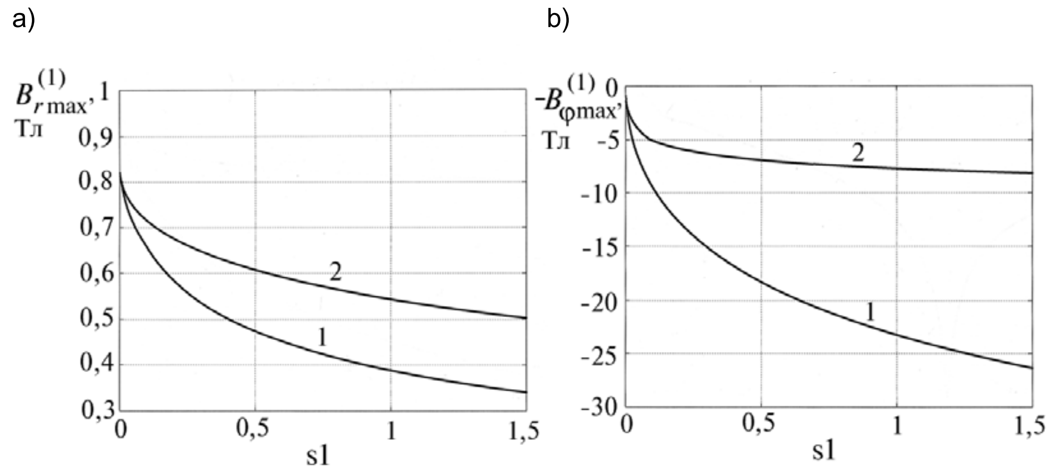


Fig. 2. Amplitude values of radial (a) and tangential (b) components of first harmonic of a magnetic flux density on a solid rotor surface in function of the rotor slip concerning a first harmonic of a magnetic field (Curve 1: $\mu = \mu_e = 100\mu_0$; - 2: $\mu = \mu_e$ accordingly to magnetisation curve of steel 3)

4. CURRENT DENSITY IN A FERROMAGNETIC ROTOR

In accordance with expressions (15), (16) complex amplitudes of a current density in a rotor will be

$$\dot{\delta}_{1z} = -j\,\omega\,\gamma\,\dot{A}_{1z} - \gamma\,\Omega\,r\,\dot{B}_{1r} \tag{34}$$

After a substitution in (34) formulas (21) and (22) accordingly for $\dot{A}_{1z}$ and $\dot{B}_{1r}$ we shall have

$$\dot{\delta}_{1z} = -j\,\gamma\,\omega\sum_{n=1}^{\infty} S_n\,\dot{C}_{1n}\,J_n\left(k\sqrt{S_n}\,r\right)e^{\mp j n \varphi}$$

Taking into account expressions (27), (28) we shall have

$$\dot{\delta}_{1z} = \pm\frac{3}{2}\frac{r}{r_1}\,\omega\,\mu\,\gamma\sum_{n=1}^{\infty}\frac{\nu_{2n}}{\nu_{1n}}\,\frac{S_n\,\dot{F}_n^{(2)}\left[J_{n-1}\left(k\sqrt{S_n}\,r\right)+J_{n+1}\left(k\sqrt{S_n}\,r\right)\right]}{a_{1n}\,J_{n-1}\left(k\sqrt{S_n}\,r_1\right)+a_{2n}\,J_{n+1}\left(k\sqrt{S_n}\,r_1\right)}\,e^{\mp j n\varphi}$$

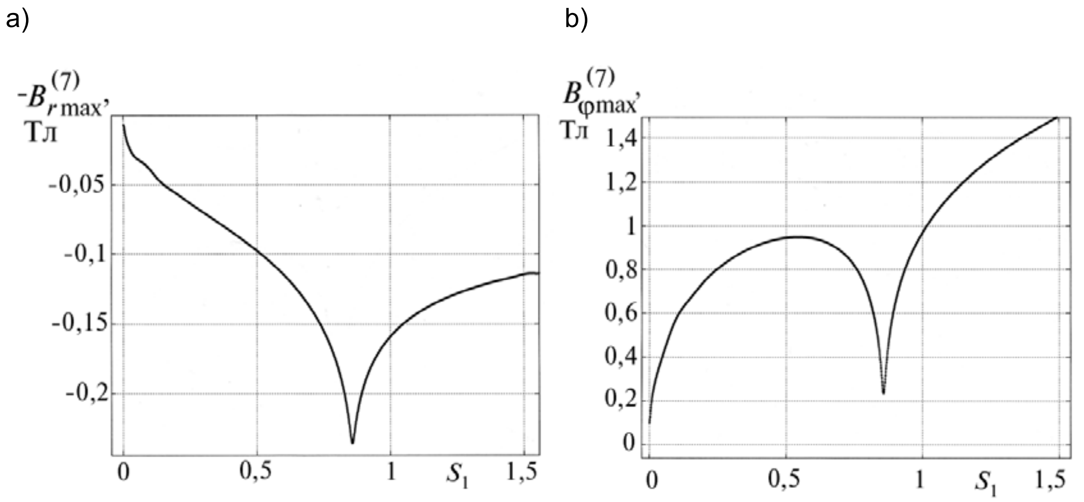


Fig. 3. Amplitude values of the seventh harmonics – radial (a) and tangential (b) components of a magnetic flux density on the solid rotor surface in a function of slip concerning the first harmonic of a magnetic field at $\mu = \mu_e = 100\mu_0$

Subsequently, our interest will be the square of the module of current density for n harmonic. From last formula, skiping indexes 1z, we have

$$\left|\dot{\delta}_n\right|^2 = \dot{\delta}_n\,\delta_n^* = \left(\frac{3}{2}\,\frac{2n}{r_1}\,\frac{\nu_{2n}}{\nu_{1n}}\dot{F}_n^{(2)}\right)^2 \omega\mu\gamma S_n \times$$

$$\times\frac{J_n\left(k\sqrt{S_n}\,r\right)J_n\left(k^*\sqrt{S_n}\,r\right)}{a_{1n}^2\left[J_{n-1}\left(k\sqrt{S_n}\,r_1\right)+\dfrac{a_{2n}}{a_{1n}}J_{n+1}\left(k\sqrt{S_n}\,r_1\right)\right]\left[J_{n-1}\left(k^*\sqrt{S_n}\,r_1\right)+\dfrac{a_{2n}}{a_{1n}}J_{n+1}\left(k^*\sqrt{S_n}\,r_1\right)\right]}$$

(35)

Cylindrical functions with considered complex argument

$$k\sqrt{S_n}\,r = \sqrt{-j\,\omega\mu\gamma S_n}\,r = j^{3/2}\,q_n\,r$$

where

$$q_n = \sqrt{\omega\mu\gamma S_n}$$

expresses through Kelvin functions

$$I_n\left(j^{3/2}\, q_n\, r\right)= ber_n\left(q_n\, r\right)+ j\, bei_n\left(q_n\, r\right)=b_n\, e^{j\beta_n}$$

where

$$b_n=\sqrt{ber_n^{\;2}\left(q_n\, r\right)+bei_n^{\;2}\left(q_n\, r\right)}\,,\qquad \beta_n=\mathrm{arctq}\frac{bei_n\left(q_n\, r\right)}{ber_n\left(q_n\, r\right)}$$

In outcome the formula for a square of a module of a current density (35) will receive a form

$$\left|\dot{\delta}_n\right|^2=\left(\frac{3n}{r_1}\,\frac{\nu_{2n}}{\nu_{1n}}\,\dot{F}_n^{(2)}\right)^2\omega\,\mu\,\gamma\, S_n\left(\frac{b_n^2\left(q_n\, r\right)}{D_n\left(q_n\, r_1\right)}\right) \qquad (36)$$

where

$$D_n\left(q_n\, r_1\right)=a_{1n}^2\left[b_{n-1}^2\left(q_n\, r_1\right)+\left(\frac{a_{2n}}{a_{1n}}b_{n+1}\left(q_n\, r_1\right)\right)^2+2\frac{a_{2n}}{a_{1n}}b_{n-1}\left(q_n\, r_1\right)b_{n+1}\left(q_n\, r_1\right)\cos\left(\beta_{n+1}-\beta_{n-1}\right)\right]$$

From obtained formulas in Figs. 4-5 are shown the main and higher harmonics amplitudes of the current density on the solid rotor surface in function of rotor slip.

