Paper

The role of time in influence diagrams

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Abstract — An influence diagram is a compact representation emphasizing the qualitative features of decision problem under uncertainty. Classical influence diagram has parameters stable in time, determined order of suggested decisions and generally is independent of time. Here we have shown some possible methods of construction of time dependent influence diagrams: with decision ordering, time-sliced segments and time consuming nodes. Such gathering of methods can help in selection of a proper solution.

Keywords* — *uncertainty, belief networks, influence diagrams, ordering in time.

1. Introduction

Graphical modelling for decision support systems under uncertainty is getting more and more widespread. It is an appealing way to think of and communicate on the underlying structure of the domain in question. Graphical models are potentially powerful because they translate a complex decision problem into an easily understood, qualitative form. Quantitative, numerical solution of the problem presented in such a form is usually much more complicated, but for most typical cases there are available not only precise algorithms but also commercial systems, computing needed results.

Probabilistic graphical models are graphs in which nodes represent random variables, and arcs (or lack of them) represent conditional independence assumptions. Undirected graphical models are used to depict *Markov networks*, but directed models, enhanced with additional nodes, can describe different sceneries of decision systems.

- If some random variables from the set C describe *state of the world*, with given prior probabilities of possible values, and these variables influence some other *chance variables* from the set C' (what is noted down as $C \to C'$), the graph can be understood as the model of simple *Bayesian network* or *believe network* [1]. For chance variables we must specify the conditional probability distribution at each node. In a more complicated case chance variables can influence another chance variables: $(C, C') \to C'$. The most common task we wish to solve using Bayesian networks is probabilistic inference.

- Sometimes belief networks are controlled by external interventions, described by decision variables (from the set D). A *decision variable* is a variable whose instances correspond to possible actions among which the intervening person can choose. The model $(C, D) \to C'$ is known as *causal graph*, chance variables are called *consequences* and appropriate graphs are used for causal reasoning [2].

- Believe networks and causal graphs may help in preparing a new kind of decisions (from the set D'), which control external actions $((C, C', D) \to (C', D'))$ or influence chance variables $((C, C', D, D') \to (C', D'))$. In many cases the differences between variables from sets C and C' or from D and D' are not important, thus putting $C \cup C' = \mathcal{C}$ (called *observations*) and $D \cup D' = \mathcal{D}$ one can describe this model by $(\mathcal{C}, \mathcal{D}) \to (\mathcal{C}, \mathcal{D})$. This is a simple version of an *influence diagram* [3].

- Each decision support system attempts to find the best possible decision, so in a graph model we need one more type of nodes: *utility nodes* (from the set $\mathcal{U}$), that represent the usefulness of the consequences of decisions and observations, measured on a numerical scale called *utility*. The model $(\mathcal{C}, \mathcal{D}) \to (\mathcal{C}, \mathcal{D}, \mathcal{U})$ illustrates a full version of *influence diagram* (ID), used as an analysis tool and a communication tool for decision support.

We normally assume that the model structure and the parameters of influence diagrams do not change, i.e. the model is time-invariant. However, in many cases ID is used to describe a proces containing the sequence of events, and time should be taken into account. In such situations we can add extra nodes to represent the current "regime", or we can repeat the basic diagram to represent time-slices [4].

Classical IDs require a linear temporal ordering of the decisions, and this is often felt as an unnecessary constraint. In reality some decisions can be taken independently of each other, and their identification (and order modification) can simplify system implementation [5], because the solution of a decision problem modeled by an ID is a sequence of decisions that maximizes the expected utility.

An ID specifies also a certain order of observations and decisions through its structure. This order is reflected in the corresponding methodology of solving ID [6].

The problems specified above confirm the significant role of time in ID. Some of these problems will be considered below in detail, but for that more precise definitions are needed.

2. Influence diagrams

An influence diagram is a directed acyclic graph $I = (\mathcal{V}, \mathcal{E})$, where the nodes (vertices) $\mathcal{V}$ can be partitioned into three disjoint subsets: *chance nodes* $\mathcal{C}$, *decision nodes* $\mathcal{D}$ and *utility* (*value*) *nodes* $\mathcal{U}$, thus $\mathcal{V} = \mathcal{C} \cup \mathcal{D} \cup \mathcal{U}$. It is a common practise to term nodes and variables in ID by the same name and use them interchangeably.

