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SOLID ROTOR INDUCTION MACHINE OPTIMISATION BY USING ANALYTICAL AND FINITE ELEMENT TECHNIQUES

SUMMARY *An optimisation procedure for solid rotor induction machines, exhibiting important advantages for micro-machine production due to the simplicity of their construction, is presented. Both non-salient and salient rotor structures have been considered and compared. Design optimisation has been performed based on the finite element method and analytical solutions in case of regular geometries combined with sensitivity analysis techniques. The developed models precision has been checked by comparing their results for the same machine configuration.*

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1. INTRODUCTION

Solid rotor induction machines can be attractive rivals of servomotors used in small scale applications, due to their simple structure. They provide important advantages such as good reliability, low cost and involve no maintenance requirements. Optimisation procedures used for synchronous machines can equally be applied in such cases [1].

The design of the induction motor as well as the evaluation of the equivalent circuit parameters can be obtained by using finite element techniques [2], [3]. However, the optimisation of the rotor geometry may require laborious and expensive numerical schemes especially when 3D configuration should be considered [4], [5]. Although both stochastic and deterministic optimisation algorithms can be implemented [6]-[9], the sensitivity analysis technique combined with finite element methods enables robust and fast convergence [1].

The optimisation of the rotor surface shape is performed by using a perturbation technique at the final design stage. For the sake simplicity only rectangular rotor slots have been considered here. The cost function used involves torque maximization.

The general moving boundary approach could have been applied in this problem but the objective function evaluation with the mesh modification render the method extremely time consuming [4]. Another important feature is that the number of design variables and constraints involved in such an optimisation procedure can be relatively high while the mesh congruency and regularity must be continuously checked. That is why a reduction of the design variables as the one adopted previously can greatly facilitate the solution procedure.

2. METHODOLOGY

The geometry optimisation of the induction motor is based on solid rotor configuration for construction simplicity purposes. The analysis problem has been solved either by using existing solutions of closed form (case of rotating field around non-salient rotor) or by 3D finite elements (slotted stator and rotor case). A particular reduced scalar potential formulation necessitating no source field calculation has been adopted to develop the 3D finite element model [10]. The analysis methods as well as the optimisation procedure are developed hereafter.

2.1. Analytical solution for solid rotor machine

The case of a non-salient rotor structure surrounded by a uniform air-gap has been considered, as shown in Fig.1. The stator presence has been represented by a sinusoidal rotating magnetic field. The rotor is consisted of concentric cylindrical layers of conducting materials. The analytical solution of Laplace equation in the air gap takes the form in terms of the complex magnetic vector potential:

$$\overline{A}_a(r,\theta)=\left[(C_1+jC_2)r+(C_3+jC_4)\left(\frac{1}{r}\right)\right]\exp[-jk(\theta-\omega t)] \tag{1}$$

where C_1, C_2, C_3, C_4 are real constants, k is the number of pole pairs and ω the angular velocity.

In a cylindrical conducting layer the analytical solution of diffusion equation for the complex magnetic vector potential is of the form:

$$\begin{aligned}\overline{A}_c(r,\theta)=\{&[D_1+jD_2][ber_k(\lambda r)+j\,bei_k(\lambda r)]+[D_3+jD_4]\cdot\\ &\cdot[\ker_k((\lambda r)+j\,kei_k(\lambda r))]\}\exp[-jk(\theta-\omega t)]\end{aligned} \tag{2}$$

where D_1, D_2, D_3, D_4 are real constants, $ber_k, bei_k, \ker_k, kei_k$ $are\, k^{th}$ order Kelvin functions of first and second kind, while $\lambda^2=(\sigma\omega\mu)$, with σ denoting the electrical conductivity and μ the magnetic permeability.

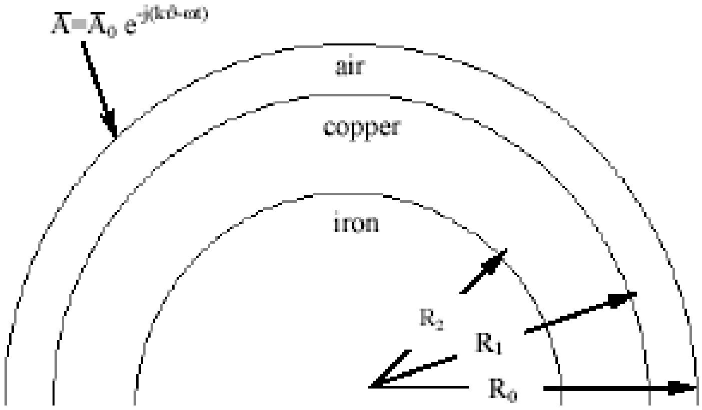


Fig.1. Geometry of the non-salient solid iron rotor structure surrounded by a sinusoidal rotating magnetic field.

