Paper

Multi-agent utility theory for ethical conflict resolution

Hiroyuki Tamura

Abstract — In this paper we try to construct a two-attribute group disutility function for two conflicting decision makers, taking into account the property of utility independence and/or convex dependence between them. Two variables in the group utility function are disutility levels of two conflicting decision makers. The disutility level of each decision maker is modeled using multiple attributes, that is disutility function of each decision maker is formulated as a multi-attribute disutility function. By using a group disutility function for two conflicting agents, we can model the mutual concessions of the two conflicting agents taking into account ethical preference of each decision maker, and hence we can expect fairer multiple agents decision making for realizing better social welfare.

Keywords — utility theory, conflict resolution, multi-agent decision making, ethical consideration, convex dependence.

1. Introduction

In planning a large public project it is quite important to obtain a consensus between two conflicting decision makers, a representative of the regional inhabitants and of the enterpriser, to create a pleasant and useful environment. This paper deals with a methodology of modeling decision analysis for consensus formation between two conflicting multiple agents, regional inhabitants and the enterpriser (developer) of a big public project such as constructing a large international airport, a freeway with heavy traffic, a refuse incineration plant, etc. For this purpose we try to construct a group disutility function for two conflicting agents, taking into account the utility independence [1, 1993] or convex dependence [2] between them. This is called the "multi-agent utility theory". By using such a group disutility function for two conflicting agents, we can model the mutual concessions of the two conflicting agents taking into account ethical preference [3] with each other, and hence we can expect fairer MADA (multiple agents decision making) for realizing better social welfare.

2. A group disutility functions for multi-agent decision making

Let $D_1 \times D_2$ be a two-attribute disutility function space and $d_1(x_1) \in D_1,\ d_2(x_2) \in D_2$ denote the disutility functions of decision maker DM1 and DM2 on the multi-attribute consequence spaces X_1 and X_2, respectively, where $x_i \in X_i\ (i = 1, 2)$ denotes a specific consequence for DMi.

For a given $d_1(x_1) \in D_1$, and $d_2(x_2) \in D_2$, a group disutility function on $D_1 \times D_2$ space is defined as

$$G(x_1, x_2) = g\big(d_1(x_1), d_2(x_2)\big) \equiv g(d_1, d_2)\,.$$

Let us assume that d_1^0 and d_2^0 denote the worst levels of disutilities of DM1 and DM2, respectively, and d_1^* and d_2^* denote the best levels of disutilities of DM1 and DM2, respectively. Given an arbitrary $d_2 \in D_2$, a normalized conditional group disutility function (NCGDF) of DM1 is defined as

$$g_1(d_1 \mid d_2) \equiv \frac{g(d_1, d_2) - g(d_1^*, d_2)}{g(d_1^0, d_2) - g(d_1^*, d_2)}\,,$$

where it is assumed that

$$g(d_1^0, d_2) > g(d_1^*, d_2)\,.$$

It is obvious that

$$g_1(d_1^0 \mid d_2) = 1, \quad g_1(d_1^* \mid d_2) = 0$$

that is, NCGDF is normalized and is a *single-attribute* group disutility function. Hence it is easily identified.
The NCGDF for DM2, that is, $g_2(d_2 \mid d_1)$ can also be defined similarly as

$$g_2(d_2 \mid d_1) \equiv \frac{g(d_1, d_2) - g(d_1, d_2^*)}{g(d_1, d_2^0) - g(d_1, d_2^*)}\,.$$

The NCGDF $g_1(d_1 \mid d_2)$ represents DM1's and $g_2(d_2 \mid d_1)$ represents DM2's subjective preference for the group disutility as a function of his own disutility level, under the condition that the disutility level of the other DM is given. If NCGDF $g_1(d_1 \mid d_2)$ does not depend on the conditional level d_2, then attribute D_1 is utility independent [1, 1993] on attribute D_2. If attributes D_1 and D_2 are mutually utility independent, the two-attribute disutility function $g(d_1, d_2)$ can be described as either a multiplicative or additive from [1, 1993].
Suppose

$$g_1(d_1 \mid d_2) \neq g_1(d_1 \mid d_2^*)$$

for some

$$d_2 \in D_2$$

that is, utility independence does not hold between two attributes D_1 and D_2. In this case we can use a property of convex dependence [2] as a natural extension of utility independence.

The property of convex dependence is defined as follows: attribute D_1 is mth order convex dependent on attribute D_2, denoted $D_1(\mathrm{CD}_m)D_2$, if there exist distinct $d_2^0, d_2^1, \ldots, d_2^m \in D_2$ and real functions $\lambda_0, \lambda_1, \ldots, \lambda_m$ on D_2 such that NCGDF $g_1(d_1 \mid d_2)$ can be written as

$$g_1(d_1 \mid d_2) = \sum_{i=0}^{m} \lambda_i(d_2)\, g_1(d_1 \mid d_2^i),$$

where

$$\sum_{i=0}^{m} \lambda_i(d_2) = 1$$

for all $d_1 \in D_1$ and $d_2 \in D_2$ where m is the smallest non-negative integer for which this relation holds.