The chance nodes (drawn as circles or ovals) correspond to *chance variables*, and represent events which are not under the direct control of the decision maker. The decision nodes (drawn as squares) correspond to *decision variables* and represent actions under the direct control of the decision maker. The utility nodes (drawn as diamonds) define *utility functions*, indicating the local utility for a given configuration of variables in their domain. The total utility is the sum or the product of the local utilities.

The arcs (links) in an ID (the pairs of nodes (x,y) from the set $\mathcal{E}$) can be partitioned into three disjoint subsets, corresponding to the type of node they go into. Arcs into utility nodes represent functional dependencies by indicating the domain of associated utility function. Arcs into chance nodes, denoted *dependency arcs*, represent probabilistic dependencies. Arcs into decision nodes, denoted *informational arcs*, imply information precedens: if there is an arc from a node x to a decision node d then the state of x is known when decision d is made.

If there is a directed link from x to y ($x,y \in \mathcal{V}$), then x is called a *parent* of y, and y a *child* of x. The sets of parents and children of x are denoted $pa(x)$ and $ch(x)$, respectively. For each utility node u the set $ch(u)$ is empty. In an ID we usually assume "no forgetting", which means that if there is a link from x to d we need not have a link from x to elements of $ch(d)$.

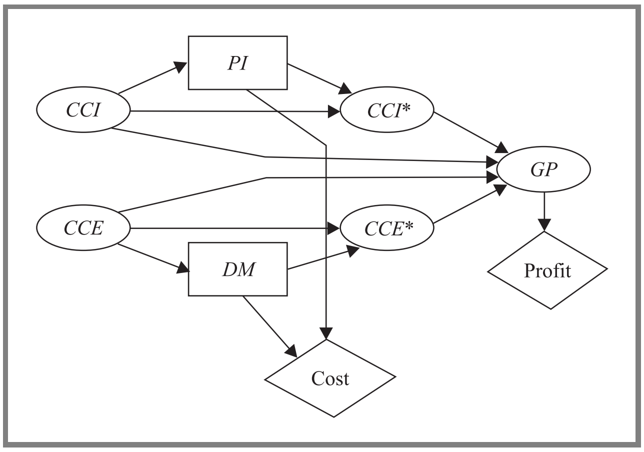

Fig. 1. Influence diagram of call center.

Simplified example of influence diagram for call center (CC) is depicted in Fig. 1. The task of this call center is to promote voice mail delivered by a telecommunication company. Call center intensity *CCI* and efficiency *CCE*, defined by known probability distribution of informative variables, influence the global performance *GP* of CC. This performance can be enlarged by external intervention, changing intensity and efficiency, namely personel increase *PI* and/or special algorithm of data mining *DM*.

Utility nodes state the *costs* of interventions (negative value) and the *profit* given by enlarged global performance (positive value). Optimal decision is determined on the basis of the sum of these utilities.

With each chance variable and decision variable x we associate a *state space* W_x which denotes the set of possible outcomes/decision alternatives for x [5]. For a set X of variables we define the state space as $W_X = \times\{W_x | x \in X\}$.

The uncertainty associated with each chance variable r is represented by a *conditional probability function* $P(r|pa(r)) : W_{\{r\} \cup pa(r)} \to [0,1]$.

When evaluating an ID we identify a *strategy* for the decision variables; a strategy can be seen as a prescription of responses to earlier observations and decisions. A strategy is then a set of functions $\Delta = \{\delta_d | d \in \mathcal{D}\}$, where δ_d is a decision function given by:

$$\delta_d : W_{pa(d)} \to W_d \,.$$

The evaluation is usually performed according to the *maximum expected utility principle*, which states that we should always choose an alternative that maximizes the expected utility. A strategy that maximizes the expected utility is termed an *optimal strategy*. It strongly depends on temporal ordering of variables and therefore time considerations are so important.

3. Time in influence diagrams

There are at least three particular cases when the value of time or ordering in time play important role.

