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SYMBOLIC CALCULATIONS – TOOL FOR FAST ANALYZING POLIHARMONIC MODELS OF SQUIRREL – CAGE MOTORS

ABSTRACT *This article focuses on the methods of: selecting the MMF space harmonics and deriving the so-called poliharmonic models of an induction machine by means of symbolical calculation. The rules for selecting the most important harmonics and the algorithm for fast deriving the poliharmonic models (using symbolical calculation) are presented.*

1. INTRODUCTION

The reasons for deformation of magnetic field space distribution in the air-gap of an induction squirrel-cage machine can be divided into following groups: deformation as a result of discrete layout of stator windings and rotor windings (cage) in slots, deformation as a result of magnetic saturation

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and histeresis, deformation as a result of non-uniform and non-smooth air-gap [1]. In this paper we focus on the first reason for magnetic field deformation and take into consideration non-sinusoidal MMF patern of the windings. The MMF space harmonic of the p^{th} order (p – number of pole pairs) generates main asynchronous torque whereas higher MMF space harmonics (of orders greater than p) are responsible for producing undesired additional torque. These additional torque are of asynchronous or pulsating (synchronous) nature and are commonly known as parasitic torque. The parasitic torque arising in a squirrel-cage motor cause many undesirable phenomena and influence the behaviour of a machine in both steady and transient states. The most important concequences are: dependence of the motor starting time on coincidental initial rotor position [2], [3], dependence of the value of the starting motor torque on coincidental initial rotor position [2] - [4], impact of parasitic torque on reversal and start transients [2] - [4], tortional vibrations of the shaft which might cause mechanical resonances in the drive systems [3]-[6]. Fig.1 shows exemplary trajectory: electromagnetic torque T_e v. rotor speed Ω_m during reversal of the drive system with an induction squirrel-cage motor in which the original rotor was replaced by the rotor with non-skewed bars, whereas Fig.2 shows the some trajectory in case of the rotor with properly skewed rotor bars.

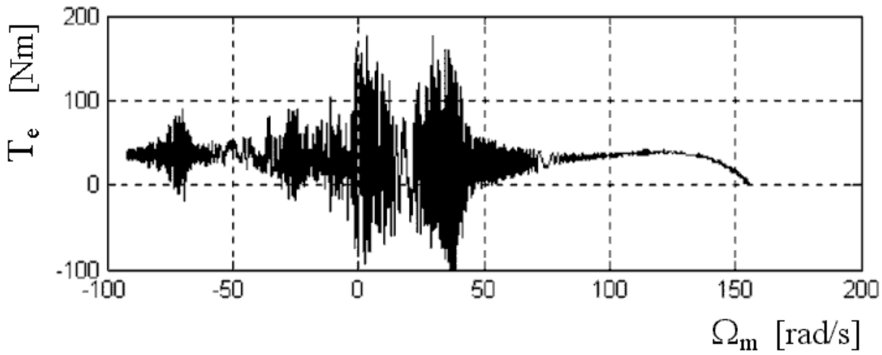

Fig.1. Motor reversal.

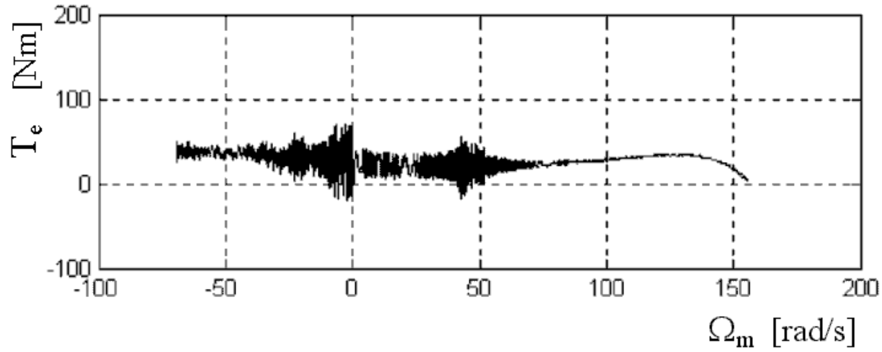

Fig.2. Motor reversal.