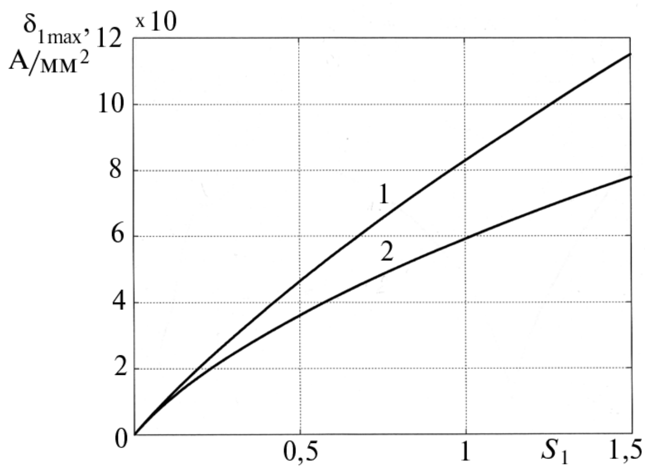


Fig. 4. Amplitude of a first harmonic of a current density on a solid rotor surface in function of slip (curve 1: $\mu=\mu_e=100\mu_0$**; - 2:** $\mu=\mu_e$ **is on a magnetization curve of steel 3)**

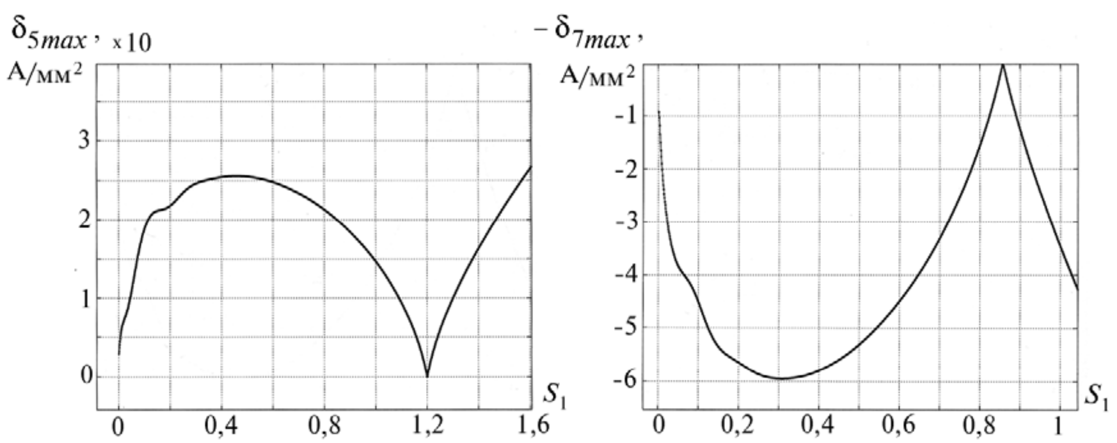


Fig. 5. Amplitudes of fifth (a) and seventh (b) harmonics of a current density on the solid rotor surface in function of slip at $\mu = \mu_e = 100\mu_0$

5. LOSSES IN A FERROMAGNETIC ROTOR

The losses in the steel cylinder from n harmonic of eddy currents in accordance with (36) are

$$P_n = \frac{l}{2\gamma}\int_0^{r_1}\int_0^{2\pi} \dot{\delta}_n\, \delta_n^*\, r\, dr\, d\varphi = \frac{\pi l}{\gamma}\int_0^{r_1} \delta_n\, \delta_n^*\, r\, dr =$$
$$= \left(\frac{3n}{r_1}\,\frac{\nu_{2n}}{\nu_{1n}}\dot{F}_n^{(2)}\right)^2 \frac{\pi\, l\, \omega\, \mu\, S_n}{D_n\left(q_n r_1\right)}\int_0^{r_1}\left(ber_n^2\left(q_n r\right) + bei_n^2\left(q_n r\right)\right)\times r\, dr \tag{37}$$

where l – active length of the rotor.

The integral in this expression is named as Lommel one. His value is known [3]. Finally

$$P_n = \frac{9\pi\, n\, l\, \omega\, \mu\, S_n}{2 D_n\left(q_n r_1\right)}\left(\frac{\nu_{2n}}{\nu_{1n}}\dot{F}_n^{(2)}\right)^2 \cdot$$
$$\cdot\left[ber_{n+1}\left(q_n r_1\right) bei_{n-1}\left(q_n r_1\right) - ber_{n-1}\left(q_n r_1\right) bei_{n+1}\left(q_n r_1\right)\right] \tag{38}$$

Using this formula the losses in a solid rotor from main and higher harmonics of an eddy current in a function of slip were calculated. The graphs of these relations are shown in Figs. 6-7.

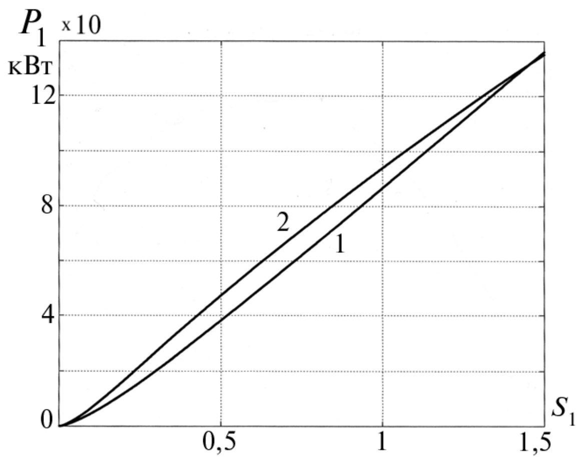


Fig. 6. Losses in a solid rotor from a first harmonic of eddy current in a function of slip (curve 1: $\mu = \mu_e = 100\mu_0$; 2 - $\mu = \mu_e$ is on a magnetisation curve of steel 3)

a) b)

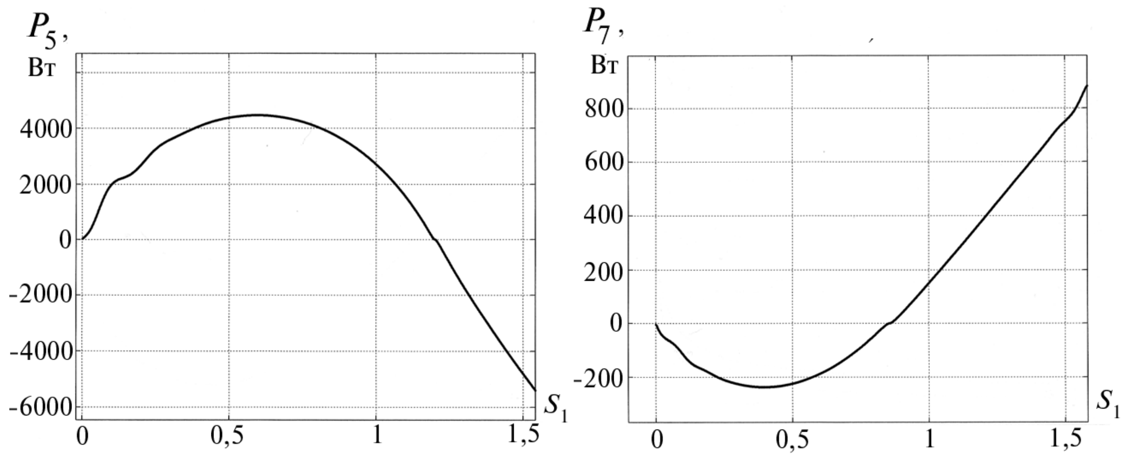


Fig. 7. Losses in a solid rotor from a fifth (a) and seventh (b) harmonics of an eddy current in a function of slip at $\mu = \mu_e = 100\mu_0$

The depth of penetration in the steel array n of a harmonics of a magnetic field can be defined according to the formula

$$\Delta_n = \sqrt{2/S_n \omega \mu \gamma} \tag{39}$$

The graph $\Delta_1 = \Delta_1(S_1)$ for a first harmonic of a magnetic field is presented in Fig. 8.

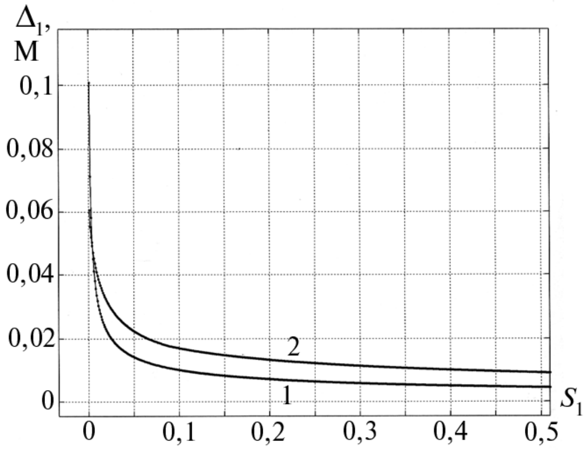


Fig. 8. Depth of penetration in the steel array of a rotor of a first harmonic magnetic fields in a function of slip (curve 1: $\mu = \mu_e = 100\mu_0$; - 2: $\mu = \mu_e$ is on a magnetization curve of steel 3)

6. THE ELECTROMAGNETIC TORQUE CALCULATION OF PERFORMANCE DATA

As is known, the area density of electromagnetic forces which are operational in a tangent direction to a cylindrical surface of rotor will be equal [4]

$$T_{\text{п}\varphi} = \left(B_{2r}\, H_{2\varphi}\right)\Big|_{r=r_1+\varepsilon}$$

where

$\varepsilon > 0$ – small constant;

r – radius of a circle flanking to a rotor.