Time is not explicitly declared. If the decision problem, modeled by an influence diagram, has not periods of time clearly stated, a diagram is constructed sequentially: at first chance variables and dependency arcs between them are introduced, then decision variables with information links are added, and next utility nodes are defined and connected with other nodes. The ID is ready, but before using it we have to term its *realization*: an attachment of functions to the appropriate variables. This means that the chance nodes and variables are associated with conditional probability functions (or prior probabilities for nodes without parents) and the utility nodes and variables are associated with utility functions. Decision nodes correspond to actions taken by external agents; ID defines information needed for each decision (by information arcs) and, sometimes, the order of decisions. An order depends on the structure of the ID and its interpretation.

A directed path $\pi = \langle x_1, x_2, \ldots, x_k \rangle$ in ID is an ordered sequence of distinct nodes such that $x_i \in pa(x_{i+1})$. The set $an(y)$, called *ancestors of* y, contains all nodes x such

that there exist a path $\langle x,\dots,y\rangle$. For ordering in time we will use notation $x \prec y$ (x before y).
It will be assumed that:

1. If there is the path from a node x to a node y then a variable x and all elements of this path are relevant for a variable y, i.e. values or decisions of y are functions of all values and decisions from the path realization. This is the simplifying assumption because in reality:

   – some values of the probability distribution can block the influence of some other values from the path,

   – some values from the path are not required for an optimal strategy, i.e. are not elements of the maximum expected utility function [5].

   Nevertheless this assumption is sensible because it helps to order in time events defined by ID in more general case, when probability distributions and utility functions are modified during the project preparation.

2. Actions represented by decision nodes cannot be taken simultaneously and therefore different nodes d_i should be related to different times t_i.

3. All parameters are stable in time.

If in the ID one can find a path containing all decision nodes then an order of decisions is forced by this path, and each earlier decision influences future decisions. If decision nodes are located in distinct paths–their ordering in time can be changed and we call them *incompatible* nodes.

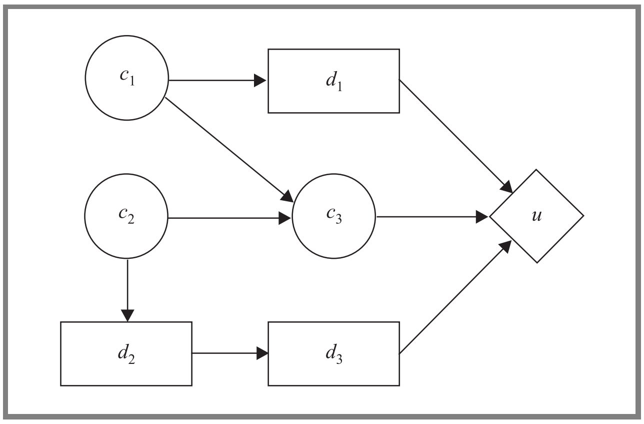

Fig. 2. Influence diagram with incompatible nodes.

Let two decision nodes belong to the set of parents of some utility nodes: $\{d',d''\} \in pa(u)$, i.e. nodes d' and d'' are incompatible. If $d' \prec d''$ then utility function of u may be partially or totally satisfied by first decision d', what influence a future decision d''; thus order is important. Since all nodes from $an(d')$ influence d'' (assumption 1), they also influence d''. In the case of two sequences with the paths of chance nodes α,β: $\langle d',\alpha,u\rangle$ and $\langle d'',\beta,u\rangle$ situation is the same if $\alpha=\beta$, but if $\alpha \neq \beta$ then real order of decisions influences total utility and depends on a delay introduced by α and β. This case will be discussed below.

The conclusion from these considerations is as follows: if $\{d',d''\} \in an(u)$ and $d' \prec d''$ and delays can be neglected then d' and all its ancestors have an impact on d''.

In the example from Fig. 2 the path $\langle c_2,d_2,d_3,u\rangle$ means that $d_2 \prec d_3$, and distinct path $\langle c_1,d_1\rangle$ means that node d_1 is incompatible with nodes d_2,d_3. If $d_3 \prec d_1$ then the nodes c_1,c_2,c_3,d_2,d_3 influence decision d_1, but if $d_1 \prec d_2$ then c_1,d_1 have an impact on remaining nodes.