These parasitic phenomena shown in Fig.1 and in less scale in Fig.2 cannot be analysed with the help of standart monoharmonic model allowing only for main MMF space harmonic, although such a model is very convenient for implementing in computer programs. Only a mathematical model taking into consideration, besides main MMF space harmonics, higher space harmonics allows us to analyse these particular phenomena. Taking into consideration all consecutive MMF space harmonics in a poliharmonic model is, of course, impossible and in actual considered cases number of MMF space harmonics are usually limited. In spite of that it is difficult to implement such a model in computer programs, because each element of inductance matrices depends on many MMF space harmonics taken into consideration [2], [3]. In addition, mutual inductances between stator and rotor windings depend on a rotor angle. It is crucial to elaborate procedure for searching MMF space harmonics generating the strongest parasitic asynchronous and synchronous torque, which enables us to implementation quickly the poliharmonic model in computer program and will result in significant reduction of numerical solution time.

2. BASIC INFORMATION ON POLIHARMONIC MODEL OF AN INDUCTION SQUIRREL-CAGE MOTOR

Mathematical model of a squirrel-cage motor in natural coordinate is formulated under the following assumptions: the stator and the rotor windings are symmetrical, the rotor winding is considered as Q_r-phase winding (Q_r - number of cage bars), the skin effect phenomena in rotor bars are neglected, the air-gap is smooth and uniform, the magnetic circuit is linear. Under the above assumptions the mathematical model of a squirrel-cage motor allows for higher MMF space harmonics and neglects permeance and saturation magnetic field space harmonics. The mathematical model of a squirrel-cage motor in natural coordinates consists of the 3+Q_r voltage equations for stator and rotor windings and motion equation (1)-(4) [2], [7]. Certain simplification of this model can be achieved by transformation of machine equations from natural coordinates to new $\alpha\beta$-dq coordinates ($\alpha\beta$- stator quantities connected with stator reference frame, dq- rotor quantities connected with rotor reference frame). The poliharmonic model of an induction machine in $\alpha\beta$-dq coordinates has the following form:

$$\frac{d}{dt}\begin{bmatrix}[i_s^{\alpha\beta}]\\ [i_r^{dq}]\end{bmatrix}=\underbrace{\left(\begin{bmatrix}[L_{\sigma s}^{\alpha\beta}] & [0]\\ [0] & [L_{\sigma r}^{dq}]\end{bmatrix}+\begin{bmatrix}[M_{ss}^{\alpha\beta}] & [M_{sr}^{\alpha\beta dq}(\vartheta)]\\ [M_{sr}^{\alpha\beta dq}(\vartheta)]^T & [M_{rr}^{dq}]\end{bmatrix}\right)^{-1}}_{A}\cdot\left\{\begin{bmatrix}[u_s^{\alpha\beta}]\\ [0]\end{bmatrix}-\left(\begin{bmatrix}[R_s^{\alpha\beta}] & [0]\\ [0] & [R_r^{dq}]\end{bmatrix}+\right.\right.$$

$$\left.\left.\underbrace{\frac{d}{dt}\begin{bmatrix}[M_{ss}^{\alpha\beta}] & [M_{sr}^{\alpha\beta dq}(\vartheta)]\\ [M_{sr}^{\alpha\beta dq}(\vartheta)]^T & [M_{rr}^{dq}]\end{bmatrix}}_{B}\right)\begin{bmatrix}[i_s^{\alpha\beta}]\\ [i_r^{dq}]\end{bmatrix}\right\} \tag{1}$$

$$\frac{d}{dt}\Omega_m=\frac{1}{J}\left(\left\{\underbrace{[i_s^{\alpha\beta}]^T\frac{d}{d\vartheta}[M_{sr}^{\alpha\beta dq}(\vartheta)][i_r^{dq}]}_{C}\right\}-T_m\right) \tag{2}$$

$$\frac{d}{dt}\vartheta=p\Omega_m \tag{3}$$

$$\vartheta=p\vartheta_m \tag{4}$$

where:

$[R_s^{\alpha\beta}]$, $[R_r^{dq}]$	– matrices of stator and rotor resistances,
$[L_{\sigma s}^{\alpha\beta}]$, $[L_{\sigma r}^{dq}]$	– matrices of stator and rotor leakage inductances,
$[M_{ss}^{\alpha\beta}]$, $[M_{rr}^{dq}]$	– matrices of stator and rotor self inductances,
$[M_{sr}^{\alpha\beta dq}(\vartheta)]$	– matrices of stator - rotor mutual inductance,
$[i_s^{\alpha\beta}]$, $[i_r^{dq}]$	– current vectors,
$[u_s^{\alpha\beta}]$	– supply voltage vector,
Ω_m	– angular speed,
ϑ	– rotor angle,
J	– moment of inertia,
p	– number of pole pairs.

In the above equations the superscript $\alpha\beta$ denotes vectors and matrices associated with $\alpha\beta$ - coordinate system and the superscript dq denotes vectors and matrices associated with dq-coordinate system. Stator-rotor mutual inductance matrix $M_{sr}^{\alpha\beta dq}(\vartheta)$ is connected with both coordinate systems. It must be stressed that during numerical integration of equation set (1)-(3) according to the standard commonly used method it is necessary to determine in every step:

inversed form of inductance hypermatrix in Eqn.(1) (denoted by **A**), time derivative from matrices of mutual inductance in Eqn.(1) (denoted by **B**), rotor angle derivative from matrices of mutual inductance in Eqn.(2) (denoted by **C**), time-varying elements of matrices in Eqns (1)-(3).

3. SELECTION OF THE MMF HARMONICS

The symmetrical stator windings generates MMF space harmonics of odd orders belonging to the set S_1 (5) or S_0 (6):

$$\{S_1\} = \{p, 5p, 7p, 11p, 13p, 17p, 19p, \ldots\} \tag{5}$$

$$\{S_0\} = \{3p, 9p, 15p, 21p, 27p, 33p, \ldots\} \tag{6}$$

If the stator windings are wye-connected the MMF space harmonics included in the set S_0 can be omitted. The symmetrical multiphase rotor winding (cage winding) generates consecutive MMF space harmonics belonging to the sets R_1, R_2, …, R_i (where i = 1, 2, …, $Q_r/2$ - for even number of rotor bars Q_r and i = 1, 2, …, $(Q_r+1)/2$ - for odd number of rotor bars) (7)(8)(9):

$$\{R_1\} = \{1, Q_r - 1, Q_r + 1, 2Q_r - 1, 2Q_r + 1, \ldots\} \tag{7}$$

$$\{R_2\} = \{2, Q_r - 2, Q_r + 2, 2Q_r - 2, 2Q_r + 2, \ldots\} \tag{8}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\{R_i\} = \{i, Q_r - i, Q_r + i, 2Q_r - i, 2Q_r + i, \ldots\}. \tag{9}$$

Consecutive stator MMF space harmonics of the orders ν = 5p, 7p, 11p, 13p, 17p, 19p,… acting upon corresponding rotor MMF space harmonics having the same orders generate parasitic asynchronous torque. The pairs of MMF space harmonics included in the product of the sets given below:

$$\{R_1\} \cap \{S_1\},\ \{R_2\} \cap \{S_1\},\ \ldots, \{R_i\} \cap \{S_1\} \tag{10}$$

are responsible for producing parasitic synchronous torque. In further analysis it is very convenient to apply the so-called **diagram of decomposition of an induction machine into elementary machines** (poliharmonic model expressed in graphical way), which is described in detail in [2], [3], [7], [8]. Fig.3 shows fragment of the diagram drawn for an exemplary induction squirrel-cage motor with the following data: number of rotor slots Q_r=8 and number of pole pairs p=1.