On this circle (at $\varepsilon \to 0$) radial component of a magnetic flux density (B_{2r}) and tangential component of the magnetic fields intensity $(H_{2\tau})$ will be equal

$$B_{2r}(r_1, \varphi) = \sum_{n=1}^{\infty} \dot{B}^{n}_{r\max}(r_1) \cos\left(\omega t \mp n\varphi\right) \tag{40}$$

$$H_{2\varphi}(r_1,\varphi)=\sum_{n=1}^{\infty}\dot{H}_{\varphi\max}^{(n)}(r_1)\cos\left(\omega t\mp n\varphi+\frac{\pi}{2}\right) \tag{41}$$

where

$$\dot{B}_{r\max}^{n}(r_1)=\frac{3}{2}\frac{\mu_1\nu_{2n}n\dot{F}_n^{(2)}}{r_1\nu_{1n}}\times\frac{J_{n-1}\left(j^{3/2}q_n r_1\right)+J_{n+1}\left(j^{3/2}q_n r_1\right)}{a_{1n}J_{n-1}\left(j^{3/2}q_n r_1\right)+a_{2n}J_{n+1}\left(j^{3/2}q_n r_1\right)}=$$
$$=\dot{B}_{r\max}^{n}(r_1)e^{j\alpha_{rn}}$$

$$\dot{H}_{\varphi\max}^{(n)}(r_1)=\mp\frac{3}{2}\frac{j\nu_{2n}n\dot{F}_n^{(2)}}{r_1\nu_{1n}}\frac{J_{n-1}\left(j^{3/2}q_n r_1\right)-J_{n+1}\left(j^{3/2}q_n r_1\right)}{a_{1n}J_{n-1}\left(j^{3/2}q_n r_1\right)+a_{2n}J_{n+1}\left(j^{3/2}q_n r_1\right)}=$$
$$=\dot{H}_{\varphi\max}^{(n)}(r_1)e^{j(\alpha_{\varphi n}+\pi/2)}$$

In accordance with the theory of tensions for the electromagnetic torque of an electric machine will be fair the expression

$$M=r_1^2 l\int_0^{2\pi}T_{\text{п}\varphi}\,d\varphi=r_1^2 l\int_0^{2\pi}B_{2r}(r_1,\varphi)\times H_{2\varphi}(r_1,\varphi)d\varphi=$$
$$=r_1^2 l\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{r\max}^{(n)}(r_1)H_{\varphi\max}^{(m)}(r_1)\int_0^{2\pi}\cos\left(\omega t\mp n\varphi+\alpha_{rn}\right)\times$$
$$\times\cos\left(\omega t\mp m\varphi+\frac{\pi}{2}+\alpha_{\varphi m}\right)d\varphi=\pi r_1^2 l\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}B_{r\max}^{(n)}(r_1)H_{\varphi\max}^{(m)}(r_1)\sin\left(\alpha_{rn}-\alpha_{\varphi m}\right)$$

(42)

With the help of this formula the electromagnetic torques from first and higher harmonics of a magnetic field in a function of slip for the mentioned above tested asynchronous motor with a solid rotor were determined. The outcomes of calculations are adduced in Fig. 9-10.

Being repelled the formula (32) for radial component of a magnetic flux density it is easy to define the stator current of the asynchronous machine with a solid rotor.

For amplitude of a first harmonic of radial component of a magnetic flux density it is fair

$$\dot{B}_{2r}^{(n)} = \frac{3}{2}\frac{n\mu_0}{r_2} v_{1n} F_n^{(2)} G_n \, e^{j\alpha_n}$$

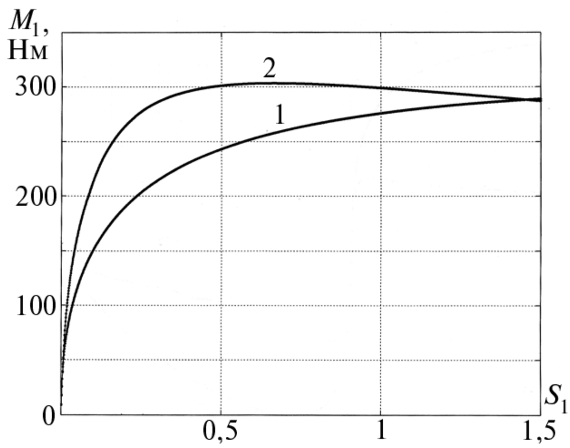


Fig. 9. The electromagnetic torque of a first harmonic of a magnetic field in a function of slip S_1 (curve 1: $\mu = \mu_e = 100\mu_0$; - 2: $\mu = \mu_e$ is on magnetisation curve of steel 3)

where

$$G_n = \sqrt{\frac{N_1^2 + N_2^2}{N_3^2 + N_4^2}} ,$$

here:

$$N_1 = \tilde{a}_{1n} ber_{n-1}(q_n r_1) + \tilde{a}_{2n} ber_{n+1}(q_n r_1)$$

$$N_2 = \tilde{a}_{1n} bei(q_n r_1) + \tilde{a}_{2n} bei_2(q_n r_1)$$

$$N_3 = a_{1n} ber_{n-1}(q_n r_1) + a_{2n} ber_{n+1}(q_n r_1)$$

$$N_4 = a_{1n} bei_{n-1}(q_n r_1) + a_{2n} bei_{n+1}(q_n r_1)$$

$$q_n = \sqrt{S_n \omega \mu \gamma}$$

$$\alpha_n = \beta_{1n} - \beta_{2n}$$

$$\beta_{1n} = \operatorname{arctq} N_2 / N_1$$

$$\beta_{2n} = \operatorname{arctq} N_4 / N_3$$

$$\dot{F}_n^{(2)} = -\left(2\sqrt{2} / \pi np\right) wk_{wn} I$$

As the magnetic flux of n harmonics of one pole is equal

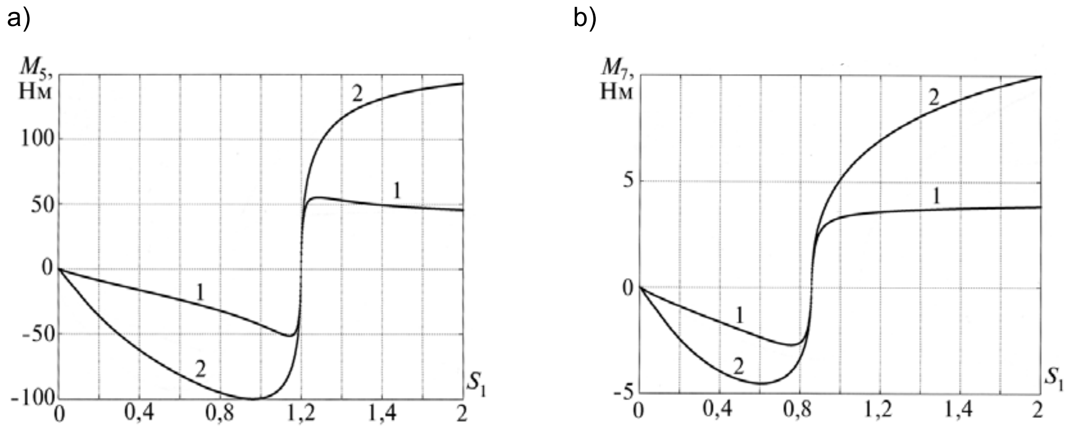
$$\dot{\Phi}_n = \frac{2}{\pi} \dot{B}_{2r}^{(n)} \frac{l\tau}{n}$$

then for an effective value of EMF vector of n harmonic of stator winding we have

$$\dot{E}_n = -j\left(2\pi / \sqrt{2}\right) f wk_{wn} \dot{\Phi}_n = -j\, Q_n\, G_n\, I e^{j\alpha_n}$$

where

$$Q_n = \frac{6}{\pi} \mu_0 \nu_{1n} \frac{l}{p^2 n} \omega \left(wk_{wn}\right)^2$$



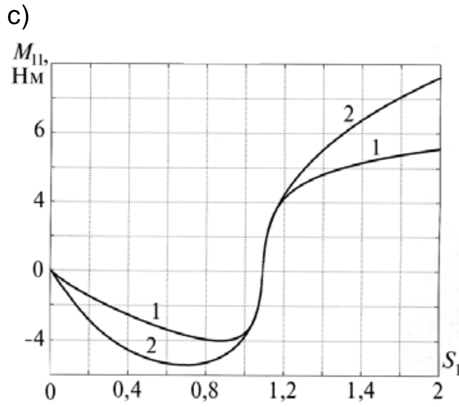


Fig. 10. The electromagnetic torque of fifth (a), seventh (b) and eleventh (c) harmonics of magnetic field in a function of slip S$_1$ (curve 1: $\mu = \mu_e = 100\mu_0$; - 2: $\mu = \mu_e$ is on a magnetisation curve of steel 3)

Considering an equilibrium equation of voltages in one phase of a stator winding circuit

$$\dot{U} = -\dot{E} + \dot{I}\, Z_1$$

where

$$\dot{E} = \sum_n \dot{E}_n$$

let's have finally for a stator current

$$I = \frac{U}{\sqrt{\left(R_1 - \sum_n Q_n G_n \sin\alpha_n\right)^2 + \left(X_{\sigma 1} + \sum_n Q_n G_n \cos\alpha_n\right)^2}}$$

and his phase angle

$$\varphi = -\mathrm{arc\,tq}\,\frac{X_{\sigma 1} + \sum_n Q_n G_n \cos\alpha_n}{R_1 - \sum_n Q_n G_n \sin\alpha_n}$$

The calculation of a stator current defined, as visible, in a function of rotor slip S_1, allows to find according to the offered above formulas all performance data of the asynchronous machine with a solid rotor.

The first harmonic of a magnetic flux density displaces on a surface of a rotor, having amplitude

$$B_{1\max}^{(r_1)} = \sqrt{\left[B_{1\max}^{(r)}(r_1)\right]^2 + \left[B_{1\max}^{(\varphi)}(r_1)\right]^2} \tag{43}$$

As the amplitudes of the nearest higher harmonics (5-th and 7-th) of flux density are much less the first one, the magnetic permeability of a surface of a rotor $\mu = \mu_e$ can be determined on a level of an effective value of a first harmonic of flux density [5].