The constants in these solutions are calculated by applying the boundary conditions and the continuity relations across the material boundaries.

2.2 Scalar potential 3D FEM technique

The use of scalar potential formulations in 3D configurations usually necessitates a prior source field calculation by using Biot-Savart's Law, which presents the drawback of considerable computational effort. We have developed a particular scalar potential formulation enabling treatment of 3D magnetostatics. It permits one to model efficiently laminated iron cores with or without air-gaps and needs no prior source field calculation.

According to our method the magnetic field strength $\boldsymbol{H}$ is conveniently partitioned to a rotational and an irrotational part as follows:

$$\boldsymbol{H} = \boldsymbol{K} - \nabla\Phi \tag{3}$$

where Φ is a scalar potential extended all over the solution domain while $\boldsymbol{K}$ is a vector quantity (fictitious field distribution), defined in a simply connected subdomain comprising the conductor, that satisfies Ampere's law and is perpendicular on the subdomain boundary. The stator of the machine, shown in Fig.2, has been modeled by such a method.

This technique has been extended for cases involving eddy currents in well-defined paths [10]. In cases of thin skin effect depth however, it is possible to express the corresponding induced surface current density $\boldsymbol{J_I}$ as follows [11].

$$\boldsymbol{J_I} = (\partial / \partial t)(\mathrm{grad}\, T x \boldsymbol{n}) \tag{4}$$

where T is a scalar quantity existing only on the surface where eddy currents are developed while $\boldsymbol{n}$ is the unit normal to the above mentioned surface. Such a representation has been adopted for the solid rotor shown in Fig.2.

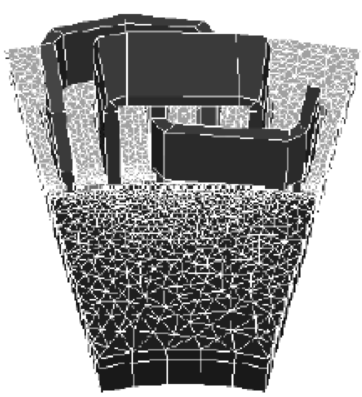


Fig.2. Representation of the solid rotor machine and adopted mesh for the 3D FEM analysis.

2.3. Optimisation Procedure

The optimisation of the rotor shape is performed by using a perturbation technique of the slot dimensions as well as copper parts. When combined with the analytical solution procedure as design variables are considered the copper layer width (R_1-R_2) the machine loading (A_0 value) as well as the number of poles (k value). When combined with the 3D FEM model in the case of slotted rotor machine, for the sake of simplicity only rectangular slot cross sections have been considered. In that case the design variables are the slot number and dimensions as well as the machine loading represented by:

$$\boldsymbol{x} = x_1, x_2, x_3, x_4 \tag{5}$$

where x_1 is the number of slots, x_2 is the slot height, x_3 is the slot width and x_4 the current in stator slots. These variables make maximum the objective function $F(x)$ representing the torque of the machine reduced to the volume of the rotor active part. The constraint conditions expressed by inequalities (geometrical constraints on the slot dimensions and current density limitation in stator slots imposed by thermal considerations) can be transformed into constraints expressed by equalities as follows:

$$g_j(x) - g_{j0} \approx 0 \tag{6}$$

where $g_j(x)$ are auxiliary variables. Then the following composite cost function can be defined:

$$P(x,\lambda) = F(x) + \sum_{j=1}^{m} \left[\lambda_j \left(g_j(x) - g_{j0} \right) \right] \tag{7}$$