This definition says that, if $D_1(\mathrm{CD}_m)D_2$, then any NCGDF on D_1 can be described as a convex combination of $(m+1)$ NCGDFs with different conditional levels where $\lambda_i(d_2)$s are not necessarily non-negative. Especially, when $m = 0$ and $D_1(\mathrm{CD}_0)D_2$, attribute D_1 is utility independent on attribute D_2.

The algorithm for constructing a two-attribute group disutility function is as follows:

Step 1. NCGDFs $g_1(d_1 \mid d_2^0)$, $g_1(d_1 \mid d_2^*)$ and $g_1(d_1 \mid d_2^{0.5})$ are assessed, where $d_2^{0.5}$ denotes the intermediate level of attribute D_2 between the worst level d_2^0 and the best level d_2^*.

Step 2. If these NCGDFs are almost identical, $D_1(\mathrm{CD}_0)D_2$ holds. Otherwise, go to Step 3.

Step 3. If the convex combination of $g_1(d_1 \mid d_2^0)$ and $g_1(d_1 \mid d_2^*)$ is almost identical with $g_1(d_1 \mid d_2^{0.5})$, $D_1(\mathrm{CD}_1)D_2$ holds. Otherwise, higher order convex dependence holds. Once the order of convex dependence is found, the decomposition form [2] two-attribute disutility function can be obtained. Single-attribute NCGDFs play a role of basic elements in the two-attribute group disutility function.

Step 4. By assessing the corner values of a group disutility function in two-attribute space, coefficients of linear terms in the two-attribute group disutility function are obtained [4]. As a result a two-attribute group disutility function is obtained.

In modeling multi-agent decision making with conflicting DMs, NCGDF plays the most important role as it can model various patterns of a DM's preference who is self-centered and selfish or flexible and cooperative, and so forth.

3. Consensus formation modeling for multi-agent decision making

Let DM1 and DM2 be

– DM1: representative of the regional inhabitants;

– DM2: representative of the enterpriser who is planning a new public project.

Suppose the disutility function d_1 for DM1 evaluates environmental impact from the public project and the disutility function d_2 for DM2 evaluates the cost to realize various countermeasures of the public project. These disutility functions are constructed by questioning the environmental specialists about each situation of DM1 and DM2.

We construct the NCGDFs by again questioning the environmental specialists about each situation of DM1 and DM2. Consequently, suppose we obtained three types of models as follows:

Model 1. Mutual utility independence holds. Both DM1 and DM2 do not think that group disutility is small unless their own disutility is also small. In this case both DM1 and DM2 are selfish and strongly insist upon their own opinion. This situation shows the initial phase of planning a new project, when the plan has just been presented to the regional inhabitants.

Model 2. Utility independence holds for DM1 and first order convex independence holds for DM2. The attitude of DM1 is almost the same as in Model 1, however, DM2 is becoming more flexible towards obtaining consensus of DM1. In this case DM1 does not have enough information on the project, however, DM2 has obtained various information. This situation corresponds to the second phase of the consensus formation process.

Model 3. Mutual first order convex independence holds. The attitude of both DM1 and DM2 is getting more flexible and cooperative. In this case both DMs have obtained sufficient information about planning the public project and the countermeasures for preventing environmental impacts from the project, and thus, show a mutual concession taking into account ethical consideration with each other. This situation corresponds to the final phase of the consensus formation process between DM1 and DM2.

Suppose the minimum value of group disutility is obtained for Model 3. This implies that the most impartial consensus formation is obtained under the situation of Model 3, which is based on convex dependence between two conflicting DMs.

As seen from the consensus formation model described above it may be used as a fundamental material for discussion when the regional inhabitants and the enterpriser of a public project regulate and adjust their opinion of each other.

4. Concluding remarks

By using a group disutility function for two conflicting agents, we could model the mutual concessions of the two conflicting agents taking into account ethical preference of each decision maker. We believe that the group disutility function proposed in this paper is the first mathematical model that can handle ethical preference of conflicting decision makers with each other. The key idea of this mathematical model is that the two-attribute group disutility function is a function of single-attribute normalized conditional group disutility function of each decision maker. Ethical preference of each decision maker is described in this NCGDF.

The consensus formation model described in this paper is expected to be used as a fundamental material for discussion when the enterpriser of a public project and the regional inhabitants regulate and adjust their opinion with other for realizing better social welfare.

References

[1] R. L. Keeney and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. 1993 (originally publised New York: Wiley, 1976).

[2] H. Tamura and Y. Nakamura, "Decompositions of multiattribute utility functions based on convex dependence", *Oper. Res.*, vol. 31, no. 3, pp. 488–506.

[3] A. Sen, *Collective Choice and Social Welfare*. Amsterdam: North-Holland, 1979.

[4] H. Tamura and T. Yoshikawa, Eds., *Large-Scale Systems Control and Decision Making*. New York: Marcel Dekker, 1990.

Hiroyuki Tamura
e-mail: tamura@sys.es.osaka-u.ac.jp
Department of Systems and Human Science
Graduate School of Engineering Science
Osaka University
1-3 Machikaneyama, Toyonaka
Osaka 560-8531, Japan