Diagrams are time-sliced. The definition of ID and known solution algorithms assume that all parameters of ID have a static nature. If we want to use an influence diagram for modelling a system with uncertain states which alter in time, we must repeat basic structure of ID and relate each instantiation with distinct moment of time. From the basic structure $\mathcal{I}$ of ID one can construct a chain $\langle(\mathcal{I}_0,t_0),(\mathcal{I}_1,t_1),\dots,(\mathcal{I}_k,t_k),\rangle$ of links $\mathcal{I}_i$, with similar structures. Usually each node $x_{j,0}$ from $\mathcal{I}_0$ has similar nodes $x_{j,1},x_{j,2},\dots$ in remaining segments of ID, but some parameters of these nodes are different, simulating parameter changes in discrete time, with a characteristic $\langle x_{j,0},x_{j,1},\dots,x_{j,k}\rangle$. Segments of such time-sliced diagram are connected by arcs (*temporal links*) which define how the distribution of time slice i depends conditionally on the distribution of the variables of time slice $i-1$. The time slices of ID are assumed to be chosen such that the ID obeys the Markov property: the future is conditionally independent of the past given the present.

Figure 3 depicts very simple structure of bit-sliced ID, with segments corresponding to weeks (in a month) or to quarters (in a year).

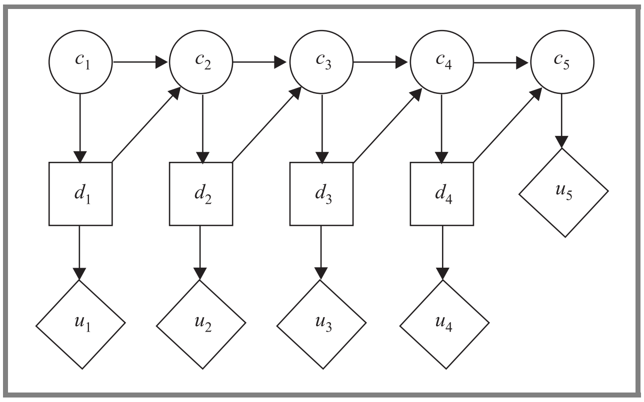

Fig. 3. Bit-sliced influence diagram.

Chance nodes may refer to the states of production (business, health etc.) during a considered period, decision nodes show external interventions improving appropriate state at the end of the period, and utility nodes valuate the results (periodic and final).

One of the segments (for example $\mathfrak{I}_x$) of a time-sliced ID can refer to the current time t_x. If all parameters up to t_x are observed then inference of parameters from $\mathfrak{I}_{x+1}, \mathfrak{I}_{x+2}, \ldots$ is known as *prediction*. On the basis of information from $\mathfrak{I}_x$ one can also compute some parameters from the past (*hindsight*).

Time-sliced ID helps in choosing the best moment for decision taking; comparing effects of the same decision in different slices one can select the most suitable time.

Time is related to nodes. Chance nodes may describe states or events with substantial length of time. Decision nodes are also related to time consuming actions, and results of decisions are sometimes very late towards the moment of decision. If in a modeled problem time is critical, all these delays should be taken into account.

Let $\tau(x)$ describe the time needed by a node x and $\tau(\pi)$ – the time necessary for a path π. When $\tau\langle d', \alpha\rangle > \tau\langle d'', \beta\rangle$ and both paths give the same result with equal costs, the decision d'' should be preferred. All other cases can be discussed easily.

As an example of these considerations we will discuss the case of the call center from the Section 1. In this case the global performance of CC can be enlarged by personel increase or the new algorithm of data mining, or both. If the time of performance improvement is critical, the total utility function depends on the times of two actions (personel recruiting and education versus algorithm preparation and implementation).

There are some other methods for introducing an impact of time in ID.

- When a value of time is uncertain we can use a special chance node to represent it, and utilize for further inference.
- If delay of action related with d decreases utility u, we can model losses with time by the linear or exponential form of utility functions [7]:

$$u(d,t) = u(d,t_0) - at, \qquad u(d,t) = u(d,t_0)e^{bt}.$$

4. Summary

We described the assessment and use of time dependent influence diagrams. It has been shown that there are many opportunities to introduce time: by ordering nodes, time slices, time dependent variables and functions. Unfortunately different approaches are devoid of common methodology helping in an ID construction. It seems that further investigations should be directed to the integration of existing methods.

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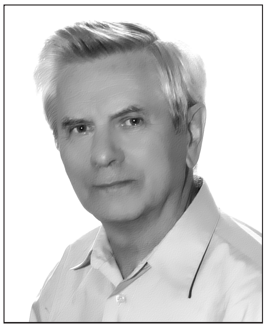

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