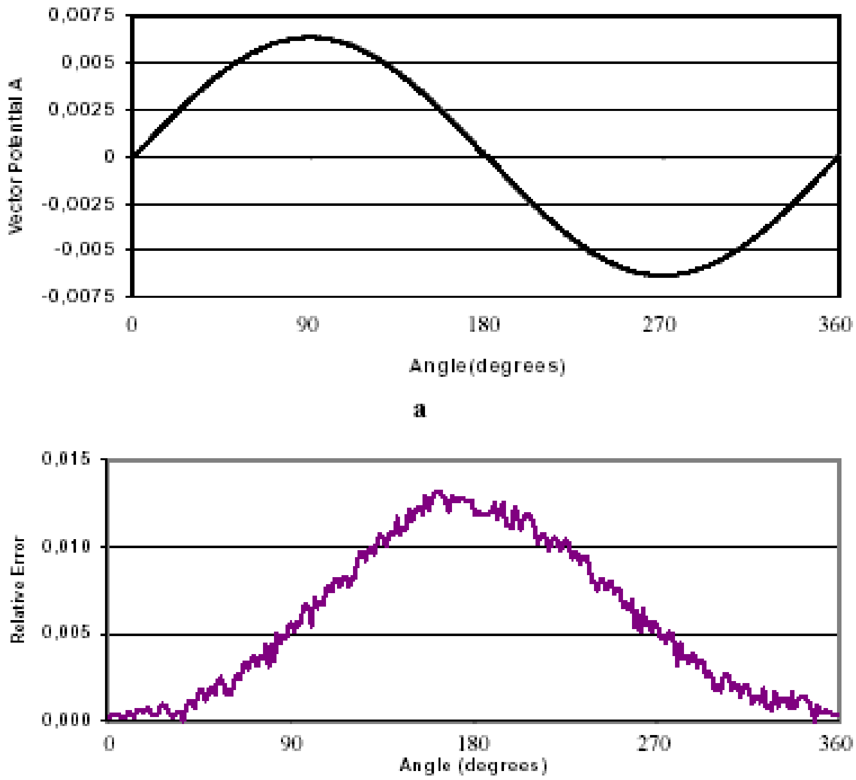
where λ_j is the j^{th} Langrange multiplier. The composite cost function extremum is obtained by the following equations:

$$\frac{\partial P}{\partial x_i} = 0, \qquad \frac{\partial P}{\partial \lambda_j} = 0 \tag{8}$$

This system of equation leads to torque maximisation for a given volume of rotor active part. The general moving boundary approach could have been applied in this problem but the objective finction evaluation with the mesh modification would have rendered the method extremely time consuming [4].

3. RESULTS AND DISCUSSION

The described procedure has been applied for the design of a 2.5 kW, eddy current induction machine with solid rotor, as shown in Fig.2. Both cases of copper and solid iron rotors have been investigated. In a first step, the analysis of the non-salient solid rotor case has been performed, by using the analytical solutions. As an example, the cases of solid iron and copper rotor configurations have been considered. The vector potential values on rotor surface for the solid iron machine calculated by a 2D finite element model and analytically have been compared, as shown in Fig.3a). Fig.3b gives the corresponding relative error illustrating the good agreement of the two methods.



b

Fig.3. Comparison of analytical and numerical results in case of sinusoidal excitation and solid iron rotor: a) potential vector on rotor surface, b) relative error.

The slotted stator cases have been considered by using the 3D FEM model for both copper and iron rotors. The respective field distributions for a frequency of 50 Hz are shown in Figs 4a) and 4b), respectively. Figure 5a compares the variation of torque with slip in the two above mentioned configurations. This figure illustrates the considerably better performance of solid copper rotor.

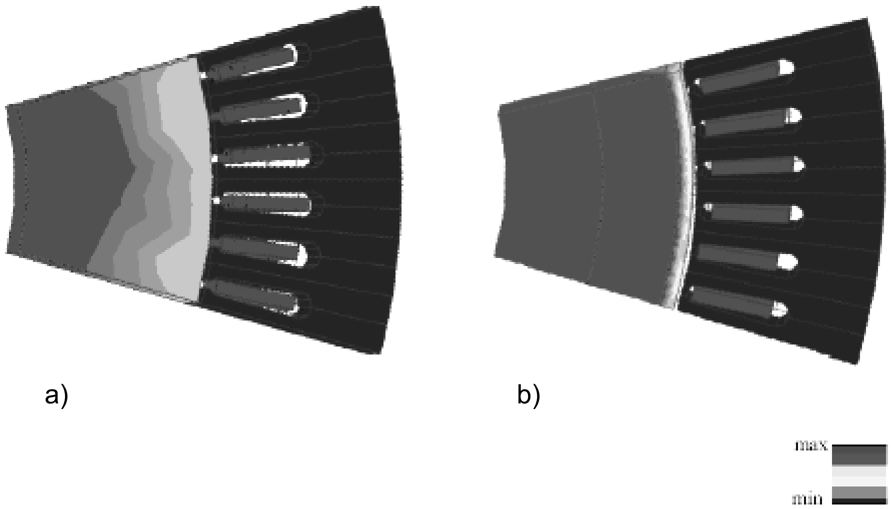


Fig.4. Eddy currents distribution in the solid rotor induction machine, calculated by the 3D FEM model: a) copper rotor, b) iron roror.