On a large part of pole pitch of a rotor surface the magnetic permeability is close to minimum value

$$\mu = \mu_e = \mu_{\min} = f\left(B_{1\text{действ}}(r_1)\right) \tag{44}$$

appropriate to effective value of a first harmonic of a magnetic flux density on the rotor surface (43). Therefore, as a first approximation, the magnetic permeability of the rotor surface can be determined by expression (44).

It is more logical to determine the permeability μ_e as average value of a curve $\mu_e = \mu_e(\alpha)$:

$$\mu_e = \mu_{e\text{ср}} = \frac{1}{\pi}\int_0^{\pi} \mu_e(\alpha)d\alpha \tag{45}$$

At this stage the magnetic permeability of whole rotor mass will be assumed also as identical and equal μ_e – as magnetic permeability of a rotor surface. Below it will be considered more accurate calculation of permeability using splitting the cylinder of a rotor into concentric annular domains.

The nature of change $\mu_e = \mu_{\min}$ functions of a rotor slip is shown in Fig. 11. Here curve 2 is obtained with the help of a magnetisation curve of steel 3 and amplitude of a first harmonic of a magnetic flux density calculated on the formulas (40), (41), (43).

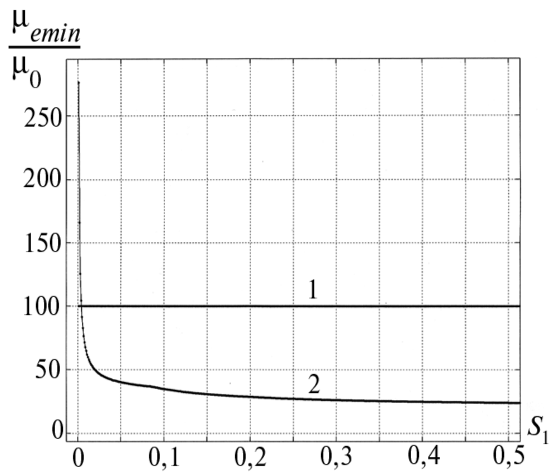


Fig. 11. Minimum value of relative magnetic permeability on a surface of the steel array of a rotor in a function of his slip (curve 1: $\mu_{e\min}/\mu_0 =$ 100; -2: $\mu_{e\min}/\mu_0$ is on a magnetization curve of steel 3)

In Fig.12 are shown computational and experimental [6] relations of the torque (a) and a stator current (b) from slip for an asynchronous motor with a solid rotor from steel 3, made on the basis mass-produced electric motor AO2-81-2 of power 40 kW with synchronous speed 3000 rpm. The curves were calculated taking into account the frontal effect of eddy currents short circuiting of a rotor by reduction the electrical conductivity γ of rotor material on followed factor [7]

$$k_л = 1 + 2\tau/\pi l$$

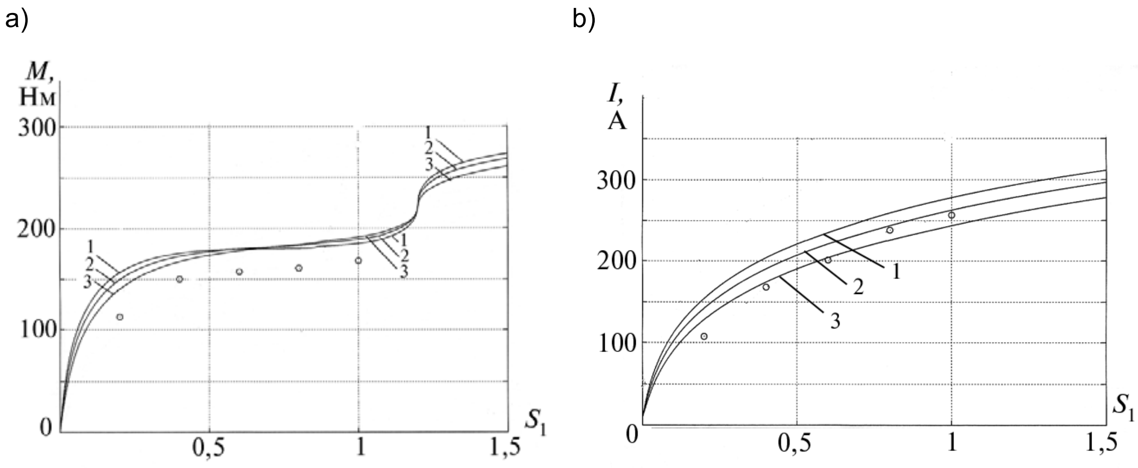


Fig. 12. The electromagnetic torque (a) and current of a stator (b) in a function of slip of an asynchronous motor with a smooth solid rotor from steel 3, made on the basis of asynchronous machine AO2-81-2 (curve 1 corresponds to $\gamma = 6 \cdot 10^6 / к_л$ (Om· m)$^{-1}$**; curve - 2:** $\gamma = 5 \cdot 10^6 / к_л$ (Om· m)$^{-1}$**; curve- 3:** $\gamma = 4 \cdot 10^6 / к_л$ (Om· m)$^{-1}$**;** $к_л = 2{,}5$**; o - experimental points**

The overstating of the calculated torque values as compared to experimental ones (approximately to 14 %) at this stage of calculation is due to neglecting the magnetic voltage in the stator core (it is assumed $\mu_3 = \infty$).

In summary we shall mark, that analytical and numerical methods of calculation of asynchronous machines with a solid rotor many activities are dedicated. The analytical approaches, naturally, are connected to those or diverse assumptions. Let's call these of them, which are assumed in this study:

1. the curvature of magnetic cores and air gap is neglected (the electromagnetic phenomena are considered in rectangular coordinate system) [6];
2. the effect of higher harmonics of magneto-motive force (MMF) on characteristics of machines is not analysed [6, 7, 8, 9];
3. the leakage of magnetic flux in an air gap is rather neglected (magnetic flux coming out of stator core is assumed as equal to magnetic flux entering from an air gap to a rotor core) [7, 9] or its calculation is made approximately [6, 8];
4. the calculation of a magnetic field in steel rotor core is made at given on his surface waves of a magnetic flux density (B_δ) and electric field strength (E_z) [6, 8];
5. the connection between EMF of a rotor and stator is on the basis of an approximated integral representation of a Faraday law of flux density [6, 8, 9].

7. CALCULATION OF MAGNETIC FIELD AT FINAL VALUE OF MAGNETIC PERMEABILITY OF THE STATOR CORE

In the previous calculations the magnetic permeability of the stator core (environment 3 in Fig. 1) was assumed infinitely large. In this case environment 3 did not exert influence on a magnetic field in an adjacent annular domain – air gap. But such effect takes place in reality, when the magnetic permeability of the core of a stator is finite.

7.1. Calculation of magnetic field at the existence of eddy currents in the stator core

Supposing the stator core electro-conductive, the calculation of a magnetic field in its can be made similarly to calculation of a magnetic field in a massive steel rotor. Pursuant to the formulas (20), (22), (23) we shall have for the environment 3

$$\dot{B}_{3r} = \mp \frac{j}{r} \sum_{n=1}^{\infty} n \left[D_{1n} J_n(k_3 r) + D_{2n} Y_n(k_3 r) \right] e^{\mp jn\varphi} \quad (46)$$

$$\dot{B}_{3\varphi} = -k_3 \sum_{n=1}^{\infty} \left[D_{1n} J'_n(k_3 r) + D_{2n} Y'_n(k_3 r) \right] e^{\mp jn\varphi} \quad (47)$$

where

$k_3 = \sqrt{-j\,\omega\,\mu_3\,\gamma_3}$, D_{1n}, D_{2n} - constants defined from boundary conditions.

In the environment 2 (air gap) on boundary circle with radius $r = r_2$ we have according with the formulas (8), (11), (12)

$$\dot{H}_{2r}(r_2,\varphi) = -\frac{3}{2 r_2} \sum_{n=1}^{\infty} n \left(\nu_{2n} \dot{F}_n^{(1)} + \nu_{1n} F_{n\Sigma}^{(2)} \right) e^{\mp jn\varphi} \quad (48)$$

$$\dot{H}_{2\varphi}(r_2,\varphi) = \pm \frac{3j}{2 r_2} \sum_{n=1}^{\infty} n \, \dot{F}_{n\Sigma}^{(2)} \, e^{\mp jn\varphi} \quad (49)$$

where

$\dot{F}_{n\Sigma}^{(2)} = \dot{F}_n^{(2)} + \Delta \dot{F}_n^{(2)}$, $\Delta \dot{F}_n^{(2)}$ – adding stipulated by a magnetic field of the stator core.