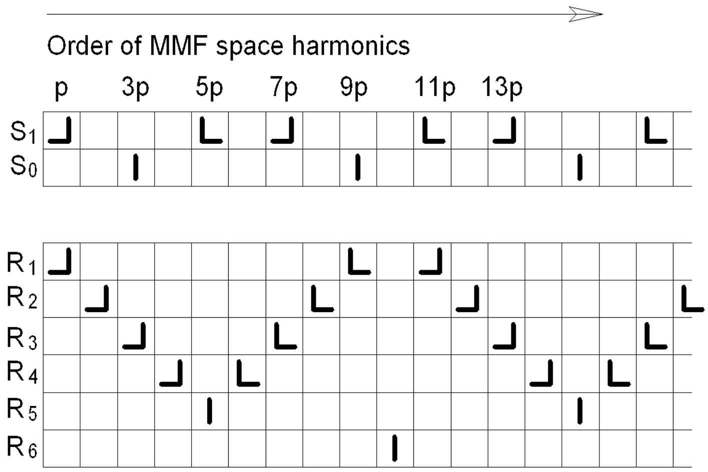

Fig.3. Exemplary diagram of decomposition.

In such a diagram the sign ┘ denotes 2-phase clockwise-oriented winding connected with single MMF space harmonic generated by stator or rotor. The sign └ denotes 2-phase anticlockwise-oriented winding and the sign | denotes 1-phase winding. Now, let us consider procedure for selection of MMF space harmonics, which play key role in dynamic behavior of an induction motor. The selection of the MMF space harmonics can be divided into two stages: natural and intentional reduction [3]. The natural reduction takes into accound the following features of electrical and magnetical circuits: symmetry of the stator and rotor windings and the number of pole pairs, connection of stator windings (if the stator windings are wye-connected the row S_0 in Fig.3. can be omitted), relations between number of pole pairs p and numbers of rotor bars Q_r (causing that the rotor windings exited by stator space harmonic do not produce certain space harmonics e.g. row R_2 and R_4 in Fig.3. can be omitted). Intentional reduction depends on the kind of transients, which will be analysed. If we consider start of the motor, we can take into accound only these harmonics which influences behaviour of a machine in motor region. Choosing the MMF harmonics according to the above - mentioned rools we get the so-called reduced diagram of the decomposition. The reduced form of the diagram presented in Fig.3 after natural reduction is shown in Fig.4.

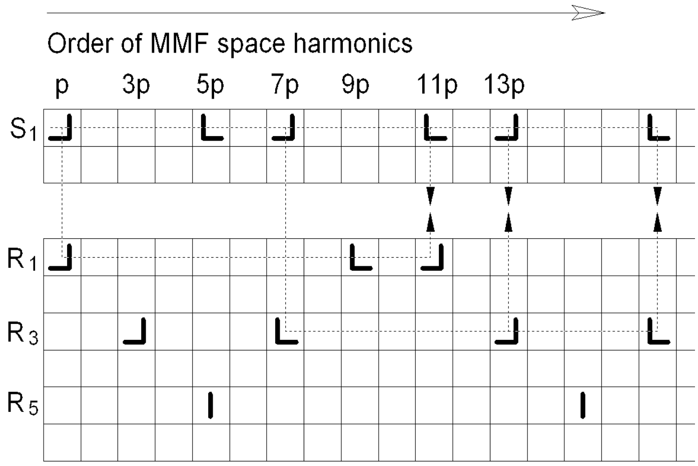

Fig. 4. Fragment of decomposition diagram

The paths of generating the parasitic synchronous torque are denoted by dotted lines in Fig.4. In the first column of the diagram the elementary machine is located which generates the fundamental space harmonic ($\nu = p$). In the 11th column of the diagram the 11th elementary machine is located, which is connected with the 11th MMF space harmonic. The 1st fundamental elementary machine ($\nu = p$) produces mainly the fundamental asynchronous torque, while the 11th elementary machine generates basically the parasitic synchronous/pulsating torque. As seen, the 11th elementary machine ($\rho = 11$) operates as synchronous machine while the 1st fundamental elementary machine ($\nu = p$), being the fundamental asynchronous machine, plays additionally the role of an exciting machine for 11th elementary synchronous machine. The synchronous speed of parasitic synchronous torque can be determined according to a number of synchronous machine ρ, number of exciting machine ν and orientation of their 2-phase stator and rotor windings [2], [3], [7], [8]. The all-possible cases of synchronous speeds are given in the Table 1.