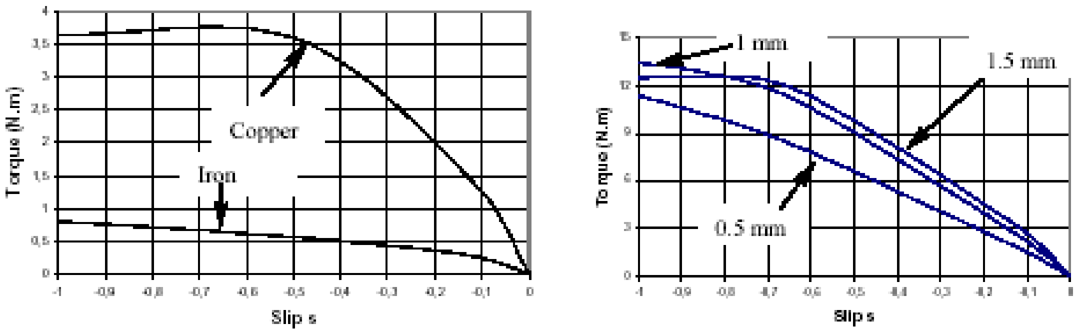
In second step the optimisation procedure has been introduced in order to provide the optimum width of uniform copper coating on a solid iron rotor. The optimum width was 0.95 mm with tolerance of 0.05 mm and was obtained after 14 iterations. A width of 1 mm enables almost four times greater torque than solid copper case. Its performance compared to copper coats of 0.5 mm and 1.5 mm respectively are shown in Fig.5b).

The optimisation procedure has been applied in conjunction with the FEM model in order to provide the optimum number of rotor slots as well as their shape dimensions. This procedure has shown that torque is almost proportional to the slot number and depth while it is no practically affected by the slot width provided that it is greater than twice the air-gap width.

An optimum of eight slots per pole has been obtained being at least 2 mm wide for an air gap width of 1 mm, after 22 iterations. Figure 6a) gives the torque variation for slot depths of 10 mm and 20 mm respectively. This figure shows that the ratio of torque for the two cases remains almost constant for all slip values.

Finally, the effects of a copper cage placed in solid iron rotor slots have been investigated, as shown in Fig.6b). The comparison of Figs. 6a) and 6b shows that the maximum torque is practically the same but the rotor cage provide a maximum torque for small slip values.

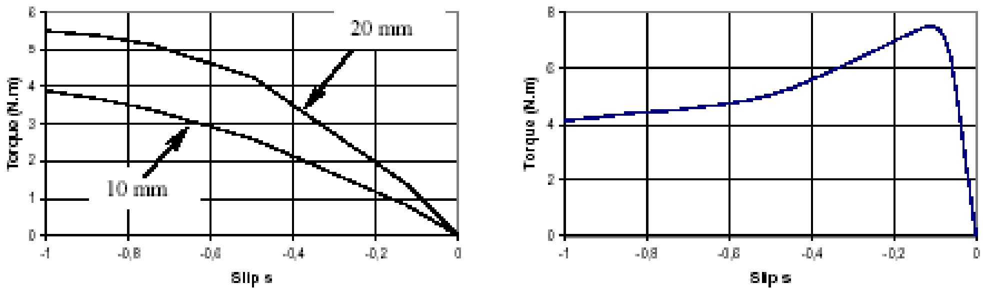
a) b)



a b

Fig.5. Torque variation with slip in the non-salient solid rotor machine case:
a) single material rotor case, b) iron rotor with different copper coating.

a) b)



a b

Fig.6. Torque variation with slip in the salient solid rotor machine case:
a) different slot depth, b) copper cage placed in rotor slots of 20 mm depth.

4. CONCLUSION

A geometry optimisation methodology based on analytical solutions and 3D finite elements combined to sensitivity analysis technique has been presented. This method has been applied to optimise the rotor geometry of a solid rotor induction motor, presenting important advantages for small scale applications due to its structure simplicity. The method involves a reduced number of design variables and constraints and requires a very limited number of iterations.

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OPTYMALIZACJA MASZYNY INDUKCYJNEJ Z WIRNIKIEM LITYM PRZY UŻYCIU METOD ANALITYCZNYCH I METODY ELEMENTÓW SKOŃCZONYCH

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STRESZCZENIE *Przedstawiono procedurę optymalizacji maszyn indukcyjnych z wirnikiem litym, wykazujących ważne zalety dla produkcji mikromaszyn ze względu na prostotę ich budowy. Uwzględniono i porównano zarówno wirniki z biegunami utajonymi jak i wydatnymi. Optymalizację budowy przeprowadzono w oparciu o metodę elementów skończonych i o rozwiązania analityczne w przypadku regularnych geometrii wirnika w połączeniu z metodami analizy wrażliwości. Dokładność opracowanych modeli sprawdzono przez porównanie ich wyników dla tej samej konfiguracji maszyny.*

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