The formulas (46)-(49) contain three unknowns of a factor D_{1n}, D_{2n}, $\Delta \dot{F}_n^{(2)}$, which are determined from three equations expressing boundary condition

$$\dot{B}_{3r}(r_2,\varphi) = \mu_0 \dot{H}_{2r}(r_2,\varphi)$$

$$\frac{\dot{B}_{3\varphi}(r_2,\varphi)}{\mu_3} = \dot{H}_{2\varphi}(r_2,\varphi) + \frac{\partial U_2(r_2,\varphi)}{r_2\,\partial\varphi}$$

$$\dot{B}_{3\,r}(r_3,\varphi) = 0$$

From these three equations it is easy to receive expressions for unknowns of factors

$$D_{1n} = \pm\frac{3}{2}\,\frac{j\,\mu_0\,\mu_3\,n\,Y_n(k_3 r_3)\left(\nu_{2n}\dot{F}_n^{(1)} + \nu_{1n}\dot{F}_n^{(2)}\right)}{-\mu_3\,n\,M + \mu_0\,\nu_{1n}\,r_2\,k_3\,N}$$

$$D_{2n} = \mp\frac{3}{2}\,\frac{j\,\mu_0\,\mu_3\,n\,J_n(k_3 r_3)\left(\nu_{2n}\dot{F}_n^{(1)} + \nu_{1n}\dot{F}_n^{(2)}\right)}{-\mu_3\,n\,M + \mu_0\,\nu_{1n}\,r_2\,k_3\,N}$$

$$\Delta\dot{F}_n^{(2)} = -\,\frac{\mu_0\,k_3\,r_2\,N\left(\nu_{2n}\dot{F}_n^{(1)} + \nu_{1n}\dot{F}_n^{(2)}\right)}{-\mu_3\,n\,M + \mu_0\,\nu_{1n}\,r_2\,k_3\,N}$$

where

$$M = J_n(k_3 r_2)\,Y_n(k_3 r_3) - J_n(k_3 r_3)\,Y_n(k_3 r_2)$$

$$N = J_n'(k_3 r_2)\,Y_n(k_3 r_3) - J_n(k_3 r_3)\,Y_n'(k_3 r_2)$$

7.2. Calculation of a magnetic field if neglected eddy currents in the stator core

The magnetic field in the environment 3 (Fig. 1) can be determined on the basis of the solution of Neumann boundary problem. In this case we assume given the radial (normal) components of magnetic field intensity on boundaries of area 3:

$$H_{3rA}(r_2,\varphi) = -\sum_{n=1} \widetilde{a}_n^{(2)} \cos n\varphi + \widetilde{b}_n^{(2)} \sin n\varphi$$

$$H_{3rA}(r_3,\varphi) = 0$$

Then the scalar magnetic potential in this area will be equal [1]

$$U_{3A}(r,\varphi) = \sum_{n=1}^{\infty} \left(\widetilde{A}_n r^n + \widetilde{C}_n r^{-n}\right) \cos n\varphi + \left(\widetilde{B}_n r^n + \widetilde{D}_n r^{-n}\right) \sin n\varphi + k \tag{50}$$

where

$$\left.\begin{aligned} \widetilde{A}_n &= -\frac{r_2^{n+1}\,\widetilde{a}_n^{(2)}}{n\left(r_3^{2n} - r_2^{2n}\right)}, \\ \widetilde{B}_n &= -\frac{r_2^{n+1}\,\widetilde{b}_n^{(2)}}{n\left(r_3^{2n} - r_2^{2n}\right)}, \\ \widetilde{C}_n &= -\frac{r_2^{n+1}\,r_3^{2n}\,\widetilde{a}_n^{(2)}}{n\left(r_3^{2n} - r_2^{2n}\right)}, \\ \widetilde{D}_n &= -\frac{r_2^{n+1}\,r_3^{2n}\,\widetilde{b}_n^{(2)}}{n\left(r_3^{2n} - r_2^{2n}\right)}, \end{aligned}\right\} \tag{51}$$

k - constant.

As regards the magnetic field of phase A is even function (Fig.1) we can accept $\widetilde{b}_n^{(2)} = 0$. Then we have $\widetilde{B}_n = \widetilde{D}_n = 0$.

Meaning that

$$-\widetilde{a}_n^{(2)} = \dot{H}_n^{(2)} \cos\omega t \tag{52}$$

and summarise magnetic fields of all three phases, we have

$$\dot{H}_{3r}(r_2,\varphi) = \frac{3}{2}\sum_{n=1}^{\infty} \dot{H}_n^{(2)} \cos\left(\omega t \mp n\varphi\right) \Leftrightarrow \frac{3}{2}\sum_{n=1}^{\infty} \dot{H}_n^{(2)} e^{\mp jn\varphi} \tag{53}$$

After the substitution of expressions (51) in the formula for SMP (50) we have for an inner boundary of area 3

$$U_{3A}(r_2,\varphi)=-r_2\sum_{n=1}^{\infty}\frac{\nu_{3n}}{n}\tilde{a}_n^{(2)}\cos n\varphi+k$$

where

$$\nu_{3n}=\left(r_3^{2n}+r_2^{2n}\right)/\left(r_3^{2n}-r_2^{2n}\right)$$

Summarising SMP of all three phases and taking into account (52) we shall receive

$$U_3(r_2,\varphi)=\frac{3}{2}r_2\sum_{n=1}^{\infty}\frac{\nu_{3n}}{n}\dot{H}_n^{(2)}e^{\mp jn\varphi}+k$$

Then the tangential component of MF intensity for $r=r_2$ will be equal

$$H_{3\varphi}(r_2,\varphi)=-\frac{\partial U_3(r_2,\varphi)}{r_2\,\partial\varphi}=\pm\frac{3j}{2}\sum_{n=1}^{\infty}\nu_{3n}\,\dot{H}_n^{(2)}e^{\mp jn\varphi} \qquad (54)$$

The boundary conditions for boundary between areas 2 and 3 are represented by two equations

$$\mu_0 H_{2r}(r_2,\varphi)=\mu_3 H_{3r}(r_2,\varphi)$$

$$H_{2\varphi}(r_2,\varphi)=H_{3\varphi}(r_2,\varphi)-\frac{\partial U_2(r_2,\varphi)}{r_2\,\partial\varphi}$$

Substituting in these the formulas (6), (48), (49), (53), (54), we shall receive two equations for finding two unknowns $\dot{H}_n^{(2)}$ and $\dot{F}_{n\Sigma}^{(2)}$:

$$-\frac{\mu_0 n}{r_2}\left(\nu_{2n}\,\dot{F}_n^{(1)}+\nu_{1n}\,\dot{F}_{n\Sigma}^{(2)}\right)=\mu_3\,\dot{H}_n^{(2)}; \qquad \frac{n}{r_2}\dot{F}_{n\Sigma}^{(2)}=\nu_{3n}\,\dot{H}_n^{(2)}+\frac{n}{r_2}\dot{F}_n^{(2)}$$

Solving above equations, we shall find

$$\dot{H}_n^{(2)}=-\frac{\mu_0\,n\left(\nu_{2n}\,\dot{F}_n^{(1)}+\nu_{1n}\,\dot{F}_n^{(2)}\right)}{r_2\left(\mu_3+\mu_0\,\nu_{1n}\,\nu_{3n}\right)} \qquad (55)$$

$$\dot{F}_{n\Sigma}^{(2)} = \frac{-\mu_0\, \nu_{2n}\, \nu_{3n}\, \dot{F}_n^{(1)} + \mu_3\, \dot{F}_n^{(2)}}{\mu_3 + \mu_0\, \nu_{1n}\, \nu_{3n}} \quad (56)$$

We see, that in an ideal case, when $\mu_3 = \infty$, we have

$$\dot{H}_n^{(2)} = 0, \quad \dot{F}_{n\Sigma}^{(2)} = \dot{F}_n^{(2)} \text{ or } \Delta \dot{F}_n^{(2)} = 0$$

8. CONSIDERING THE CHANGE OF MAGNETIC PERMEABILITY IN MASSIVE FERROMAGNETIC ROTOR

The relation of magnetic permeability to a diving depth of a magnetic field in a body of a rotor is easy for taking into account by splitting his array into elementary segments shaped of concentric rings (Fig. 13). To each i elementary segment there will correspond the value of magnetic permeability equal ${}^{i}\mu_1$.

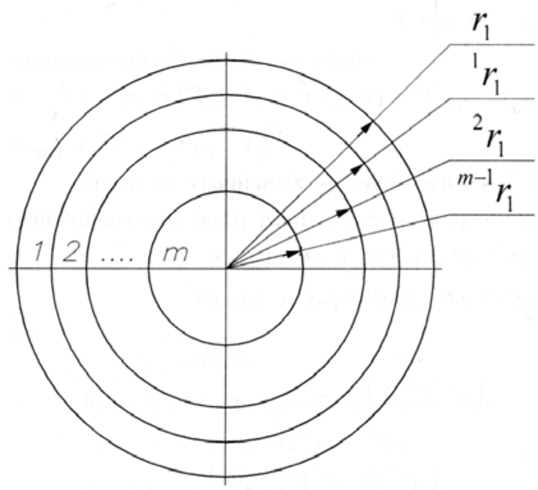


Fig. 13. Elementary segments of a rotor in the form of concentric rings with numbers 1, 2, …, m

On boundaries of elementary segments the magnetic permeability will jump. Inside an elementary ring the parameters of the environment ${}^{i}\mu$ and ${}^{i}\gamma$ are invariable, i.e. environment will be homogeneous (linear), and the initial differential equation (18) for a vector potential in it will be linear. Therefore for such elementary segments there is no necessity initially to consider magnetic permeability as a continuous function of space coordinates. That it, apparently, becomes, if number of elementary concentric rings to take infinitely large.