TABLE 1

ν \ ρ	┘ ┘	┘ └	└ ┘	└ └
┘ ┘	0	$\frac{2\Omega_0}{\rho+\nu}$	$\frac{-2\Omega_0}{\rho-\nu}$	0
┘ └	$\frac{2\Omega_0}{\rho+\nu}$	0	0	$\frac{-2\Omega_0}{\rho-\nu}$
└ ┘	$\frac{2\Omega_0}{\rho-\nu}$	0	0	$\frac{-2\Omega_0}{\rho+\nu}$
└ └	0	$\frac{2\Omega_0}{\rho-\nu}$	$\frac{-2\Omega_0}{\rho+\nu}$	0

In searching for the most predominant parasitic synchronous torque generated by pairs of MMF space harmonic (elementary machines) it is very helpful to define the so-called **total factor of space harmonic pair (ν, ρ) referred to the number of considered row in diagram of decomposition**. This factor is given by the following formula:

$$\left| k_{(\nu,\rho)} = \frac{k_{ws\nu} k_{ws\rho} k_{wr\nu} k_{wr\rho} k_{sq\nu} k_{sq\rho}}{\nu \cdot \rho \cdot n} \right| \tag{11}$$

where:

k_{ws}, k_w – stator and rotor winding factors for considered harmonics of orders ν and ρ;

$k_{sq\nu}$, $k_{sq\rho}$ – skew factor for harmonics of orders ν;

and

ρ; n – number of row in the diagram of decomposition.

Calculated values of the total factor for the exemplary machine in point (Q_r=8, p=1) are put together in Fig. 5.

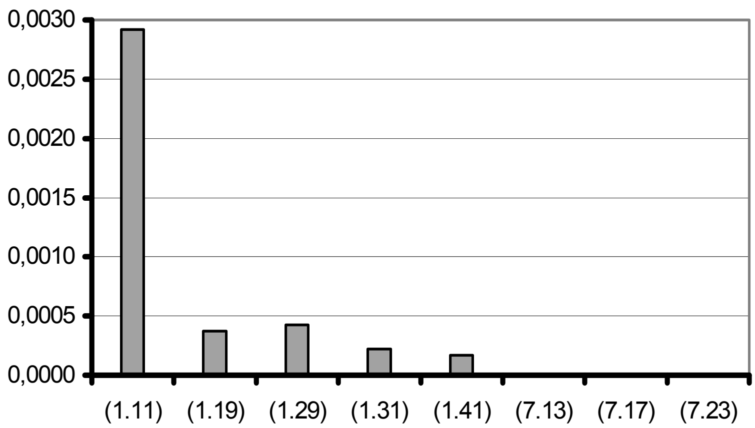

Fig.5. Total factors for choosen pairs (ν, ρ) of MMF space harmonics.

It results from Fig.5 that the most predominant parasitic synchronous torque are generated by the pairs of harmonics (p, 11p), (p, 19p), (p, 29p), (p, 31p), (p, 41p) connected with the first row in diagram of decomposition. Such a model takes into acount parasitic synchronous torque arising in brake region: synchronous speed - Ω_0/5 rad/s – pair of space harmonics (1,11), synchronous speed - Ω_0/20 rad/s – pair of space harmonics (1,41), in motor region: synchronous speed Ω_0/10 rad/s – pair of space harmonics (1,19), and at start: synchronous speed Ω_m= 0 rad/s – pairs of space harmonics (1,29) and (1,31).