Pursuant to the formulas (17), (20) we shall have following expressions for components of complex amplitude n of a harmonics of a magnetic flux density in an elementary segment

$${}^{i}\dot{B}_{1r}^{(n)} = \mp \frac{j}{r}\, n \left[{}^{i}C_{1n}\, J_n\left({}^{i}k\sqrt{S_n}\, r\right) + {}^{i}C_{2n}\, Y_n\left({}^{i}k\sqrt{S_n}\, r\right)\right] \tag{57}$$

$${}^{i}\dot{B}_{1\varphi}^{(n)} = -\,{}^{i}k\sqrt{S_n}\left[{}^{i}C_{1n}\, J_n'\left({}^{i}k\sqrt{S_n}\, r\right) + {}^{i}C_{2n}\, Y_n'\left({}^{i}k\sqrt{S_n}\, r\right)\right] \tag{58}$$

where

$$i = 1,2,\ldots,m,\ {}^{i}k = \sqrt{-\,j\,\omega\,{}^{i}\mu_1\,\gamma}$$

For last (m) of a segment shaped circle, it is necessary to accept

$${}^{m}C_{2n} = 0$$

The unknown factors ${}^{i}C_{1n}$, ${}^{i}C_{2n}$ and $\dot{F}_n^{(1)}$ with the help of the formulas (12), (13) for elementary segments will be determined from boundary conditions (24).

For example, from boundary conditions of an air gap and outside surface of a rotor the equations follow

$$\begin{aligned}{}^{1}C_{1n}\, J_n\left({}^{1}k\sqrt{S_n}\, r_1\right) + {}^{1}C_{2n}\, Y_n\left({}^{1}k\sqrt{S_n}\, r_1\right) \mp \left(3\,j\,\mu_0\,\nu_{1n}/2\right)\dot{F}_n^{(1)} = \\ = \pm\left(3\,j\,\mu_0\,\nu_{2n}/2\right)\dot{F}_n^{(2)}\end{aligned} \tag{59}$$

$${}^{1}C_{1n}\, J_n'\left({}^{1}k\sqrt{S_n}\, r_1\right) + {}^{1}C_{2n}\, Y_n'\left({}^{1}k\sqrt{S_n}\, r_1\right) \pm \left(3\,j\,n\,{}^{1}\mu_1/2\, r_1^{1}\, k\sqrt{S_n}\right)\dot{F}_n^{(1)} = 0 \tag{60}$$

Let's enter into consideration vector of unknowns

$$\overline{x} = \left[x_1\, x_2 \ldots x_{2m}\right]^T$$

The components which one will be equal

$$\begin{aligned}&x_1 = {}^{1}C_{1n},\ x_2 = {}^{1}C_{2n},\ x_3 = {}^{2}C_{1n},\ x_4 = {}^{2}C_{2n},\ldots,x_{2m-3} = \\ &= {}^{(m-1)}C_{1n},\ x_{2m-2} = {}^{(m-1)}C_{2n},\quad x_{2m-1} = {}^{m}C_{1n},\quad x_{2m} = \dot{F}_n^{(1)}\end{aligned}$$

Then from boundary conditions i and (i+1) of elementary segments having a boundary circle of radius $r = {}^{i}r_1$, we shall receive equations

$$a_{11}x_1 + a_{12}x_2 + a_{1,2m}x_{22}a_{2m} = b_1 \quad (61)$$

$$a_{21}x_1 + a_{22}x_2 + a_{2,2m}x_{22}a_{2m} = 0 \quad (62)$$

$$a_{2i+1,2i-1}\,x_{2i-1} + a_{2i+1,2i}\,x_{2i} + a_{2i+1,2i+1}\,x_{2i+1} + a_{2i+1,2i+2}\,x_{2i+2} = 0 \quad (63)$$

$$a_{2i+2,2i-1}\,x_{2i-1} + a_{2i+2,2i}\,x_{2i} + a_{2i+2,2i+1}\,x_{2i+1} + a_{2i+2,2i+2}\,x_{2i+2} = 0 \quad (64)$$

where

i = 1, 2, ..., (m-1)

$$a_{11} = J_n\left({}^{1}k\sqrt{S_n}r_1\right), \qquad a_{12} = Y_n\left({}^{1}k\sqrt{S_n}r_1\right), \qquad a_{1,2m} = \mp 3j\mu_0\nu_{1n}/2$$

$$a_{21} = J'_n\left({}^{1}k\sqrt{S_n}r_1\right), \qquad a_{22} = Y'_n\left({}^{1}k\sqrt{S_n}r_1\right), \qquad a_{2,2m} = \pm 3jn^{1}\mu_1/2r_1{}^{1}k\sqrt{S_n}$$

$$a_{2\,i+1,2i-1} = J_n\left({}^{i}k\sqrt{S_n}\;{}^{i}r_1\right),\; a_{2\,i+1,2i+1} = -J_n\left({}^{i+1}k\sqrt{S_n}\;{}^{i}r_1\right),\; a_{2\,i+1,2i} = Y_n\left({}^{i}k\sqrt{S_n}\;{}^{i}r_1\right)$$

$$a_{2\,i+1,2i+2} = -Y_n\left({}^{i+1}k\sqrt{S_n}\;{}^{i}r_1\right), \qquad a_{2\,i+2,2i-1} = {}^{(i+1)}\mu_1\,{}^{i}k\,J'_n\left({}^{i}k\sqrt{S_n}\;{}^{i}r_1\right)$$

$$a_{2\,i+2,2i} = {}^{(i+1)}\mu_1\;{}^{i}kY'_n\left({}^{i}k\sqrt{S_n}\;{}^{i}r_1\right), \qquad a_{2\,i+2,2i+1} = -\,{}^{i}\mu_1\;{}^{(i+1)}kJ'_n\left({}^{(i+1)}k\sqrt{S_n}\;{}^{i}r_1\right)$$

$$a_{2\,i+2,2i+2} = -{}^{i}\mu_1\;{}^{(i+1)}kY'_n\left({}^{(i+1)}k\sqrt{S_n}\;{}^{i}r_1\right),$$

$$a_{2m-1,2m} = 0, \quad a_{2m,2m} = 0, \qquad Y'_n(z) = \frac{1}{2}\left[Y_{n-1}(z) - Y_{n+1}(z)\right]$$

The unknown constants of rings will be determined from a vector-matrix equation

$$\mathbf{Ax} = \mathbf{b} \quad (65)$$

where

$$b_1 = \pm(3j\mu_0\nu_{2n}/2)\dot{F}_n^{(2)}; \;\; b_k = 0, \;\; k = 2,3,...,2m$$

The matrix **A** has following frame

$$[\mathrm{A}] = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 & 0\ldots & 0 & 0 & 0 & a_{1,2m} \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0\ldots & 0 & 0 & 0 & a_{2,2m} \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 0\ldots & 0 & 0 & 0 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0\ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} & 0\ldots & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{63} & a_{64} & a_{65} & a_{66} & 0\ldots & 0 & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0\ldots & a_{2m-1,2m-3} & a_{2m-1,2m-2} & a_{2m-1,2m-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0\ldots & a_{2m,2m-3} & a_{2m,2m-2} & a_{2m,2m-1} & 0 \end{bmatrix}$$

The solution of a set of equations (65) allows to determine components of complex amplitude *n* of a harmonics of a magnetic flux density in ring segments of a rotor according to the formulas (57), (58). Therefore, there are modules of amplitudes of a first harmonic of flux density in these segments

$$^{i}B_{1\max}^{(1)} = \sqrt{\left[{}^{i}B_{1r\max}^{(1)}\right]^2 + \left[{}^{i}B_{1\varphi\max}^{(1)}\right]^2}$$

on which ones (turning towards their effective values [5]) with the help of a magnetisation curve of a rotor material – the magnetic permeability i of a segment ${}^{i}\mu_1$ will be determined.

If to enter the denotations

$$\left.\begin{aligned} {}^{i}a_{1n} = \mathrm{Re}\,{}^{i}C_{1n},\ {}^{i}b_{1n} = J_m\,{}^{i}C_{1n} \\ {}^{i}a_{2n} = \mathrm{Re}\,{}^{i}C_{2n},\ {}^{i}b_{2n} = J_m\,{}^{i}C_{2n}, \end{aligned}\right\} \tag{66}$$

$$\mathrm{her}_n x = \frac{2}{\pi}\mathrm{kei}_n x, \quad \mathrm{hei}_n x = -\frac{2}{\pi}\mathrm{ker}_n x, \tag{67}$$

where

$$x = \sqrt{\omega\,{}^{i}\mu_1\,{}^{i}\gamma\left|S_n\right|}\,{}^{(i-1)}r_1, \qquad i = 1, 2, ..., m,\ {}^{(0)}r_1 = r_1 \tag{68}$$

it is possible to give amplitude values of radial and tangential harmonics of flux density on outside boundaries of annular domains of a rotor with radiuses ${}^{(i-1)}r_1$ $(i = 1, 2, ..., m)$ in form

$$ {}^{i}B_{1r}^{(n)} = \frac{n}{{}^{(i-1)}r_1} \sqrt{{}^{r}C_n^2 + {}^{r}D_n^2} \tag{69} $$

$$ {}^{i}B_{1\varphi}^{(n)} = \frac{1}{2} \sqrt{\frac{\omega\, {}^{i}\,{}^{i}\mu_1\, {}^{i}\gamma_1 |S_n|}{2}} \sqrt{{}^{\varphi}C_n^2 + {}^{\varphi}D_n^2} \tag{70} $$

where expression for functions C_n and D_n depend on the sign of slip S_n [1)].

1) $S_n > 0$

$$ {}^{r}C_n(x) = {}^{+r}C_n(x) = -{}^{i}a_{1n}\, {}^{i}\mathrm{bei}_n x - {}^{i}b_{1n}\, {}^{i}\mathrm{ber}_n x + {}^{i}a_{2n}\left(-{}^{i}\mathrm{ber}_n x + {}^{i}\mathrm{her}_n x\right) + {}^{i}b_{2n}\left({}^{i}\mathrm{bei}_n x - {}^{i}\mathrm{hei}_n x\right) \tag{71} $$

$$ {}^{r}D_n(x) = {}^{+r}D_n(x) = {}^{i}a_{1n}\, {}^{i}\mathrm{ber}_n x - {}^{i}b_{1n}\, {}^{i}\mathrm{bei}_n x + {}^{i}a_{2n}\left(-{}^{i}\mathrm{bei}_n x + {}^{i}\mathrm{hei}_n x\right) - {}^{i}b_{2n}\left({}^{i}\mathrm{ber}_n x - {}^{i}\mathrm{her}_n x\right) \tag{72} $$

$$ \begin{aligned} {}^{\varphi}C_n(x) = {}^{+\varphi}C_n(x) = {}^{i}a_{1n}\left({}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x + {}^{i}\mathrm{bei}_{n-1} x - {}^{i}\mathrm{bei}_{n+1} x\right) + \\ + {}^{i}b_{1n}\left(-{}^{i}\mathrm{bei}_{n-1} x + {}^{i}\mathrm{bei}_{n+1} x + {}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x\right) + {}^{i}a_{2n}\left(-{}^{i}\mathrm{bei}_{n-1} x + {}^{i}\mathrm{bei}_{n+1} x + {}^{i}\mathrm{hei}_{n-1} x - \right. \\ \left. - {}^{i}\mathrm{hei}_{n+1} x + {}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x - {}^{i}\mathrm{her}_{n-1} x + {}^{i}\mathrm{her}_{n+1} x\right) + {}^{i}b_{2n}\left(-{}^{i}\mathrm{ber}_{n-1} x + {}^{i}\mathrm{ber}_{n+1} x + {}^{i}\mathrm{her}_{n-1} x - \right. \\ \left. - {}^{i}\mathrm{her}_{n+1} x - {}^{i}\mathrm{bei}_{n-1} x + {}^{i}\mathrm{bei}_{n+1} x + {}^{i}\mathrm{hei}_{n-1} x - {}^{i}\mathrm{hei}_{n+1} x\right) \end{aligned} \tag{73} $$

$$ \begin{aligned} {}^{\varphi}D_n(x) = {}^{+\varphi}D_n(x) = {}^{i}a_{1n}\left(-{}^{i}\mathrm{ber}_{n-1} x + {}^{i}\mathrm{ber}_{n+1} x + {}^{i}\mathrm{bei}_{n-1} x - {}^{i}\mathrm{bei}_{n+1} x\right) + \\ + {}^{i}b_{1n}\left({}^{i}\mathrm{bei}_{n-1} x - {}^{i}\mathrm{bei}_{n+1} x + {}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x\right) + {}^{i}a_{2n}\left({}^{i}\mathrm{bei}_{n-1} x - {}^{i}\mathrm{bei}_{n+1} x - {}^{i}\mathrm{hei}_{n-1} x + {}^{i}\mathrm{hei}_{n+1} x + \right. \\ \left. + {}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x - {}^{i}\mathrm{her}_{n-1} x + {}^{i}\mathrm{her}_{n+1} x\right) + {}^{i}b_{2n}\left({}^{i}\mathrm{ber}_{n-1} x - {}^{i}\mathrm{ber}_{n+1} x - {}^{i}\mathrm{her}_{n-1} x + {}^{i}\mathrm{her}_{n+1} x - \right. \\ \left. - {}^{i}\mathrm{bei}_{n-1} x + {}^{i}\mathrm{bei}_{n+1} x + {}^{i}\mathrm{hei}_{n-1} x - {}^{i}\mathrm{hei}_{n+1} x\right) \end{aligned} \tag{74} $$

[1)] Bessel cylindrical functions of the second kinds with complex arguments

$$ k\sqrt{S_n}\, r = \sqrt{-j\omega\mu\gamma S_n}\, r = q_n r e^{j3\pi/4} = x e^{j3\pi/4}, S_n > 0 $$

$$ k\sqrt{S_n}\, r = \sqrt{j\omega\mu\gamma |S_n|}\, r = q_n r e^{j\pi/4} = x e^{j\pi/4}, S_n < 0 $$

where $q_n = \sqrt{j\omega\mu\gamma |S_n|}$

are expressed through Kelvin functions:

$$ Y_n\left(x e^{j3\pi/4}\right) = -\frac{2}{\pi}\mathrm{ker}_n x - \mathrm{bei}_n x + j\left(-\frac{2}{\pi}\mathrm{kei}_n x + \mathrm{ber}_n x\right) $$

$$ Y_n\left(x e^{j\pi/4}\right) = \left[\frac{2}{\pi}\mathrm{ker}_n x - \mathrm{bei}_n x - j\left(\frac{2}{\pi}\mathrm{kei}_n x + \mathrm{ber}_n x\right)\right]\cos \pi n . $$

2) $S_n < 0$

$${}^{r}C_n(x)={}^{-r}C_n(x)={}^{i}a_{1n}\,{}^{i}\mathrm{bei}_n x-{}^{i}b_{1n}\,{}^{i}\mathrm{ber}_n x+{}^{i}a_{2n}\left({}^{i}\mathrm{ber}_n x+{}^{i}\mathrm{her}_n x\right)+{}^{i}b_{2n}\left({}^{i}\mathrm{bei}_n x+{}^{i}\mathrm{hei}_n x\right) \quad (75)$$

$${}^{r}D_n(x)={}^{-r}D_n(x)={}^{i}a_{1n}\,{}^{i}\mathrm{ber}_n x+{}^{i}b_{1n}\,{}^{i}\mathrm{bei}_n x-{}^{i}a_{2n}\left({}^{i}\mathrm{bei}_n x+{}^{i}\mathrm{hei}_n x\right)+{}^{i}b_{2n}\left({}^{i}\mathrm{ber}_n x+{}^{i}\mathrm{her}_n x\right) \quad (76)$$

$$\begin{aligned}{}^{\varphi}C_n(x)={}^{-\varphi}C_n(x)={}^{i}a_{1n}\left({}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x+{}^{i}\mathrm{bei}_{n-1}x-{}^{i}\mathrm{bei}_{n+1}x\right)+\\ +{}^{i}b_{1n}\left({}^{i}\mathrm{bei}_{n-1}x-{}^{i}\mathrm{bei}_{n+1}x-{}^{i}\mathrm{ber}_{n-1}x+{}^{i}\mathrm{ber}_{n+1}x\right)+{}^{i}a_{2n}\left(-{}^{i}\mathrm{bei}_{n-1}x+{}^{i}\mathrm{bei}_{n+1}x-{}^{i}\mathrm{hei}_{n-1}x+{}^{i}\mathrm{hei}_{n+1}x+\right.\\ \left.+{}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x+{}^{i}\mathrm{her}_{n-1}x-{}^{i}\mathrm{her}_{n+1}x\right)+{}^{i}b_{2n}\left({}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x+{}^{i}\mathrm{her}_{n-1}x-{}^{i}\mathrm{her}_{n+1}x+\right.\\ \left.+{}^{i}\mathrm{bei}_{n-1}x-{}^{i}\mathrm{bei}_{n+1}x+{}^{i}\mathrm{hei}_{n-1}x-{}^{i}\mathrm{hei}_{n+1}x\right)\end{aligned} \quad (77)$$

$$\begin{aligned}{}^{\varphi}D_n(x)={}^{-\varphi}D_n(x)={}^{i}a_{1n}\left({}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x-{}^{i}\mathrm{bei}_{n-1}x+{}^{i}\mathrm{bei}_{n+1}x\right)+\\ +{}^{i}b_{1n}\left({}^{i}\mathrm{bei}_{n-1}x-{}^{i}\mathrm{bei}_{n+1}x+{}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x\right)+{}^{i}a_{2n}\left(-{}^{i}\mathrm{bei}_{n-1}x+{}^{i}\mathrm{bei}_{n+1}x-{}^{i}\mathrm{hei}_{n-1}x+{}^{i}\mathrm{hei}_{n+1}x-\right.\\ \left.-{}^{i}\mathrm{ber}_{n-1}x+{}^{i}\mathrm{ber}_{n+1}x-{}^{i}\mathrm{her}_{n-1}x+{}^{i}\mathrm{her}_{n+1}x\right)+{}^{i}b_{2n}\left({}^{i}\mathrm{ber}_{n-1}x-{}^{i}\mathrm{ber}_{n+1}x+{}^{i}\mathrm{her}_{n-1}x-{}^{i}\mathrm{her}_{n+1}x-\right.\\ \left.-{}^{i}\mathrm{bei}_{n-1}x+{}^{i}\mathrm{bei}_{n+1}x-{}^{i}\mathrm{hei}_{n-1}x+{}^{i}\mathrm{hei}_{n+1}x\right)\end{aligned} \quad (78)$$