4. DERIVING POLIHARMONIC MODEL OF AN INDUCTION MOTOR WITH THE HELP OF SYMBOLIC CALCULATION

The procedure of formulating a poliharmonic model of a squirrel-cage motor with the help of symbolic calculations consists of the following steps:

step 1. Selecting the predominant space harmonics. Basing on: diagram of decomposition of 3-phase squirrel-cage machine into elementary machines, paths of generating parasitic synchronous torque (dotted lines in Fig.4) and values of total factor choose the most important MMF space harmonics (elementary machines) and determin their winding orientation according to Table I (clockwise - ┘ or anticlockwise - └). Establish all necessary symbolic variables.

step 2. Defining the elementary matrices. With the help of symbolic calculations define vectors and matrices included in Eqns (1)-(2). The stator-rotor mutual inductance matrix is build from the sum of 2×2 dimensional elementary matrices connected with MMF space harmonics taken into consideration. For determination the stator-rotor mutual inductances for chosen space harmonics Table 2 including all possible cases is very helpful.

TABLE 2
Elementary stator rotor matrices

Stator harmonics	Rotor harmonics ┘	Rotor harmonics └
┘	$\begin{bmatrix} \cos \nu\vartheta_m & -\sin \nu\vartheta_m \\ \sin \nu\vartheta_m & \cos \nu\vartheta_m \end{bmatrix}$	$\begin{bmatrix} \cos \nu\vartheta_m & \sin \nu\vartheta_m \\ \sin \nu\vartheta_m & -\cos \nu\vartheta_m \end{bmatrix}$
└	$\begin{bmatrix} \cos \nu\vartheta_m & -\sin \nu\vartheta_m \\ -\sin \nu\vartheta_m & -\cos \nu\vartheta_m \end{bmatrix}$	$\begin{bmatrix} \cos \nu\vartheta_m & \sin \nu\vartheta_m \\ -\sin \nu\vartheta_m & \cos \nu\vartheta_m \end{bmatrix}$

step 3. Symbolic determination of B, C, A matrices. Employing the result of the „**step 2**” we can determine symbolical form of:

- time derivative of matrix **B** in Eqn.(1) and rotor angle derivative of matrix **C** in Eqn.(2) (using “*diff*” command in symbolical Matlab language),
- inversed form of inductance hypermatrix **A** in Eqn.(1) (using “*inv*” command in symbolical Matlab language).

step 4. Symbolical simplification of the inversed hypermatrix A. Symbolical simplification of inductance hypermatrix **A** can be obtained with help of commands: “simple”, “collect” and “subs” (in symbolical Matlab language). For space harmonics belonging to the product of the sets $\{R_1\}\cap\{S_1\}$ we have:

$$[A]^{-1} = \frac{1}{\det\left([A]\right)}\left[\begin{array}{c|c} [M_1] & [M_2] \\ \hline [M_2]^T & [M_3] \end{array}\right] \tag{12}$$

and for space harmonics belonging to the product of the sets $\{R_1\}\cap\{S_1\}$ and $\{R_3\}\cap\{S_1\}$ we have:

$$[A]^{-1} = \frac{1}{\det\left([A]\right)}\left[\begin{array}{c|c|c} [M_1] & [M_2] & [M_4] \\ \hline [M_2]^T & [M_3] & [M_6] \\ \hline [M_4]^T & [M_6]^T & [M_5] \end{array}\right] \tag{13}$$

step 5. Implementation of the obtained equations.