The expressions (57), (58) rather easily allow to calculate a two-layer rotor [10] (first layer – massive cylinder from alloy of iron and copper, the second layer – the laminated core from electric machine’s steel) or to take into account availability of the coopering of the solid rotor surface enriching performance data of an asynchronous motor [6, 8]. In this case for one or several first concentric rings it is necessary to accept

$${}^{i}\mu={}^{i}\mu_e=\mu_0 \text{ и } {}^{i}\gamma=\gamma_{\mathrm{Cu(Al)}}$$

If there are homogeneous concentric ring elementary segments the formulas (35), (38) for a current density and losses in a rotor will be other. Pursuant to initial expression (34) for a current density and, considering the formula (57) for radial component *n* of a harmonics of a magnetic flux density for *i* of a ring, we shall have for complex amplitude of a current density in it

$${}^{i}\dot{\delta}_1^{(n)}=-j\,{}^{i}\gamma\omega S_n\left[{}^{i}C_{1n}J_n\left({}^{i}k\sqrt{S}r\right)+{}^{i}C_{2n}Y_n\left({}^{i}k\sqrt{S}r\right)\right] \quad (79)$$

According to denotations above introduced in (66)-(68), (71), (72), (75), (76) it is fair

$$j\left[{}^{i}C_{1n}J_{n}\left({}^{i}k\sqrt{S_{n}}\,r\right)+{}^{i}C_{2n}Y_{n}\left({}^{i}k\sqrt{S_{n}}\,r\right)\right]={}^{\pm r}C_{n}(x)+j\,{}^{\pm r}D_{n}(x)$$

where

$x=\sqrt{\omega\,{}^{i}\mu_{1}\,{}^{i}\gamma_{1}S_{n}}\,r={}^{i}q_{n}r$; the sign (+) corresponds to $S_n>0$; the sign (-) – $S_n<0$.

Therefore, for a module of amplitude n of a harmonics of a current density in i ring we have

$$^{i}\delta^{(n)}={}^{i}\gamma_{1}\omega S_{n}\sqrt{{}^{\pm r}C_{n}^{2}\left({}^{i}q_{n}r\right)+{}^{\pm r}D_{n}^{2}\left({}^{i}q_{n}r\right)} \quad (80)$$

The losses in a rotor from n harmonic of a current density should be found according to the formula

$$P_{n}=\pi l\int_{0}^{r_{1}}\frac{\left[{}^{i}\delta^{(n)}\right]^{2}rdr}{{}^{i}\gamma_{1}}=\pi l(\omega S_{n})^{2}\sum_{i=1}^{m}{}^{i}\gamma_{1}\int_{{}^{i}r_{1}}^{{}^{(i-1)}r_{1}}\left[{}^{\pm r}C_{n}^{2}\left({}^{i}q_{n}r\right)+{}^{\pm r}D_{n}^{2}\left({}^{i}q_{n}r\right)\right]rdr \quad (81)$$

In Figs. 14-16 some parameters of a magnetic field of ring concentric segments of a solid rotor in a function of slips calculated on the formulas (65), (69), (70) are presented.

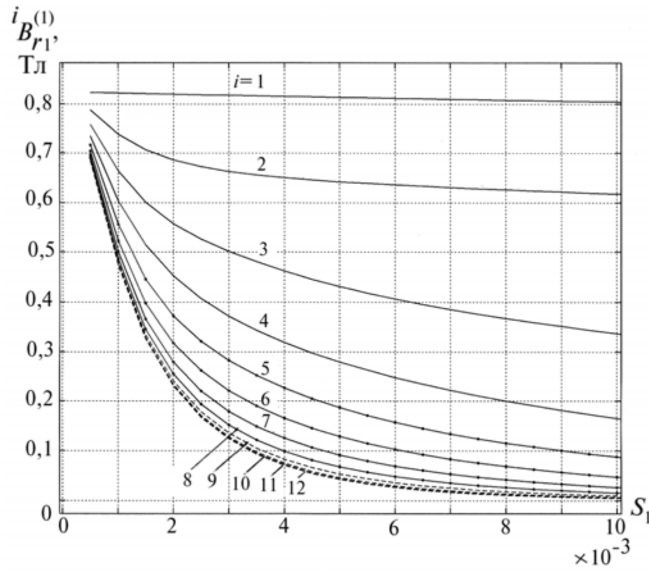


Fig. 14. Amplitude of a first harmonic of radial component of a magnetic flux density on outer boundaries of ring concentric segments (m = 12) of a solid rotor as a function of slip

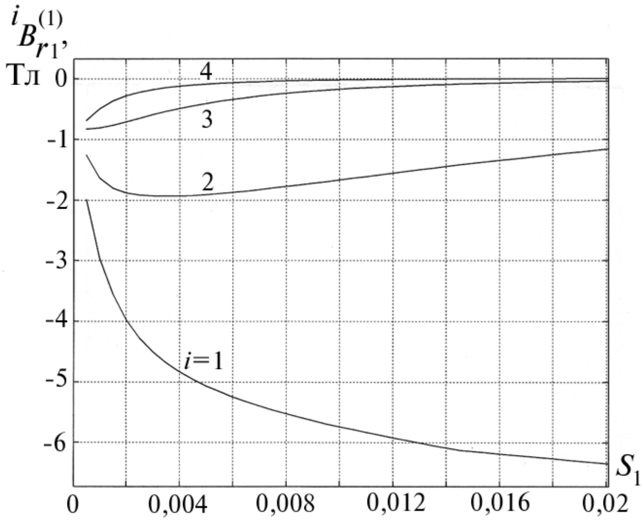


Fig. 15. Amplitude of a first harmonic of radial component of a magnetic flux density on outer boundaries of ring concentric segments (m = 4) of a solid rotor as a function of slip

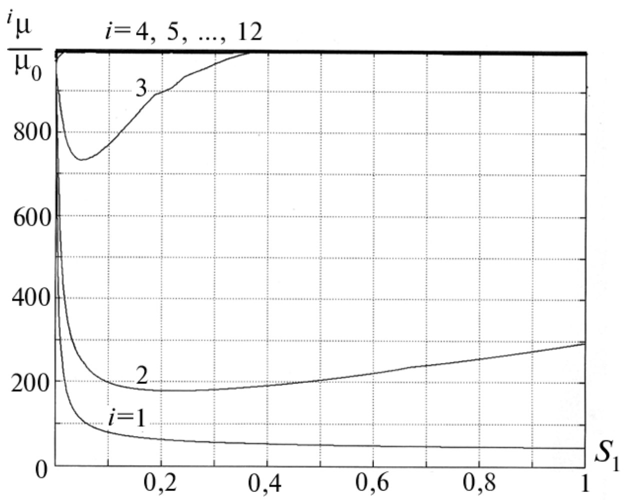


Fig.16. Magnetic permeability of concentric ring segments (m = 12) of a solid rotor in a function of slip

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ANALITYCZNE PODEJŚCIE DO OBLICZANIA MASZYN ELEKTRYCZNYCH W OPARCIU O ROZWIĄZANIE PROBLEMÓW BRZEGOWYCH DOMEN PIERŚCIENIOWYCH METODĄ ROZDZIAŁU ZMIENNYCH FOURIERA

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STRESZCZENIE *Szczelina powietrzna obustronnie uzębiona, będąca obszarem przejścia, może być konformalnie odwzorowana jako pierścień kołowy; tradycyjnie tę procedurę przeprowadza się w przybliżeniu przy pomocy współczynników Cartera. Przekroje poprzeczne rdzeni stojana i wirnika mają również kształt pierścieni.*

Wirujące pole magnetyczne w obszarach przewodów (w żłobkach, w oknach między biegunami) może być przedstawione przez sumę pól potencjału i pewnego dodatkowego (łatwego do obliczenia) pola.

[1)] All in Russian

Umieszczenie dodatkowego pola w miejscu, gdzie są przewody wymaga rozmieszczenia w brzegowej części żłobka (w części przy dnie lub klinie) nieskończenie cienkich warstw prądu tak, aby łączne prądy były równe pełnym prądom żłobków.

Użycie skalarnych magnetycznych potencjałów (SMP) warstw prądu pozwala otrzymać analityczne rozwiązanie problemu Dirichleta dla kołowych pierścieni wspomnianych trzech obszarów (szczelina powietrzna, rdzeń magnetyczny stojana i wirnika) przy założeniu, że przenikalności magnetyczne są stałe.

Dla uzyskania lepszego uogólnienia problemu zakładamy, że rdzeń wirnika jest wykonany z litej stali. Wybranie tego materiału czyni model matematyczny bardziej uniwersalnym, gdyż przy przejściu do wysokich wartości przenikalności magnetycznej i rezystywności, materiał lity uzyskuje własności stali blachowanej.

SMP warstw prądu uzwojeń statora i wirnika przedstawia się za pomocą szeregów Fouriera.

Parametry pola elektromagnetycznego w litym wirniku (promieniowa i styczna składowa indukcji, gęstość prądu i straty mocy) otrzymuje się na podstawie analitycznego rozwiązania różniczkowych równań Bessela dla wektorowego potencjału magnetycznego metodą rozdziału zmiennych i przedstawia się je przez odpowiednie funkcje Bessela i Kelvina.

Obliczone dane dla silnika asynchronicznego z litym wirnikiemi porównano z wynikami prób. Zmiany przenikalności magnetycznej w stali wirnika uwzględniono przez podzielenie go na współosiowe pierścienie o stałej przenikalności w każdym z nich.