5. EXEMPLARY INVERSED MATRICES AND SIMULATION RESULTS

Employing the above-described procedure we get for the exemplary induction machine: Q_s=36, Q_r=28, p=2 (chosen pairs of MMF space harmonics are: (p, 13p)) the poliharmonic model: including the following elementary matrices of inversed inductance hypermatrix:

$$[M_1] = \left[\begin{array}{c|c} a_1 + b_1\cos 28\vartheta & b_1\sin 28\vartheta \\ \hline b_1\sin 28\vartheta & a_1 - b_1\cos 28\vartheta \end{array}\right] \tag{14}$$

$$[M_2] = \left[\begin{array}{c} b_2\cos 2\vartheta + c_2\cos 26\vartheta \\ b_2\sin 2\vartheta + c_2\sin 26\vartheta \end{array}\right. \\ \left.\begin{array}{c} -b_2\sin 2\vartheta + c_2\sin 26\vartheta \\ b_2\cos 2\vartheta - c_2\cos 26\vartheta \end{array}\right] \tag{15}$$

$$[M_3] = \left[\begin{array}{c|c} a_3 + b_3\cos 24\vartheta & b_3\sin 24\vartheta \\ \hline b_3\sin 24\vartheta & a_3 - b_3\cos 24\vartheta \end{array}\right] \tag{16}$$

$$\det\left([A]\right) = \left(L_1L_2 - \left(L_{sr2} + L_{sr26}\right)^2\right)\cdot \left(L_1L_2 - \left(L_{sr2} - L_{sr26}\right)^2\right) \tag{17}$$

where:

$a_1 = L_2\left(L_1L_2 - L_{sr2}^2 - L_{sr26}^2\right)$,
$b_1 = 2L_2L_{sr2}L_{sr26}$,
$c_2 = L_{sr26}\left(L_{sr26}^2 - L_{sr2}^2 - L_1L_2\right)$,
$b_3 = 2L_1L_{sr2}L_{sr26}$,
$b_2 = L_{sr2}\left(L_{sr2}^2 - L_{sr26}^2 - L_1L_2\right)$,
$a_3 = L_1\left(L_1L_2 - L_{sr2}^2 - L_{sr26}^2\right)$,
L_1, L_2 – stator and rotor self inductances,
L_{sr2}, L_{sr26} – stator rotor mutual inductances for 2nd and 26th MMF harmonics.

The poliharmonic models for one pair of space harmonics (p,13p) and for two pairs of space harmonics (p,13p), (5p, 23p) derived with help of standard and symbolical method are compared from view-point of solving times (the Runge Kutta 45 method). The simulation parameters are: simulation steps – variable step, maximum step size – 5.10^{-6} s, time interval – 0 – 0.5 s, relative tolerance – 1.10^{-6}, absolute tolerance – 1.10^{-6}.

The simulations results are shown in Table 3.

TABLE 3
Solving time.

Model with:	Pair of harmonics (p, 13p)		Pairs of harmonics (p, 13p), (5p, 23p)	
Method:	Symbolical	Standard	Symbolical	Standard
Processor Celeron 300MHz	12,4 s	230,5 s	28,8 s	302,1 s
Processor Pentium III 866MHz	5,3 s	90,7 s	10,5 s	139 s

6. CONCLUSIONS

Use of symbolic calculation in deriving the mathematical poliharmonic model of an induction machine allows us to eliminate inversion of the inductance hypermatrix in each integration step. The inductance hypermatrix is symbolically inversed only once. The symbolical inversion of an inductance hypermatrix:

- eliminate the numerical errors during the hypermatrix inversion,
- decrease the number of multiplication and divisions in one integration step of the machine equation,
- decrease simulation times.

Employing the symbolical calculations gives chance to formulate the general mnemonic methods for fast deriving poliharmonic models of induction machines.

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RACHUNEK SYMBOLICZNY – NARZĘDZIE DO SZYBKIEJ ANALIZY POLIHARMONICZNEGO MODELU SILNIKA Z WIRNIKIEM KLATKOWYM

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STRESZCZENIE *Artykuł przedstawia metody: wyboru harmonicznych przestrzennych SMM i tworzenia tak zwanych poliharmonicznych modeli maszyny indukcyjnej przy użyciu rachunku symbolicznego. Przedstawiono zasady wyboru najważniejszych harmonicznych oraz algorytm szybkiego otrzymywania modeli poliharmonicznych (przy użyciu rachunku symbolicznego).*

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